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SOME FEATURES OF WAVE DISTRIBUTION IN THE THIN-WALL WAVEGUIDE

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Abstract

In this report we derive rigorous and approximate dispersion relations for the round resistive thin-wall waveguide. The features of the distributions of dispersion curves of the waveguide axisymmetric TM modes are obtained. Cases of splitting and degeneracy of modes under consideration are detected and regularities of their behaviours are established.

INTRODUCTION

Thin-walled metal waveguides were considered, as for example, in [1]. The impedance properties of such waveguides were studied and in results of which some of their features were revealed. Meanwhile, the dispersion characteristics of such waveguides are not enough investigated. Typically, the dispersion relations for waveguides were used in the construction of their surface impedance [2] underlying the approximate solutions for the electromagnetic fields propagation in waveguides with thick outer walls and various thin inner coating (dielectric, rough, metal). For thin single-layer waveguide the concept of impedance boundary conditions is inapplicable, however, it is important to know its properties for understanding the mechanisms of wave propagation and, in particular, for the decomposition of the fields on the eigenmodes in such waveguides. It will be useful in determining the maximum possible thickness of the undulator vacuum chambers and characteristics of the external radiation, for example, with a view to shielding.

Metal waveguides may be named thin walled if along with a small wall thickness the metal filling of walls has a low conductivity. For sufficiently high conductivity even at low wall thickness, the waveguide behaves as a thick-walled on the most part of the frequency spectrum except at very low frequencies in which the skin layer is thicker than the wall and where the notion of the skin layer is not applicable. In turn, the thick-walled waveguide with walls of low conductivity may possess all the properties of thin-wall waveguide.

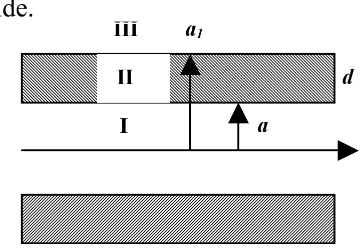

Figure 1: Geometry of problem.

In this paper we consider a round metallic waveguide with an inner radius a, wall thickness d and outer radius $a_1=a+d$ (Fig.1). Permittivity of a non-magnetic metal (magnetic permeability metal is magnetic permeability of vacuum μ_0) wall determined by the conductivity of the metal and dielectric permittivity of vacuum ε_0: $\varepsilon = \varepsilon_0 - j\sigma/\omega$.

DISPERSION RELATIONS

Fundamental solutions of the homogeneous Maxwell equations of the described structure are searched using cross-linking of partial solutions for each area of the waveguide, marked in Figure 1: in the internal region of the waveguide (I - a vacuum) in the metal wall (II - metal) and in the region outside the waveguide (III - vacuum). Non-zero tangential electric and magnetic components of axisymmetric partial solutions for each area are presented as follows:

$$(I) \quad E_z = A_m J_0(\nu_{0m} r) F_m,$$
$$H_\theta = A_m \frac{jk}{Z_0 \nu_{0m}} J_1(\nu_{0m} r), \quad 0 \le r \le a$$
$$(II) \quad E_z = \{B_m J_0(\nu_{1m} r) + C_m H_0(\nu_{1m} r)\} F_m,$$
$$H_\theta = \frac{jk\varepsilon_1}{Z_0 \varepsilon_0 \nu_{1m}} \{B_m J_1(\nu_{1m} r) + C_m H_1(\nu_{1m} r)\} F_m,$$
$$a \le r \le a + d$$
$$(III) \quad E_z = D_m H_0(\nu_{0m} r) F_m,$$
$$H_\theta = D_m \frac{jk}{Z_0 \nu_{0m}} H_1(\nu_{0m} r) F_m, \quad r \ge a + d \tag{1}$$

In (1): $Z_0 = 120\pi\,\Omega$, ν_{0m}, $\nu_{1m} = \sqrt{-j\omega\sigma\mu_0 + \nu_{0m}^2}$ - transverse eigenvalues (wave numbers) of TM_{0m} mode $(m = 1,2,3,\ldots)$ in a vacuum and in the metal wall, respectively and F_m the transverse propagation factor.

Values of the tangential field components at the boundaries of each of the areas should be equal to each other:

$$E_z^{(I)} = E_z^{(II)}|_{r=a}, \; H_\theta^{(I)} = H_\theta^{(II)}|_{r=a},$$
$$E_z^{(II)} = E_z^{(III)}|_{r=a+d}, \; H_\theta^{(II)} = H_\theta^{(III)}|_{r=a+d} \tag{2}$$

The result is a homogeneous linear system of four equations with four unknown coefficients (A_m, B_m, C_m and D_m):

$$A_m J_0(\xi_{0m}) - B_m J_0(\nu_{1m} a) - C_m H_0(\nu_{1m} a) = 0 \tag{3}$$
$$A_m \frac{1}{\xi_{0m}} J_1(\xi_{0m}) - \frac{1}{\chi_m} \{B_m J_1(\nu_{1m} a) + C_m H_1(\nu_{1m} a)\} = 0$$

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$$B_m J_0(\nu_{1m} a_1) + C_m H_0(\nu_{1m} a_1) - D_m H_0(\xi_{0m}\, a_1/a) = 0$$
$$\frac{1}{\chi_m}\{B_m J_1(\nu_{1m} a_1) + C_m H_1(\nu_{1m} a_1)\} - D_m \frac{1}{\xi_{0m}} H_1(\xi_{0m}\, a_1/a) = 0 \quad (3)$$

Here we have introduced the notations: $\xi_{0m} = a\nu_{0m}$, $\chi_m = \varepsilon_0 \nu_{1m} a/\varepsilon_1$. The existence of non-trivial solutions of (3) conditioned by vanishing of its determinant:

$$H_1(\xi_{0m}\, a_1/a)\chi_m\left(J_0(\xi_{0m})\xi_{0m}U_{10} - J_1(\xi_{0m})\chi_m U_{00}\right) + H_0(\xi_{0m}\, a_1/a)\xi_{0m}\left(-J_0(\xi_{0m})\xi_{0m}U_{11} + J_1(\xi_{0m})\chi_m U_{01}\right) = 0 \quad (4)$$

where

$$U_{ij} = J_i(\nu_{1m} a) H_j(\nu_{1m} a_1) - J_j(\nu_{1m} a_1) H_i(\nu_{1m} a), \quad i,j = 0,1 \quad (5)$$

For large values of the arguments, equation (4) can be rewritten approximately $(b = a_1/a)$:

$$tg(d\nu_1) = \frac{\xi_{0m}}{\chi_m} \frac{\dfrac{J_0(\xi_{0m})}{J_1(\xi_{0m})} - \dfrac{H_0(\xi_{0m} b)}{H_1(\xi_{0m} b)}}{1 + \left(\dfrac{\xi_{0m}}{\chi_m}\right)^2 \dfrac{J_0(\xi_{0m})}{J_1(\xi_{0m})} \dfrac{H_0(\xi_{0m} b)}{H_1(\xi_{0m} b)}}, \quad (6)$$

and for the approximation of $\varepsilon_1 \approx -j\sigma/\omega$ and $\nu_{1m} \approx \sqrt{-j\omega\sigma\mu_0}$ one obtains:

$$\frac{tg\left((1-j)\alpha\sqrt{\kappa}\right)}{1-j} = -\frac{\kappa^{3/2}\left(J_0(\xi_{0m})H_1(b\xi_{0m}) - J_1(\nu_0)H_0(b\xi_{0m})\right)}{\kappa^3 J_1(\xi_{0m})H_1(b\xi_{0m}) - 2j\xi_{0m}^2 J_0(\nu_0)H_0(b\xi_{0m})} \quad (7)$$

Here $\kappa = ks_0$ the dimensionless wavenumber, $s_0 = (2a^2 c\varepsilon_0/\sigma)^{1/3}$ the characteristic size of unbounded resistive tube [3] and $\alpha = ad/s_0^2$. The frequency range of applicability of equation (7) is determined by double inequality:

$$(a^2 Z_0 \sigma)^{-1} << k << Z_0\sigma\,. \quad (8)$$

For the small wall thickness the further simplification of the equation (14) is possible:

$$tg\left((1-j)\alpha\sqrt{\kappa}\right) = -\frac{1}{\pi}\frac{2(1+j)\kappa^{3/2}}{\kappa^3 J_1(\xi_{0m})H_1(\xi_{0m}) - 2j\xi_{0m}^2 J_0(\xi_{0m})H_0(\xi_{0m})} \quad (9)$$

MODE TM$_{01}$

Equations (6)-(9) have a countable number of solutions that are continuous functions defined by initial conditions $\xi_{0m} = j_{0m}$ $(m = 1,2,3,\ldots)$, where j_{0m} - the roots of the Bessel function of the first kind and zero order $(J_0(j_{0m}) = 0)$.

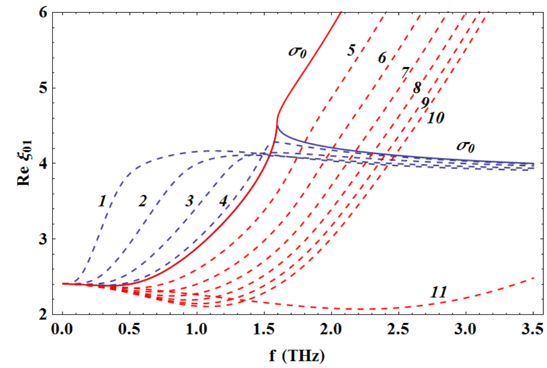

Figure 2: The frequency dependence of the real component of the transverse eigenvalues; the waveguide radius a = *1cm*, wall thickness d=*1μm*; solid curves: σ_0 =4.0753 $\Omega^{-1}\text{m}^{-1}$; dashed curves: $\sigma = 10^5 i\ \Omega^{-1}\text{m}^{-1}$, 1-4, i=1,2,3,4 ($\sigma < \sigma_0$, blue); 5-11, i=5,6,7,8,9,10,100 ($\sigma > \sigma_0$, red).

Examples of the solutions of the dispersion equation (6) for the fundamental mode at a fixed wall thickness for different values of the conductivity of the wall material are shown in Figure 2.

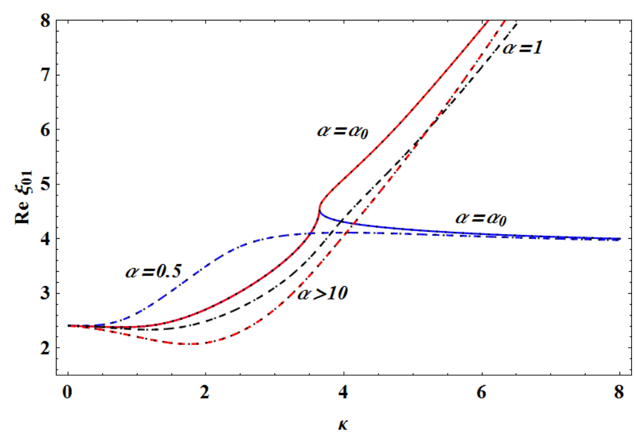

Figure 3: Dependence of the real part of transverse eigenvalues of a thin-walled circular metal waveguide from the dimensionless wavenumber κ (mode TM$_{01}$); solutions of equation (9) (dashed lines), corresponding solution of equation (6) for a = 1cm and d =1μm; solid lines: (red - upper branch, blue - lower branch) correspond to the critical value of the parameter; $\alpha = \alpha_0$; branching point $\kappa = \kappa_0 = 3.64$.

From the Figure one can trace the transformation of the dispersion curves as a function of the change in conductivity of the wall material.

It should be initially noted that the presence of the phenomenon of mode degeneration on the fixed value of the conductivity, which is manifested in the bifurcation (splitting) of the dispersion curve with a branch point at

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$f = f_0$ ($f_0 \approx 1.6THz$) and becomes two-valued function when $f > f_0$. The critical (branching) dispersion curve $\sigma = \sigma_0$ is represented in Figure as a solid one: red (upper brunch) and blue (lower brunch). Behavior of the dispersion curves at $\sigma < \sigma_0$ and $\sigma > \sigma_0$ is significantly different. In the first case, with increasing frequency the dispersion curves tend to the value j_{11} (the first root of the Bessel function $J_1(x)$). In the second case, the real part of the transverse eigenvalue tends to infinity (similarly to the case $\alpha > 10$ (Fig.3) of a single-layer unbounded resistive waveguide [4]). Transition from the first to the second case at $\sigma = \sigma_0$ occurs abruptly.

The common nature of the phenomenon of splitting or degeneracy of the dispersion curve which is due to the finite wall thickness appears from dispersion equation written in approximations (7) or (9). In particular, in the form of (9) the dispersion equation has a universal character: by varying the parameter α, one can get the dispersion characteristics for the arbitrary combinations of *a, d,* and σ. Note that parameters κ and s_0 used in this formula, have been used previously only for the thick-walled waveguides [3]. In the approximation (9) branching point on the dispersion curve occurs when $\alpha = \alpha_0$, where $\alpha_0 = 0.8385 \approx 0.5^{1/4}$ (Fig. 3).

HIGH AXISYMMETRIC MODES

Equations (6) - (9) allow us to determine the dispersion curves for the higher axisymmetric TM modes. Figure 4 shows the curves for the critical values of the parameter α for the first five axisymmetric modes.

As can be seen from this Figure, the higher modes, in contrast to the main TM_{01} mode, have two branch points (κ_1 and κ_2, in Tab.1) and in pairs are decomposed into four branches. The upper branch of each pair with increasing frequency tends to infinity. Lower branches at high frequency limit tend to the various finite eigenvalues j_{1i} equal to the roots of first order Bessel function. Two branch points of consecutive modes coincide with each other (see Fig. 4 and Table 1). Thus, the branching point of TM_{01} mode is cincided with the lower branching point of TM_{02} mode, while the upper branching point of this mode coincides with the lower branching point of the next, TM_{03} mode, and so on (Table 1). Branches of the dispersion curves of two consecutive modes with coincided branch points also pairwise overlap. Thus, when $\alpha = \alpha_{1,2}$ in the frequency ranges $\kappa \geq \kappa_{1,2}$ there is a merging of the respective branches of the dispersion curves of two consecutive mode: for example, the upper branch of the upper pair of TM_{02} mode merges with the upper branch of the lower pair of mode TM_{03}, i.e., when $\kappa \geq \kappa_{1,2}$ and at the critical value of $\alpha = \alpha_{1,2}$ the degeneration (merger) of these two modes takes place.

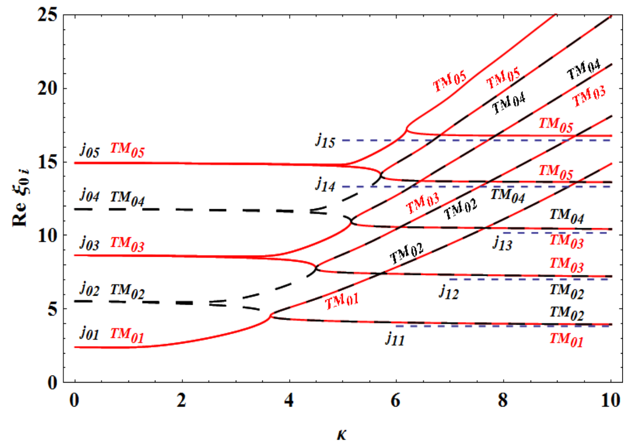

Figure 4: The real components of the dispersion curves for the first five axisymmetric modes versus dimensionless wavenumber κ; mode with an odd index i (red, solid); mode with even index *i* (black, dotted).

Table 1: Critical Parameters

Mode	α_1	α_2	κ_1	κ_2
TM_{01}	0.8385		3.65	
TM_{02}	0.8387	0.5878	3.65	4.49
TM_{03}	0.5878	0.4754	4.49	5.15
TM_{04}	0.4754	0.4073	5.15	5.70
TM_{05}	0.4073	0.3603	5.70	6.19

CONCLUSION

The phenomenon of splitting and degeneration of axisymmetric transverse magnetic (TM) eigenmodes of a thin-walled circular metallic waveguide is described. The phenomenon is due to the finite thickness of the wall of the waveguide and occurs at certain critical values of conductivity of the metal wall. Degeneration of two consecutive modes occurs due to coincidence of critical values of conductivity (parameters $\alpha_{1,2}$) for these two modes.

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