ISBN: 978-3-95450-132-8 doi:10.18429/JACoW-IPAC2014-TUPRI009

STUDY OF RESONANCE CROSSING IN NON-SCALING FFAGS USING THE S-POD LINEAR PAUL TRAP

D. J. Kelliher*, S. Machida, C.R. Prior, S.L. Sheehy,
STFC Rutherford Appleton Laboratory, ASTeC
K. Fukushima, K. Ito, K. Moriya, H. Okamoto, T. Okano,
Graduate School of Advanced Sciences of Matter, Hiroshima University, Higashi-Hiroshima, Japan

Abstract

Experiments on EMMA have shown that with rapid acceleration a linear non-scaling FFAG can accelerate through several integer tunes without detrimental effects on the beam [1]. Proton and ion applications such as hadron therapy will necessarily have a slower acceleration rate, so their feasibility depends on how harmful resonance crossing is in this regime. A simple and useful tool to answer such fundamental questions is the Simulator of Particle Orbit Dynamics (S-POD) linear Paul trap (LPT) at Hiroshima University, which can be set up to simulate the dynamics of a beam in an FFAG. We report here results of experiments to explore different resonance crossing speeds, quantify beam loss and study nonlinear effects. We also discuss the implications of these experimental results in terms of limits on acceptable acceleration rates and alignment errors.

INTRODUCTION

In a linear non-scaling FFAG the tune varies, typically by several integers, over the momentum range. This is not a problem when acceleration is rapid, as demonstrated in EMMA [1]. Indeed the large dynamic aperture allowed by the linear magnets, combined with the use of the serpentine channel for acceleration, make this an ideal device for muons. However, when acceleration is slower (for instance in the case of protons), the crossing of resonances can lead to unacceptably high orbit distortion and beam loss. Consequently, it is of interest to study the effects of much slower crossing rates than can be achieved with the current EMMA setup. Here we use a LPT to simulate EMMA and study resonance crossing.

In a LPT, a non-neutral plasma is confined radially by applying a sinusoidal voltage to four electrodes to create a rf quadrupole field and axially by applying a static voltage to two end plates to create a potential well. The S-POD device is about 20cm in length with a 1 cm aperture and is operated at 1 MHz. Argon-40 is usually chosen for the plasma. For this study, a relatively low number of ions are confined to ensure space-charge effects are negligible. After performing an experiment, the remaining number of ions is measured using a micro-channel plate (MCP) positioned at one end of the trap.

The Hamiltonian of an ideal LPT in the low intensity limit is given by

$$H = \frac{p_x^2 + p_y^2}{2} + \frac{1}{2}\kappa_{rf}(\tau)\left(x^2 - y^2\right) \quad (1)$$

where $\kappa_{rf}(\tau)$, the focusing waveform applied to the plasma, is proportional to the rf voltage applied to the electrodes and the independent variable $\tau = ct$. The above Hamiltonian is identical in form to that describing the transverse dynamics of a beam in a focusing channel.

The linear non-scaling FFAG EMMA consists of 42 cells each containing a quadrupole doublet. Each quadrupole is offset horizontally to provide a dipole component to bend the beam. The tune varies by about 7 integers over the momentum range in both transverse planes.

S-POD can be set up to simulate EMMA simply by choosing the appropriate focusing waveform $\kappa_{rf}(\tau)$. Each period of the sinusoidal focusing waveform shown in Fig. 1 represents a cell in EMMA. The fact that the sinusoid approximates a FODO rather than a doublet structure is not important for this study. The rf voltage is chosen to set the tune of the device over a large range as in EMMA.

In one type of experiment, the tune can be kept constant for a certain number of focusing periods, the plasma extracted and the number of ions measured. By repeating for a range of tunes, the stopband distribution can be measured. Alternatively by ramping the rf voltage the tune can be monotonically increased or decreased and the number of ions measured as before. This allows resonance crossing experiments to be conducted.

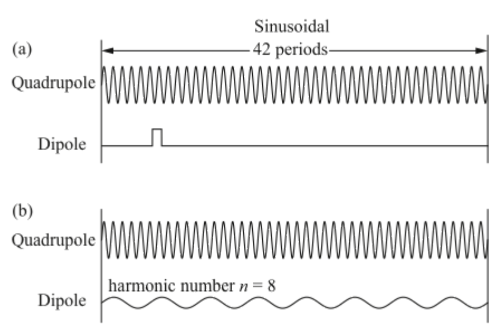

Figure 1: Illustration of the rf quadrupole waveform that simulates a single turn in EMMA. A dipole perturbation in two forms is also shown; namely, (a) a piecewise constant voltage emulating the local dipole field error and (b) a sinusoidally varying voltage corresponding to a single Fourier harmonic of the pulse voltage in (a).

* David.Kelliher@stfc.ac.uk

ISBN: 978-3-95450-132-8 doi:10.18429/JACoW-IPAC2014-TUPRI009

STOPBAND MEASUREMENT

The stopband distribution was previously measured as described above for the case of an ideal lattice [2]. Here we consider the addition of a dipole term to excite stopbands at integer resonances. A dipole term is added in S-POD by applying a second waveform to two of the four electrodes superimposed on the focusing waveform. This adds a driving term to the equation of motion that follows from Eqn. 1

$$\frac{d^2x}{d\tau^2} + \kappa_{rf}(\tau)\,x = -\frac{q}{mc^2 r_0} V_D(\tau) \qquad (2)$$

where V_D is the voltage of the dipole perturbation and r_0 is the device radius. This driving term can be equated to the dipole kick in an accelerator. After some derivation [3], it is found that setting V_D according to the following relation

$$V_D \approx \frac{mc^2 r_0}{q}\left(\frac{2\pi R}{P\lambda}\right)^2 \frac{\Delta B}{B\rho} \qquad (3)$$

ensures a similar distortion is produced in S-POD as in EMMA by a dipole perturbation ΔB. P is the number of rf periods of wavelength λ per "turn" in S-POD and R is the average radius of EMMA. A piecewise constant voltage waveform may be applied, emulating a localised error source in EMMA (Fig. 1 (a)). This single local error source results in the activation of stopbands at each integer tune. Alternatively, the application of a sinusoidal waveform ($V_D = w_n cos(n\theta + \phi_n)$) allows a single harmonic to be excited (Fig. 1 (b)). Measurements confirm that, in this case, a stopband is produced at the corresponding integer tune only [4].

RESONANCE CROSSING

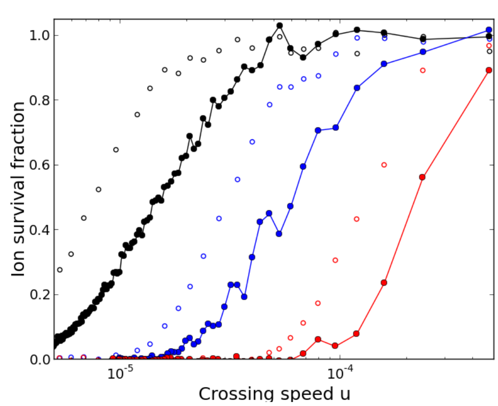

Figure 2: Ion survival rate following single (open symbols) and double (filled symbols) resonance crossings for a range of crossing speeds u and for perturbation voltages set to 0.05 V (black), 0.1 V (blue) and 0.2 V (red). In the single resonance crossing case, a single perturbation harmonic w_8 was applied while in the double resonance crossing case, both w_8 and w_9 were applied with equal magnitudes. In both cases, the tune was varied from 9.5 to 7.5.

Both single and multiple integer resonance crossing cases were investigated. In the former case, the resonance at integer 8 was activated by adding a sinusoidal dipole perturbation as described in the previous section. The tune was then dynamically reduced from 9.5 to 7.5 by varying the voltage appropriately (the tune was decreased to simulate acceleration in EMMA). As shown in Fig. 2, the number of ions surviving resonance crossing was measured for a range of crossing speeds and perturbation voltages w_n. The crossing speed parameter u is the total change in cell tune divided by the number of rf periods taken to complete the crossing; in the case of rapid acceleration in EMMA it is typically $\sim 5 \times 10^{-4}$. As expected, fewer ions survive for slower crossing speeds and for higher perturbation voltages.

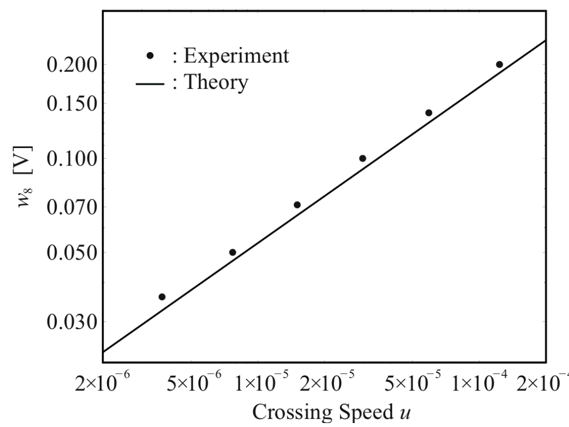

Figure 3: Critical perturbation voltage which at each crossing speed results in 50% ions loss indicating the coherent oscillation is equal to the aperture radius. The theoretical prediction given by Eqn. 5 is also shown.

Guignard derived the following equation to describe coherent amplitude growth caused by resonance crossing [5]

$$\Delta\sqrt{\epsilon} = \frac{\pi}{\sqrt{Q_\tau}} \frac{2R}{B\rho} \left| \frac{1}{2\pi} \int_0^{2\pi} \sqrt{\beta}\Delta B e^{in\theta} d\theta \right| \qquad (4)$$

where ϵ is the action of coherent excitation of dipole motion and Q_τ is the rate of change of tune per turn. Making use of Eqn. 3, the above expression can be recast in terms of S-POD parameters

$$\Delta A_n = g_n^G \frac{w_n}{\sqrt{u}} \qquad (5)$$

where ΔA_n is the coherent amplitude growth following the crossing of a resonance at integer n and the parameter g_n^G is given by

$$g_n^G = \frac{q\lambda}{2\pi mc^2 r_0} \beta_{rf}^{max} \left| \int_0^{2\pi} \sqrt{\beta_{rf}} cos(n\theta + \phi_n) e^{in\theta} d\theta \right| \qquad (6)$$

where the betatron function β_{rf} in a LPT is analogous to that in an accelerator. In S-POD the coherent amplitude cannot be directly measured. However, when the ion loss fraction ξ after resonance crossing reaches 0.5 (i.e. when half the

ISBN: 978-3-95450-132-8 doi:10.18429/JACoW-IPAC2014-TUPRI009

plasma is lost outside the aperture), it can be assumed that the amplitude approximately equals the device aperture radius. For each crossing speed u, we therefore search for the critical perturbation voltage for which $\xi = 0.5$. In Fig. 3, the results can be seen to be in good agreement with theory.

In the case of EMMA, many integer tunes are crossed during acceleration. Double resonance crossing was studied in S-POD by imposing both a harmonic 8 and 9 sinusoidal perturbation. As before, the tune was dynamically varied from 9.5 to 7.5, this time crossing two stopbands. In Fig. 2, it can be seen that crossing two resonances results in a greater ion loss than when crossing a single resonance (except where the crossing speed is slow enough to result in a complete loss of ions in both cases). In addition, fine structure is evident in the former case which is absent in the latter. We surmise that this is because the effect of the second resonance depends on the relative phase between two integer crossings; adjusting the crossing speed has the effect of varying this relative phase.

In Fig. 4, keeping the crossing speed fixed, the effect of the initial relative phase φ_r between the dipole harmonics was studied at various perturbation voltages. It can been be seen that at certain φ_r, the effects of the two resonance crossings tend to cancel. For example, when $w_8, w_9 = 0.2$, the maximum ion loss occurs when $\varphi_r \sim 250°$, whereas almost no ions are lost at $\varphi_r \sim 150°$.

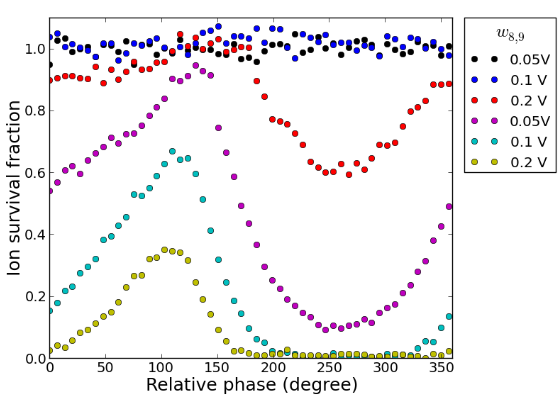

Figure 4: Dependence of the ion loss fraction following double resonance crossing on the initial relative phase between the harmonic 8 and 9 dipole perturbations. The results are shown for various perturbation strengths, where in each case the two dipole harmonics have equal voltages.

DISCUSSION

S-POD experiments were conducted exploring the crossing of integer resonances across a wide parameter range. The results of the single resonance crossing experiment show that ion losses depend on perturbation voltage and crossing speed as predicted by theory. In the case of double resonance crossing, the results imply that the amplitude of the coherent oscillation excited by the first integer resonance crossing can be increased or reduced depending on the phase

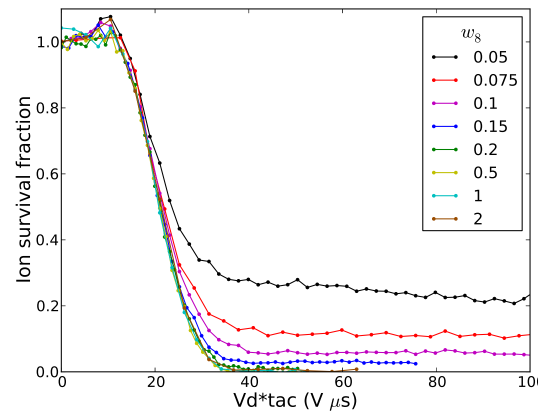

Figure 5: Time evolution of ion losses when the tune is set to integer for various harmonic perturbation voltages. The time axis is scaled by the perturbation voltage.

of the oscillation when the beam comes to the second integer resonance (Fig. 4).

The above picture holds so long as linearity can be assumed. The fine structure seen in the double resonance crossing case (Fig. 2) disappears at low crossing speeds, indicating that the phase dependence of the second resonance crossing is lost. A plausible explanation is that decoherence caused by non-linearities in the LPT result in the plasma smearing out in phase space. A similar effect is expected to occur at low crossing speeds in EMMA, though in that case the decoherence is caused by chromaticity and momentum spread.

Evidence of non-linearity is also found when the tune is fixed on an integer. In that case, when a dipole perturbation of the corresponding harmonic is applied, it is expected in a linear system that the resulting coherent oscillation will grow without bound. However, non-linearities can result in amplitude dependent detuning and, hence, a finite amplitude growth as the tune moves away from resonance [6]. This detuning may explain the nonzero number of ions that remain after the initial rapid drop (up to about 40 μs) for low perturbation voltages in Fig. 5. Non-linearities in S-POD arise from misalignments of the electrodes.

REFERENCES

[1] S.Machida et al, Nature Physics, **N8**, 243-257 (2012).

[2] H. Takeuchi et al, Phys. Rev. ST Accel Beams **15**, 074201 (2012).

[3] K. Moriya et al, to be published (2014)

[4] S. Sheehy et al, Proceedings of IPAC13, Shanghai, China (2013)

[5] G. Guignard, CERN 77-10 (1977).

[6] S.R. Mane and W.T Weng, Nucl. Instr. Meth. A306 (1991) 9-20.