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COMPLETE 6-DIMENSIONAL MUON COOLING CHANNEL FOR A MUON COLLIDER*

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Abstract

We describe a complete 6D rectilinear cooling scheme for use in a Muon Collider. This scheme uses separate 6D cooling channels for the two signs of particle charge. In each, a channel first reduces the emittance of a train of 21 muon bunches until it becomes possible to merge them into a single bunch, one of each sign. The single bunches are then sent through a second rectilinear channel for further cooling towards the requirements of a Muon Collider. We adopt this approach for a new cooling lattice design for the Muon Collider, and for the first time present a end-to-end simulation. We review key parameters such as the required focusing fields, absorber lengths, cavity frequencies and rf gradients.

INTRODUCTION

A high-luminosity muon collider requires a reduction of the six-dimensional emittance of the captured muon beam by a factor of 10^6. Ionization cooling is the only scheme that can reduce the emittance of the muon beam in times short compared to its life time.

Cooling in the longitudinal direction, requires emittance exchange. In this process a wedge absorber is placed in a region with dispersion so that higher momentum particles see more energy loss that the lower ones. Cooling lattices are typically formed by sequences of identical lattice cells wherein the solenoids are slightly tilted to generate the required dispersion. Ring shaped coolers [1] have shown to provide an impressive two orders of magnitude reduction of the 6D phase-space volume with a transmission above 50%. This design evolved without loss in performance to a helical channel [2], in order to avoid serious problems with injection and extraction of large emittance beams as well as to avoid issues related to absorber overheating. Later it was found that the cooling efficiency can be improved if the channel becomes tapered [3] so that, parameters such as cell length, absorber length, rf frequency, rf gradient and focusing strength progressively change from stage to stage. This ensures that the beam emittance is always large compared to the equilibrium emittance.

There are several challenges in designing ionization cooling channels based on vacuum rf technology. First, in a helical channel the radius of curvature becomes less than 1.5 m at the last stages [3] making the design increasingly difficult. An added complication is that stray fields from one pitch can influence those before and after causing the beam to be heavily distorted. For this reason it seems prudent, to begin investigating a rectilinear six-dimensional channel [4] that in view of its simple geometry may offer substantial advantages over a Guggenheim helix.

In this study, with the aid of numerical simulations we show that a rectilinear channel not only maintains a transmission comparable to a helical channel, but also it achieves a notable 6D emittance decrease by six orders of magnitude as exactly required by a Muon Collider. We also review important lattice parameters, such as the required focusing fields, absorber length, cavity frequency, and rf gradient.

RECTILINEAR COOLING CHANNEL

The overall layout of the proposed rectilinear channel is shown in Fig. 1. The lattice consists of a sequence of identical cells with two or four solenoids in each cell with opposite polarity. The coils (yellow) on either side of the absorber are closer together in order to minimize the beta function at that location. A series of rf cavities (dark red) are used to restore the momentum along the longitudinal axis. The dispersion necessary for emittance exchange is provided from the bend field generated by tilting the axes of the solenoids.

As presented in Fig. 1, essentially the same cells from a ring or a helix, including their coil tilts and resulting upward dipole fields, are laid out in straight (rectilinear) geometry. The solenoid focusing is so strong compared with dipole deflections that the closed orbits are merely displaced latterly, but continue down the straight lattice. The performance is essentially the same as with rings or helix, but with greatly simplified engineering

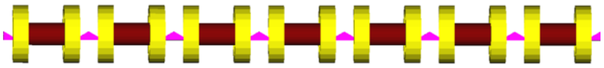

Figure 1: Top view of a 6D rectilinear cooling channel.

Cooling a Muon Bunch Train

This section cools the muon bunches after they are aligned in energy by the front-end phase-rotator [5]. We consider a 4 stage tapered channel, where each stage consists of a number of identical cells. The parameters are summarized in Table 1.

At the first stage of the tapered channel the focusing will be relatively weak to avoid excessive angular divergence that can arise from the large transverse

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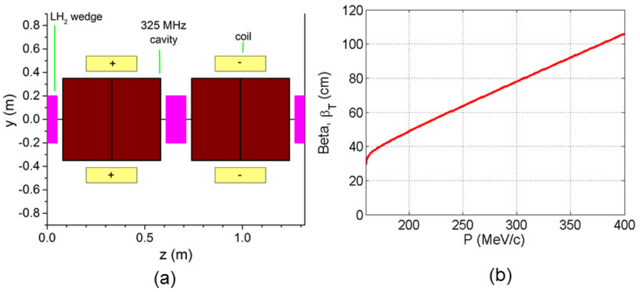
emittance of the initial muon beam. This stage is then terminated and we couple into the next stage that has a lower beta. This is achieved by simultaneously scaling down the cell dimensions and raising the strength of the on-axis solenoidal field. As a result this will produce a piecewise constant multi-stage channel where each stage will be a fixed-parameter straight channel consisting of a series of identical cells similar to the one shown in Fig. 1. All four stages consist of equally space alternating solenoids (known as FOFO lattices) so that they can operate above the π resonance and thus have the highest possible momentum acceptance.



(a) (b)

Figure 2: Cooling before the bunch merge: (a) One cell of stage 2, and (b) momentum acceptance. Note that all particles with momentum > 160 MeV/c are accepted.

Figure 2(a) shows the side view of one cell at an early stage of the channel (Stage 2). This stage consists of a sequence of 130 identical 1.5 m cells, each containing four 0.24 m-long 325 MHz pillbox cavities, and a wedge-shaped liquid hydrogen (LH) absorber, with 5.9 cm central thickness and 44 deg. opening angle, to assure energy loss. Moreover, each cell contains two two-pair solenoid coils with the two pair having opposite polarity, yielding an approximate sinusoidal variation of the magnetic field in the channel with a peak on axis value of 3.5 T and providing transverse focusing with a peak beta value of ~ 54.8 cm. The peak accelerating gradient of the rf cavities is 22.0 MV/m, while each operates at a synchronous phase 22 deg. off the 0-crossing point. Figure 2(b) shows the transverse beta function versus momentum for the aforementioned stage. The last two stages use a lattice essentially the same with the first two, but with twice the field strength, half the geometric dimensions, and 650 MHz rf instead of 325 MHz rf cavities. Both irises were covered by a thin stepped Be window in order to reduce surface gradients and improve shunt impedances. The minimum Be thickness assumed was 125 μm. The absorber material in all stages was LH and it was enclosed in Al safety windows ranging in thickness from 200 μm (Stage 1) to 50 μm (Stage 4).

The performance of the cooling channel was simulated using the ICOOL code [6]. The code includes all relevant physical processes (e.g. energy loss, straggling, multiple scattering) and includes muon decay. The input distribution in the simulation is the output from front-end phase-rotator and corresponds to a beam with a normalized transverse emittance of 48.6 mm and a normalized longitudinal emittance of 17.0 mm, while the average longitudinal momentum is 265 MeV/c. We tracked 65,000 macro-particles and included decay of muons. The evolution of the transverse phase-space at different locations is shown in Figure 3. It is worth noting that after a distance of 480 m the 6D emittance has fallen by a factor of 1000 with a transmission of 50%. In addition, at the end of the channel the transverse emittance decreased by a factor of ~11, while the longitudinal emittance shrank by a factor of ~18.

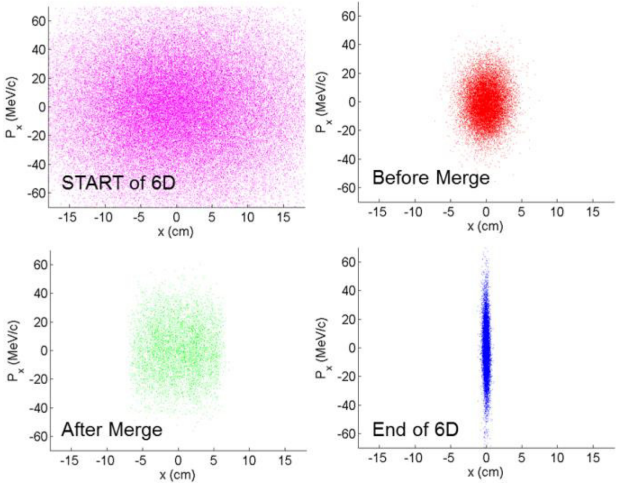


Figure 3: Snapshots of the transverse phase-space at various positions along our proposed 6D rectilinear cooling channel.

Cooling Single Muon Bunches

After the bunch recombination, the longitudinal and transverse emittances of the muon beam have increased. It can thus be taken through the same cooling system described earlier. A key difference is that for the regime before the bunch merging system only a modest transverse cooling to ~ 1.5 mm is required, while for the post-bunch-merging regime a ~ 0.3 mm rms normalized emittance is desired. We consider a 8 stage tapered channel, where each stage consists of a number of identical cells. The main parameters of our under consideration channel are listed in Table 2.

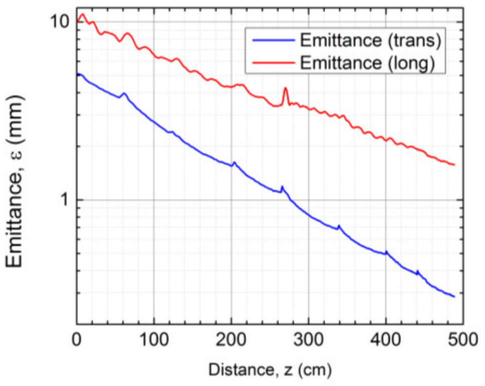


Figure 4: Cooling performance of our 6D cooling channel. The final normalized rms emittances are 0.28 mm (trans.) and 1.57 mm (long.). The transmission is near 40% with muon decays included.

Table 1: Main lattice parameters of a tapered rectilinear channel before the bunch merge.

Stage	**Cell length [m]**	**RF freq. [MHz]**	**RF grad. [MV/m]**	**RF #**	**Min. beta [cm]**	**Absorber type**	**Coil tilt [deg.]**	**Wedge angle [deg.]**
1	2.00	325	22.0	6	81.9	LH	3.13	40
2	1.32	325	22.0	4	54.8	LH	1.80	44
3	1.00	650	28.0	5	38.3	LH	1.60	100
4	0.80	650	30.0	5	30.3	LH	0.70	110

Table 2: Main lattice parameters of a tapered rectilinear channel after the bunch merge.

Stage	**Cell length [m]**	**RF freq. [MHz]**	**RF grad. [MV/m]**	**RF #**	**Min. beta [cm]**	**Absorber type**	**Coil tilt [deg.]**	**Wedge angle [deg.]**
1	2.750	325	19.0	6	42.0	LH	0.90	120
2	2.000	325	19.5	5	27.4	LH	1.30	117
3	1.500	325	21.0	4	20.2	LH	1.10	113
4	1.270	325	22.5	3	14.0	LH	1.10	124
5	0.995	650	27.0	4	8.1	LIH	0.66	61
6	0.806	650	28.5	4	5.9	LIH	0.70	90
7	0.806	650	26.0	4	4.2	LIH	0.80	90
8	0.806	650	26.0	4	3.0	LIH	0.80	90

In order to achieve the desired low emittances the beta at the last cooling stages becomes very small (≈ 3.0 cm). Since good cooling requires that the absorber half-length be comparable with the minimum value of the beta function, this becomes impractical with liquid hydrogen and we switched to using Lithium Hydride (LiH) as the absorber material for Stages 5-8.

The transverse and longitudinal emittances and the transmission are shown as function of distance along the channel in Fig. 4. After a distance of 480 m (8 Stages) the 6D emittance has fallen by a factor of 1000 with a transmission of 40%. The final transverse emittance is 0.28 mm, while the final longitudinal emittance is 1.57 mm. The final emittances match the desired baseline values for the Muon Collider. A detailed theoretical study that predicts and evaluates the effectiveness of our cooling channel can be found in Ref. 7.

A technical challenge may arise as the operating current on the conductor should not exceed the critical current corresponding to the peak field in the coil. Note from Table 2 that there is an increase of the magnet operating current with stage number. This is required to ensure enough cooling. While in the last stage, the maximum field on the coil is 15 T, our findings indicate that even with inclusion of reasonable safety factors, the needed fields are consistent with the critical limits of existing conductor technology. However, the last stage is barely within the limits of Nb_3Sn and therefore it is critically important to the development of a Muon Collider that a well thought-out test program to be continued. A more detailed magnet feasibility study for the last cooling stage of our 6D channel can be found elsewhere [8].

CONCLUSIONS

Ionization cooling is the only feasible option for cooling the beam within the short muon lifetime (τ_0=2.2 μs) for a Muon Collider. Using a tapered scheme in which parameters of the structure, such as the cell length, focusing, and radius of curvature, change progressively with distance, so that the beam emittance is always larger than the equilibrium emittance we described a complete scheme for 6D ionization cooling channels. We have identified the key needed parameters such as absorber length, rf gradient and focusing field. We showed that relatively modest magnetic fields ($B\sim15$ T, peak on axis) and a small number of different frequencies, namely 325 and 650 MHz, are enough to cool towards the baseline cooling requirements of a Muon Collider.

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