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NONLINEAR OSCILLATIONS OF A SHEET ELECTRON BEAM

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Abstract

The nonstationary model is considered allowed to describe the relativistic electron beam dynamics with nonuniform current density profile in collisionless approximation. The kinetic distribution function is used dependent on the particle motion integral, so the distribution function automatically satisfies to Vlasov equation. The equation for envelope oscillations is solved, the equilibrium and asymptotic solutions are found.

INTRODUCTION

For a lot of accelerator projects the investigation of nonlinear beam oscillations is an important task because of possible beam mismatching. Usually nonlinear beam dynamics is studied by means of the beam dynamics simulation, but analitical investigation of the dynamics by means of the simple mathematical models is more attractive because it allows to obtain the knowledge of the beam behaviour with most physical generality. First such a model was proposed by I. M. Kapchinsky and V. V. Vladimirsky (KV-model) in 1959 [1]. KV-model gives a full kinetic beam description due to the suggestion that the kinetic distribution function is a function of particle motion integral and hence automatically satisfies to Vlasov equation. Yarkovoy's model should be mentioned too which allows to describe nonstationary 2D-beams without axial symmetry. Another examples of the models one can find in [2-6]. All the models mentioned above describe the linear beam dynamics. The models taking into account the nonuniform charge density were proposed in [7-10]. In [9-10] only self-similar beam oscillations are studied, in contrast to [7, 8], where the particle distributions are not stationary.

To study nonlinear oscillations in a relativistic electron beam that would be interesting for numerous projects including ILC the analogous model [7,8] is applied to the case of a sheet continuos non-hollow beam with nonuniform charge density in the beam cross-section. The model doesn't require the particle distribution to be stationary and allows to investigate the beam envelope behaviour with time.

MODEL DESCRIPTION AND NUMERICAL CALCULATIONS

Let us consider a quasistationary relativistic intense electron beam. For the mathematical simplicity the sheet geometry of the beam is applied. Since the beam lifetime is significantly more than the time of transition processes in the beam one can describe the beam behaviour by means of a smooth function $R(z)$, where $R(z)$ – the beam tranverse size, z – longitudinal coordinate. In the case of the beam with uniform charge density KV- invariant looks as:

$$I = (R'x - Rx')^2 + \frac{\varepsilon_0 x^2}{R^2}, \quad (1)$$

where x' is derivative of x with respect to z, R' – derivative of R with respect to z, ε_0- beam rms emittance squared, x – transverse coordinate.

If we suppose that the charge density distribution $n(x,z)$ in the beam cross-section has parabolic character, which is a good approximation for the density distribution of the real non-hollow continuous beam:

$$n(x,z) = a_0(z) - a_2(z)x^2 . \quad (2)$$

We can obtain the equation for the particle transverse motion:

$$x'' = -\alpha_1(z) + \alpha_3(z)x^3 , \quad (3)$$

here $\alpha_1(z) = ka_0(z)$, $\alpha_3(z) = ka_2(z)/3$, $k = 4\pi e^2/mc^2$.

For equation (3) the integral I as analogue to KV-invariant (1) may be constructed with the help of the next relation:

$$x'(x,z,I) = \sum_{k=0}^{\infty} a_k(z,I)x^k \pm \left(\sum_{k=0}^{\infty} b_k(z,I)x^k\right)^{1/2} , \quad (4)$$

wherein we will neglect all summands with 5th power and higher.

Then let us introduce a kinetic distribution function as

$$f(I) = 2n_0\sigma(1 - I) ,$$

here n_0 – the time-independent normalization constant, σ – Heaviside function. So one can obtain for the beam charge density:

$$n = (n_0/u)(1 - \varepsilon_0^2 x^2 / 2u^2)\sigma(R - |x|) , \quad (5)$$

where

$$R = u\sqrt{2/\varepsilon_0}\left(1 + \sqrt{1 + (\varepsilon_0' u^2)' u^2 / 3\varepsilon_0^2}\right)^{-1/2} , \quad (6)$$

and function u is the solution of equation

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$$u'' = -\alpha_1(z)u + \varepsilon_0(z)/u^3 \ . \qquad (7)$$

The whole current conservation should be taken into account.

So for dimensionless beam radius and effective emittance the next equation system may be obtained:

$$\begin{aligned}(\beta'\alpha^2)' &= 12(1-\beta)/\alpha^2 \\ \alpha'' + 1 &= \beta/\alpha^3\end{aligned}\ , \qquad (8)$$

here α and β are dimensionless radius and rms emittance respectively; $\alpha = u(l_0/l_1)^{2/3}$, $\beta = \varepsilon_0 l_0{}^2$, $l_1 = c/\omega_p$, $l_0 = J/2evn_0L$, J is the whole beam current, L- the width of the beam, ω_p is the plasma frequency, corresponding to the density value n_0, v is the beam velocity.

In (8) time-dependence of rms emittance was obtained in self-consistent manner, because function *f(I)* automatically satisfies to Vlasov equation, and relation (5) for the density, i.e. for zero moment of the distribution function, has a parabolic dependence from *x*.

From the system (8) one can find the stationary equilibrium state for the beam, that coresponds to the beam radius

$$R = l_0(c/\omega_p l_0)^{2/3} \ ,$$

and effective emittance

$$\varepsilon_0 = \eta / l_0^2 \ ,$$

here η is the normalization constant.

The system (8) corresponds to the beam envelope equation of 4th order unlike the envelope equations in [9,10]. It is solved numerically by means of Runge-Kutta-Feldberg method of 4th order. The results are presented at Fig. 1-3.

Figure 1 and Figure 2 indicate the envelope oscillation build-up possibility.

From Fig. 1-2, it is evident that in the case of a strong deviation of the beam initial parameters from equilibrium ones the essential growth of rms emittance is observed. The reason of the phenomenon is the filamentation appearence.

Figure 3 indicates that the range of the beam parameters exists which corresponds to asymptotic restriction of the nonlinear oscillation amplitude growth. At the distance of few plasma wavelenghts the growth of rms emittance is stopped.

One should note again that the system (8) and its solutions are obtained in a self-consistent manner.

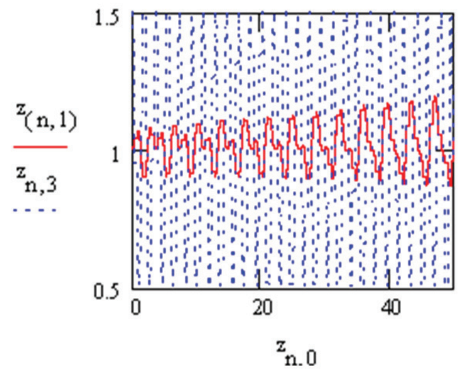

Figure.1: dimensionless rms radius $z_{n,1}$ vs dimensionless longitudinal coordinate $z_{n,0}$.

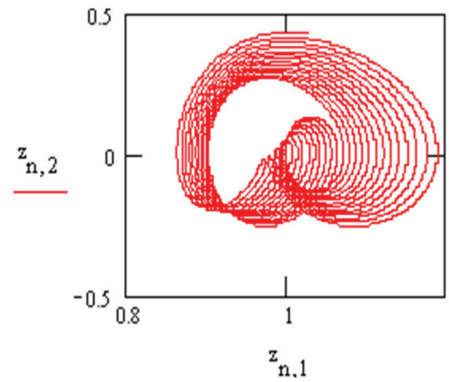

Figure 2: phase curves in the case of strong nonlinearity

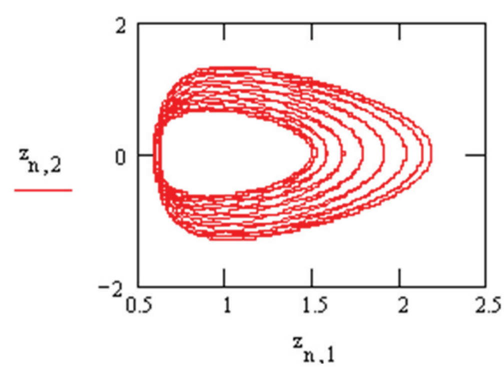

Figure 3: phase curves of the beam in asymptotic case.

CONCLUSIONS

Nonlinear oscillations of a sheet relativistic electron beam are studied in collisionless approximation. Transverse current nonuniformity leads to essentially nonlinear particle transverse oscillations, but the range of the beam parametes exists corresponding to the asymptotic limitation of the effective emittance growth that allows to simplify the beam-channel matching problem. Depending on nonlinearity power the growth of effective emittance can be observed at a time corresponding to about a quarter of the maximum plasma wavelength.

The exact beam parameters exist corresponding to the case of the beam equilibrium when the effective emittance and the beam transverse size does not grow.

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The results obtained are valid under the condition $l_0 > c/\omega_p$, i.e. when minimum system linear size is more than maximum beam plasma wavelength.

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