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DRIVE BEAM BREAK-UP CONTROL AND PRACTICAL GRADIENT LIMITATION IN COLLINEAR DIELECTRIC WAKEFIELD ACCELERATORS*

C. Li[1,3#], W. Gai[1], C. Jing[1,4], J.G. Power[1], C.X. Tang[3], and A. Zholents[2]
[1] High Energy Physics Division, Argonne National Laboratory, Argonne, IL 60439, USA
[2] Advanced Photon Source, Argonne National Laboratory, Argonne, IL 60439, USA
[3] Department of Engineering Physics, Tsinghua University, Beijing, 100084, China
[4] Euclid Techlabs LLC, Solon, OH 44139, USA

Abstract

Dielectric wakefield accelerator (DWA) concept has gained significant attention for the need of the future large scale facilities. For a practical machine, one needs to overcome a major challenge for the DWA that is the efficient energy extraction and stable propagation at the same time for the drive beam. Typically, a slightly off axis beam become unstable in the dielectric channel due to transverse wakefield excitation, that could be controlled if a strong external alternating magnetic focusing channel applied at the same time. However, there is limitation on the practical magnetic field in the focusing channel (typically < 1 Tesla), thus imposing operating point for the DWA. In this article, we explore the operating point of the DWA for various structure frequencies and drive beam charge, particularly on the gradient and total acceleration distance, and provide guidance on the DWA design.

INTRODUCTION

Future large-scale accelerator facilities will significantly benefit from accelerating techniques with high acceleration gradients. In the collinear Dielectric Wakefield Acceleration (DWA) method considered here, a leading, high-charge, Q, drive bunch loses energy while traversing a dielectric-lined waveguide and generates wakefields. The longitudinal wakefield, W_z, is used to accelerate a low-charge witness bunch which trails collinearly through the same structure. While high accelerating gradients in excess of 1GV/m have been demonstrated [1] in DWA structures, sustained acceleration, as required for high final witness bunch energy, has not. The final energy is the product of the gradient, QW_z, and the distance, L, that the drive bunch and the witness bunch have propagated in the structure. The overall wall-plug efficiency of the accelerator is proportional to the fraction of energy extracted from the drive bunch, $\eta=1-KE_f/KE_0$, where KE_0 is initial kinetic energy, KE_f a final energy and η is proportional to the propagation length of drive bunch. For example, consider a KE_0=1GeV drive bunch that generates a gradient of 1GV/m and decelerates at 0.5GeV/m. If it only propagates L=0.1m then KE_f =0.95GeV thus η=5%, which is unacceptable for a large-scale accelerator facility.

As seen above, to obtain both high final witness bunch energy and high efficiency requires a long propagation distance, L. However, large longitudinal wakefields are accompanied by large transverse wakefields and these later wakes drive the single bunch Beam BreakUp (SBBU) [2] instability. This causes the loss of the drive bunch in the DWA structure (i.e. short L) before its energy can be fully extracted. The longitudinal wakefield scales as $E_z \propto Q/a^2$ where Q is the drive bunch charge and a is radius of the structure channel. This scaling implies that high gradient can be achieved by either increasing Q or decreasing a. The transverse wakefield scales as $F_\perp \propto Q/a^3$, ($\perp = x,y$) thus decreasing a to raise the gradient will increase the SBBU even more quickly. Since large longitudinal wakefields are desired, efficient control of SBBU is essential for a practical DWA-based facility. SBBU control is complicated by the fact that the drive bunch energy spread is approaching 100% towards the end of the structure.

In this paper, we derive a scaling law for the high gradient limit of a DWA due to SBBU. Furthermore, we show that it is not possible to operate at high acceleration gradients, close to the material breakdown threshold [1], unless the accelerator is operated at low efficiency.

PROPAGATION OF THE DRIVE BEAM IN THE DIELECTRIC STRUCTURES

The wakefields generated by a single ultrarelativistic particle in a DWA structure has already been solved [3,4]. In our previous work [5], simulations showed that SBBU effects could be controlled by means of a tapered FODO lattice around the DWA structure of inner and outer radius of a and b [Figure 1]. In this paper, we extend this work to determine the maximum acceleration gradient achievable as a function of channel radius, a, when η is considered.

We begin by considering a 2-particle model of SBBU in the DWA structure imbedded into FODO lattice before presenting the multiparticle simulations results. Assume that both the *head*, “1”, and *tail*, “2”, particles have the same velocity $v_1=v_2\approx c$ and initial transverse position $x_1(0)=x_2(0)=x_0$ and angel $x'_1(0)=x'_2(0)=0$. The longitudinal wakefields cause both particles to lose energy linearly along z, $\gamma_1(z)=\gamma_0(1-\alpha_1 z)$, $\gamma_2(z)=\gamma_0(1-\alpha_2 z)$, where α_1 and α_2 are the rates at which the energy decreases. The quadrupole field gradient B' is decreased in proportion to the *tail* particle's energy for better SBBU control so that

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#st.leechen@gmail.com

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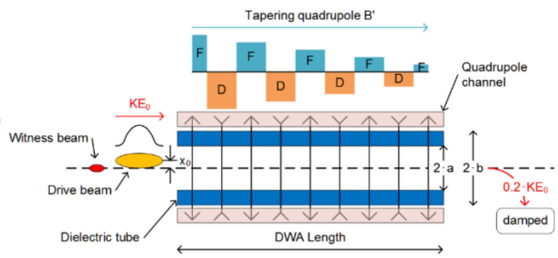

Figure 1: Schematic of a DWA linac surrounded by a FODO lattice. The quadrupole field gradient B' is tapered to match the energy loss of the drive bunch along the linac. The final energy of the drive bunch at the end of the structure is 20% of its initial energy.

$\eta/L=\alpha_2$, L is the total length of the DWA linac. Finally, we replace discreet focusing with continuous focusing to obtain the equations of motion for the *head* and *tail* particles respectively,

$$\begin{aligned} x_1'' - \frac{\alpha_1}{1-\alpha_1 z} x_1' + k_\beta^2 \left(\frac{1-\alpha_2 z}{1-\alpha_1 z} \right) x_1 = 0 \\ x_2'' - \frac{\alpha_2}{1-\alpha_2 z} x_2' + k_\beta^2 x_2 = \frac{e(Q/2)W_x}{\gamma_0 mc^2} \frac{x_1}{1-\alpha_2 z} \end{aligned} \quad (1)$$

where $\gamma_0 mc^2$ is the initial particle energy, $k_\beta = \sqrt{eB_0'/4\gamma_0 mc}$ is the initial betatron wave number, e is the elementary charge, Q is the bunch charge, and W_x is the transverse wakefield in units *V/C/m/m*. The approximate solution for the trajectories of the head and tail particles can be written as,

$$x_1(z) \approx \frac{x_0}{1-\alpha_1 z} \cos k_{\beta *} z$$

$$x_2(z) \approx \frac{x_0}{\sqrt{1-\alpha_2 z}} \cos k_\beta z + \frac{eQW_x}{2\gamma_0 mc^2} \frac{x_0}{k_\beta^2 - k_{\beta *}^2} \frac{1}{\sqrt{1-\alpha_2 z}} \quad (2)$$

$$\cdot \left(\frac{1}{1-\alpha_1 z} + \frac{1}{\sqrt{1-\alpha_2 z}} \right) \sin\left(\frac{k_\beta + k_{\beta *}}{2} z \right) \sin\left(\frac{k_\beta - k_{\beta *}}{2} z \right)$$

with $k_{\beta *} = k_\beta \left[1 - 3(\alpha_2 - \alpha_1) z / 4\pi (1-\alpha_1 z) \right]$. Note that when both particles have the same rate of energy decrease ($\alpha_1=\alpha_2$), then Eqn. (2) will approach the results presented in Ref. [6].

Inspection of the analytic trajectory of *tail* (x_2) reveals the sources of the amplitude growth. The first term in Eqn. (2) for x_2 corresponds to the free betatron oscillation with adiabatic growth, which does not significantly contribute to the amplitude growth. The second term for x_2 is due to the wakefield force from the *head* (x_1) particle and reveals beating with a maximum amplitude growth when $z=\pi/\Delta k_\beta$, where $\Delta k_\beta = k_\beta - k_{\beta *}$. Preliminary work has shown that other distributions, with higher transformer ratios, may propagate longer distances when given an initial energy chirp. (The specific shape plays an important role in the BBU control as well). This will be presented in future work.

Controlling SBBU in a DWA is now seen as a matter of controlling the amplitude growth of the tail particle due to the wakefield force (second term in Eqn.(2)). Making the approximation $\sin(k_{\beta *}-k_\beta)z/2 \approx (k_{\beta *}-k_\beta)z/2$, we can express the amplitude of the tail, $x_2(z)$, as

$$A(z) = \frac{eQW_x x_0}{2\gamma_0 mc^2 k_\beta} \frac{z}{2} \frac{1}{\sqrt{1-\alpha_2 z}} \frac{1}{2 - \frac{3(\alpha_2 - \alpha_1)z}{4\pi(1-\alpha_1 z)}} \left(\frac{1}{\sqrt{1-\alpha_2 z}} + \frac{1}{1-\alpha_1 z} \right) \quad (3)$$

We now present a case study of a high efficiency DWA structure with η = 80%, thus $\alpha_2 L=\eta=0.8$. The head particle *(Q/2)* sees half of the maximum longitudinal wakefield generated by itself, thus $\alpha_1 L=0.5\alpha_2 L=0.4$. And the total propagation length is $L=\eta\gamma_0 mc^2/eE_z^{dec}$, the deceleration gradient is $E_z^{dec}=(Q/2)W_z$. The amplitude of the tail particle *(Q/2)* at $z=L$ is,

$$A(L) = 1.896 W_x x_0 / W_z k_\beta \quad (4)$$

Thus, the amplitude at the end of the DWA linac is a function of the following 4 parameters: transverse wakefields W_x, longitudinal wakefields W_z, initial offset x_0, and betatron oscillation wave number.

Next, we parameterize Eqn. (4) in terms of only the bunch charge Q and the channel radius, a, in order to develop scaling laws for the maximum acceleration gradient.

(1) Wakefields. The ratio of the outer and inner radii of the DWA strucutre b/a was kept so as to keep the ratio of fundamental to higher-order-modes constant. Thus,

$$\begin{aligned} W_x &= W_{x1} \cdot a_1^3 / a^3 \\ W_z &= W_{z1} \cdot a_1^2 / a^2 \end{aligned} \quad (5)$$

(2) Free betatron oscillation in the FODO channel. First we have the relation between the quadrupole focal length and the FODO cell length with a betatron phase per cell $\varphi = 90°$ as $L_0/2f = \sin(\varphi/2) = \sqrt{2}/2$ [7], where $2L_0$ is the length of a FODO cell, $f = \gamma_0 mc/eB_0' L_q$ is the focal length of a quadrupole and L_q is quadrupole length. Because stronger focusing in the FODO lattice allows for improved SBBU control, we take the upper limit that $L_q= L_0$ and $B'= B_{sat}/a$, where B_{sat} is saturation magnetization at pole tips and is equal to 1.0 Tesla in this paper (A new type of focusing quadrupole whose B' may reach 3000 T/m is under development [8]).

$$\begin{aligned} L_q &= L_0 = \sqrt{\sqrt{2}\gamma_0 mca/eB_{sat}} \\ k_\beta &= \sqrt{eB_{sat}/4\gamma_0 mca} \end{aligned} \quad (6)$$

(3) Initial x_0 relative to axis. We have assumed that the x_0 is proportional to a bunch RMS size σ_x i.e ~ $0.5\sigma_x$. We assume the dependence of emittance on bunch charge is approximately linear, $\varepsilon_{xn}=C_0 Q$, with a typical value of the coefficient $C_0 = 1\mu m/nC = 1000 m/C$.

$$\left. \begin{aligned} x_0 &= 0.5\sqrt{\varepsilon_{xn}\beta_x/\gamma_0} \\ \varepsilon_{xn} &= C_0 Q \\ \beta_x &= \lambda_\beta/2\pi = 4L_0/\pi \end{aligned} \right\} \Rightarrow x_0 = \left(\frac{\sqrt{2} mca C_0^2 Q^2}{\pi^2 \gamma_0 e B_{sat}} \right)^{1/4} \quad (7)$$

Applying the above equations to Eqn. (4) we have an expression that relates the amplitude of the *tail* (x_2) at $z=L$ to the charge and the channel radius,

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$$A(Q,a) = \frac{4.135}{\sqrt{\pi}} \frac{a_1 W_{x1}}{W_{z1}} \left(\frac{\gamma_0 m^3 c^3 C_0^2}{e^3 B_{sat}^3} \right)^{1/4} \frac{Q^{1/2}}{a^{1/4}} \quad (8)$$

Propagation requires that $A(Q,a) \le a$, i.e. that the bunch does not hit the wall. Finally, one can determine an analytic expression for the maximum charge that can be propagated through the DWA structure,

$$Q_{\max}(a) = \frac{\pi}{17.10} \left(\frac{W_{z1}}{a_1 W_{x1}} \right)^2 \left(\frac{e^3 B_{sat}^3}{\gamma_0 m^3 c^3 C_0^2} \right)^{1/2} a^{5/2} \quad (9)$$

All the parameters in the above equation are in SI units.

Before we can numerically determine how Q_{max} scales with a, several steps remain. First, we numerically calculate the wakefields, W_z and W_x, [3,4] for a particular value of a; we call this the reference case. For the reference case, we choose a DWA structure with a_1=1*mm*, b_1=1.06*mm* and ε=3.75 which uniquely determines the fundamental frequency, f_1=300*GHz*. Next, we set the RMS length of a Gaussian bunch according to σ_z/λ=0.2 for all cases. For this reference case we have $\lambda_1 = c/f_1$=1*mm*. This value of σ_z/λ achieves high acceleration gradient and high transformer ratio [9]. Numerical calculation gives a maximum transverse wake inside the Gaussian drive bunch as W_{x1}=4.5*MV/m/mm/nC* and a maximum deceleration wake W_{z1}=8.5*MV/m/nC*. These numerical values are used to scale the wakefields according to Eqn. (5). The initial bunch energy is taken as a constant, 150 *MeV*, so KE_0 is not scaled with a. Given the above values, one can use Eqn. (9) to find the maximum charge is 17 *nC*.

Multiparticle simulations were done with a user-written particle pushing code to simulate beam dynamics including SBBU in the DWA structure surrounded by a FODO lattice. The basic mathematical model is the same as ref. [5]. It applies the force due to wakefield and quadrupoles to the macroparticles and tracks them in the two-dimensional coordinates (x, z). The wakefields excited by a single particle in the DWA was decomposed into discrete but infinite numbers of waveguide modes (*mn* in Ref. [3,4]). Unless specifically mentioned, the wakefield calculations used only the first three modes of both the monopole and dipole wakes (due to the finite length of the bunch) but convergence studies were done in all cases. Space charge effects are much weaker than wakefields and were excluded in simulations. Our simulations were checked to be in good agreement with both Elegant [10] and BBU3000 [11].

The scaling of charge with radius (Q_{max} vs. a) according to multiparticle simulations was determined for several radii. Simulation results are in good agreement with Eqn. (9) from the 2-particle model (Figure 2).

The scaling of the maximum acceleration gradient, E_z^{max}, with structure radius, a, for both the 2-particle model and the numerical simulations can now be simply obtained from their above charge scaling results. Given $E_z^{max} = Q_{max} W_z^{acc}$, then for the reference case (a_1=1*mm*, $Q_{\max}$=17*nC* and $W_z^{acc} = 2W_{z1}$=17*MV/m/nC*) we find E_z^{max}=290*MV/m*. This result can be used with $E_z \sim Q/a^2$ to obtain the 2-particle model prediction of the scaling of maximum gradient with radius,

$$E_z^{\max}\left[MV/m\right] \approx 290 \times \sqrt{a[\text{mm}]} \quad (10)$$

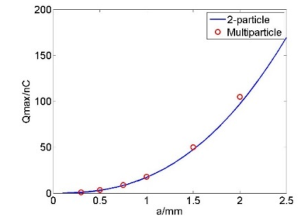

Figure 2. Maximum bunch charge as a function of the radius in the DWA linac for η = 80%. The blue line is the 2-particle model result and the red circles are the multiparticle beam dynamics simulation results.

Comparison of the scaling of E_z^{max} with radius shows good agreement between the simulation results and the 2-particle model (Figure 3). For reference, we plot the gradient scaling with a when SBBU is ignored for various fixed charges using, $E_z \sim Q/a^2$, (dashed colored lines).

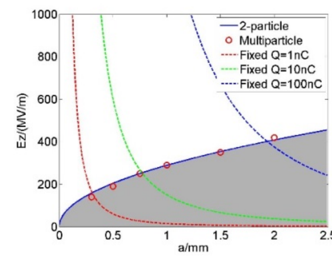

Figure 3. Maximum acceleration gradient as a function of the radius in the DWA linac for η = 80%. The blue line is the maximum gradient and the grey area underneath is the region of permissible gradients.

The above results (Figure 3 and Eqn.(10)) are the key results of this paper and can be used to guide the accelerator designer in the choice of the DWA structure. If high charge is available (e.g. Ref. [12] has drive bunch of *Q=100nC*) then a maximum gradient of 400 *MV/m* with structure radius a = *2mm* is obtainable. At the other end of the spectrum, if the drive bunch charge available is only 1.0 *nC*, as is typical for an SRF linac, then the maximum gradient is 150 *MV/m* for *a=0.3mm*.

The maximum acceleration gradient, E_z^{max}, can be increased if one is willing to sacrifice the efficiency (by lowering the requirement for η) and the total energy gain of the accelerator.

CONCLUSION

In summary, collinear wakefield acceleration in the dielectric waveguide has the potential to enable future accelerator science but the SBBU instability imposes a significant limit on the highest acceleration gradient and must be carefully controlled with a FODO channel. Since the achievable focusing is limited by the attainable magnetic field at the tip of the magnetic pole, our study shows that the maximum accelerating gradient increases with structure radius, $\propto \sqrt{a}$. This trend is the opposite of what would be expected by the simple scaling, $\propto Q/a^2$.

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