ISBN: 978-3-95450-132-8 doi:10.18429/JACoW-IPAC2014-TUPRI038

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THE LOW ENERGY PARTICLE WAKEFIELD RADIATION FROM THE OPEN END OF INTERNALLY COATED METALLIC TUBE*

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Abstract

The radiation of the non-relativistic electron beam from the open end of the resistive circular waveguide is presented. The angular and spectral characteristics of the radiation are determined. The possibility of producing the focused guasi-monochromatic radiation is discussed. The principal scheme of the experiments for 5 and 20 MeV AREAL RF photogun linac is presented.

INTRODUCTION

Proposed in [1] single-mode and single-resonance structure: two-layer round metal waveguide with low-conducting internal thin metallic coating (Fig. 1), due to the unique properties of the wakefield radiation formed by charge that following along the axis of the structure opens up opportunities for its practical application. One of such opportunities is extracting the radiation from the open end of a semi-infinite waveguide to produce the narrow-directed monochromatic radiation [2]. The problem in [2] was considered in the ultra-relativistic approximation, i.e. the velocity of a point charged particle generating wakefield radiation was equal to the speed of light in vacuum. Such an approximate treatment for cases of high energy particle simplifying picture of the process, at the same time with sufficient accuracy describes all the main features of its dynamics for high-energy beams.

However, ultrarelativistic approximation is not applicable to the beams of low-energy particles. In [3] the characteristic features of impedance and wake functions for the low energy particles for different configuration of proposed structure were examined. In particular, strict dependence of frequency and spatial - temporal characteristics of wakefield on its energy has been shown.

The energy range from 5 to 20 MeV required for experimental verification of the results at the accelerator complex AREAL [4] has been investigated and acceptable (in terms of the experiment) parameters of the waveguide, which is already at a sufficiently small particle energies (20 MeV) impedances and wake functions acquire resonance properties typical for ultrarelativistic case, studied in [1], have been determined.

In this paper we present the main results of theoretical calculations of radiation from the open end of a semi-infinite waveguide with the proposed structure.

STATEMENT OF THE PROBLEM

We consider (Fig.1) the problem of radiation from the open end of a semi-infinite circular metallic waveguide (internal radius a_1) with perfectly conducting walls and a thin internal low-conducting metal coating (thickness d and conductivity $\sigma = \sigma_1$).

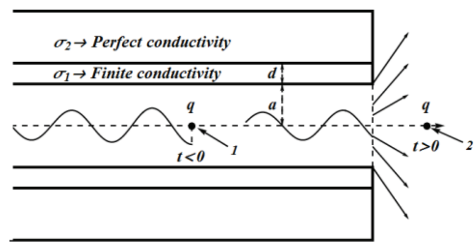

Figure 1: Geometry of the problem.

Electromagnetic fields are generated by the point particle, moving along the axis of the waveguide with arbitrary velocity v. The far radiation fields are obtained using the near field to far-field recovery technique [5] that is extended for the non monochromatic waves. It is shown that the radiation has a narrow-band and narrow directional character.

NEAR-FIELD TO FAR-FIELD TRANSITION

If the distribution tangential components $E_x(x,y)$, $E_y(x,y)$ (Fig.2) of the field of an arbitrary monochromatic (with harmonic time dependence $e^{j\omega t}$) waves on a flat surface $z=0$ (Figure 2) is given, the far field at a point R can be represented by these distributions as follows [5]:

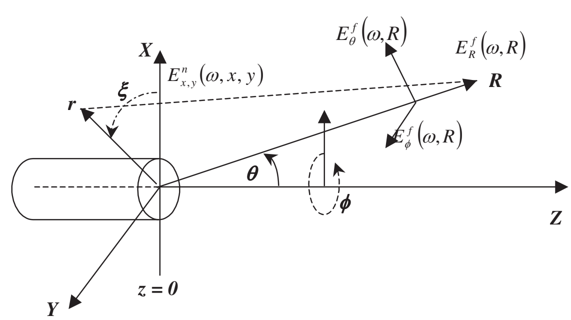

Figure 2: The radiation from the open end of the waveguide. Coordinate system and notations.

$$E^f(\omega,R) = k\cos\vartheta A(k_x,k_y)\frac{e^{jkR}}{R} \quad (1)$$

*This work was supported by State Committee of Science MES RA, in frame of the research project № SCS 13YR-1C0056
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ISBN: 978-3-95450-132-8 doi:10.18429/JACoW-IPAC2014-TUPRI038

where $k = \omega/c$ - wavenumber, ω - frequency, c - speed of light in vacuum, and

$$A_{x,y}\left(k_x, k_y\right) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} E_{x,y}^{n}(\omega, x, y) e^{-jk_x x - jk_y y} dxdy$$
$$A_z\left(k_x, k_y\right) = -k_x^{-1}\left\{k_x A_x\left(k_x, k_y\right) + k_y A_y\left(k_x, k_y\right)\right\} \quad (2)$$

the two-dimensional Fourier transform of the initial distribution of the Cartesian components of the fields,

$$k_x = k\sin\theta\cos\phi, \;\; k_y = k\sin\vartheta\sin\phi$$
$$k_z = \sqrt{k^2 - k_x^2 - k_y^2} = k\cos\theta \quad (3)$$

The radiation field in the far zone has a single polar electrical component:

$$E_\theta^f\left(R,\omega\right) = -2\pi j\frac{\omega}{c} I(\omega,\theta)\frac{e^{i\omega(R/c-t)}}{R} \quad (4)$$

where

$$I(\omega,\theta) = \int_0^a E_r^n(\omega, r) J_1(kr\sin\theta) rdr \quad (5)$$

with [6]

$$E_r = -\frac{jI_1(\lambda r)}{\tau I_0(\lambda a_1)} E_z = \frac{Z_0}{G(x,y)} I_1(\lambda r) \quad (6)$$

$$G(x,y) = 2\pi a_1 \beta\, x I_0(x)\left(\frac{I_1(x)}{x} + \frac{d}{a}\frac{\varepsilon_1}{\varepsilon_0} I_0(x)\,\mathrm{cth}\, y/y\right) \quad (7)$$

Here $x = ak_v\tau$, $y = \chi_1 d$, $\chi_1 = \sqrt{k_v^2\tau^2 - j\omega\mu_0\sigma}$, $\lambda = k_v\tau$, $\tau = \gamma^{-1}$, $k_v = \omega/v$. Taking into account (1)-(7), we obtain the spectral distribution of the radiation field:

$$E_\theta^f\left(R,\omega\right) = -j\Lambda(\theta)\frac{F(\theta,\omega)}{I_0(x)J(\omega)}\frac{e^{i\omega(R/c-t)}}{R} \quad (8)$$

where $u = ka\sin\vartheta$; in (6)-(8) $I_n(x)$ are the modified Bessel function of the first kind,

$$F(\theta,\omega) = I_0(x)J_2(u) + I_2(x)J_0(u)$$
$$J(\omega) = \frac{I_1(x)}{x} + \frac{d}{a}\frac{\varepsilon_1}{\varepsilon_0} I_0(x)\,\mathrm{cth}\, y/y \quad (9)$$

and

$$\Lambda(\theta) = \frac{Z_0}{2}\frac{\beta\sin\theta}{1-\beta^2\cos^2\theta}, \;\; \beta = v/c \quad (10)$$

an angular distribution of the transition radiation field of the point particle on a perfectly conducting flat surface.

Thus, the total radiation field is characterized by a factor of the angular distribution of transition radiation on a perfectly conducting surface (10), modulated by far range form-factor structure $\tilde{F}(\vartheta,s)$

$$E_\theta^f\left(R\right) = -j\Lambda(\theta)\tilde{F}(\vartheta,s), \quad (11)$$
$$\tilde{F}(\vartheta,s) = \mathrm{I}_1 + \mathrm{I}_2, \;\; \mathrm{I}_{1,2} = \int_{-\infty}^{\infty} Q_{1,2}\frac{J_{2,0}(u)}{J(\omega)}\frac{e^{i\omega(R/c-t)}}{R}d\omega$$
$$Q_1 = 1, \;\; Q_2 = I_2(x)/I_0(x)$$

FAR FIELD STRUCTURE

For small parameter y function $\mathrm{cth}\, y/y$ in (7) can be replaced by the first two terms of its expansion: $\mathrm{cth}\, y/y = y^{-2} + 1/3$. In this case, the factor $J(\omega)$ can be reduced to the form without discontinuity in the complex plane ω:

$$J(\omega) = \frac{I_1(x)}{x} + \frac{d_1}{a_1}\frac{\varepsilon_1}{\varepsilon_0} I_0(x)\left(3^{-1} + y^{-2}\right) \quad (12)$$

Further transforms include substitution in formulae (11) the integral forms of the Bessel functions of the first kind

$$J_n(z) = \frac{2(z/2)^n}{\sqrt{\pi}\Gamma(n+1/2)}\int_0^1\left(1-t^2\right)^{n-1/2}\cos(zt)dt,$$

changing the order of integration and replacement the first N terms of series expansion (see [3]) in the small parameter τ of the denominators in the integrand $\mathrm{I}_{1,2}$ and in the numerator of the integrand I_2. Mentioned procedure leads to analytisation of integrands in (11): their denominators are the algebraic polynomials of the variable ω of the order $N+3$ with the complex roots $\omega = \omega_{1(2),k}$ respectively.

Explicitly, the radiation field in the far zone at an arbitrary point R of observation and an arbitrary time t can be written in the following form $(s = R - ct)$:

$$\tilde{E}_\theta^{f(\;)}(R) = -j\frac{\Lambda(\theta)}{2R}\sum_{l=1}^{2}\sum_{k=1}^{N+3} A_{l,k}\left(F_{l,k} + G_{l.k}^{\pm}\right)e^{-j\frac{\omega_{l,k}}{c}s}$$

where: for $ct > R + a\sin\vartheta$ ("-"), and for $ct < R + a\sin\vartheta$ ("+"):

$$F_{1,k} = 2\left(1 \quad Sign\left(\mathrm{Im}\,\omega_{1,k}\right)\right)J_2\left(\frac{\omega_{1,k}}{c}Sin\vartheta\right)\Big/\omega_{1,k}^2$$

$$F_{2,k} = 2\left(1 \quad Sign\left(\mathrm{Im}\,\omega_{2,k}\right)\right)J_0\left(\frac{\omega_{2,k}}{c}Sin\vartheta\right), \quad G_{l,k}^{\pm} = 0;$$

and for the middle region:

$$F_{l,k} = \left(1 \quad Sign\left(\mathrm{Im}\,\omega_{l,k}\right)\right)\left(\mathrm{I}_{l,k}(0,1) + \mathrm{I}_{l,k}^{\pm}(0,\pm\chi)\right),$$
$$G_{l.k}^{\pm} = \left(1 \pm Sign\left(\mathrm{Im}\,\omega_{l,k}\right)\right)\mathrm{I}_{l,k}(\quad\chi,1), \quad \chi = (R-ct)/a\,Sin\vartheta$$

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Here the upper sign corresponds to the region $0 < ct < R + a\sin\vartheta$, and the bottom one to the region $R - a\sin\vartheta < ct < 0$ with

$$I^{\pm}_{1,k}(t_1,t_2)=\frac{2j}{3R}\frac{a^2}{c^2}\sin^2\theta\int_{t_1}^{t_2}\left(1-\tilde{t}^2\right)^{3/2}e^{j\omega_k l_\pm}d\tilde{t}\ ,$$

$$I^{\pm}_{2,k}(t_1,t_2)=\frac{2j}{R}\int_{t_1}^{t_2}\left(1-\tilde{t}^2\right)^{-1/2}e^{j\omega_k l_\pm}d\tilde{t}\ ,$$

$$l_\pm = \pm\tilde{t}\,a\sin\theta/c-(R/c-t).$$

Amplitudes $A_{l,k}$ arise during of expansion of the outside-exponential factors in the integrands terms (11) in the form of sums of the type $U=\sum_{k=1}^{N+3}A_{l,k}/(\omega-\omega_{l,k})$. Thus, the far field is formed by the sum of contributions from a discrete set of quasi-monochromatic patterns, frequency characteristics of which are due to the resonant frequencies $\omega_{l,k}$.

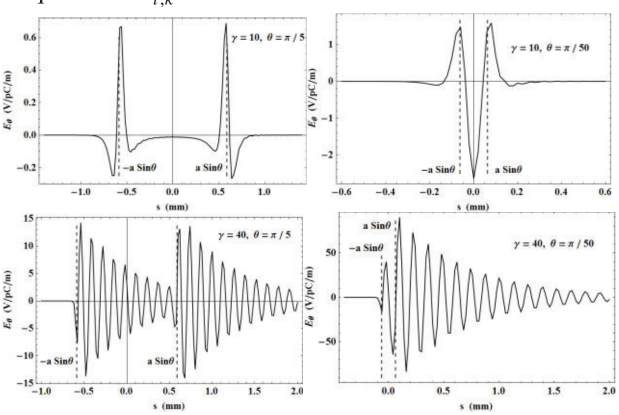

Figure 3: Temporal $(R = 1m)$ distribution of the radiation fields for different particle energy at fixed angles of observation. Lorentz factors of particles and viewing angles are given on the figures, $a = 1mm$, $d = 1\mu m$, $\sigma = 5\times10^3\Omega^{-1}m^{-1}$.

As it can be seen (Fig.3), in the first case (top, $\gamma = 10, 5MeV$) the radiation field generated set of frequencies and dominant frequency cannot be allocated. In the second case (bottom, $\gamma = 40, 20MeV$), the field has the quasi-monochromatic character and is irradiated onto a specific frequency.

ANGULAR DISTRIBUTION OF RADIATED ENERGY

The most simple and relatively easy feasible scheme of the experiment is a film located at some distance from the aperture of the waveguide perpendicular to its axis. Degree of blackening of the film in a given point is proportional to the density of energy emitted by the waveguide aperture in given direction during the entire experiment duration (Fig.4). For the bottom case $(\gamma = 40)$ we have a narrow-directed radiation: its angle of maximum density $\theta_{\max}$ substantially less than the maximum orientation of the transition radiation $\theta_{tr} \approx \gamma^{-1}$. As in the first $(\gamma = 10)$ and in the second $(\gamma = 40)$ cases the ring-form blackening with the radius $\sim R\sin\vartheta_{\max}$ formed on the screen.

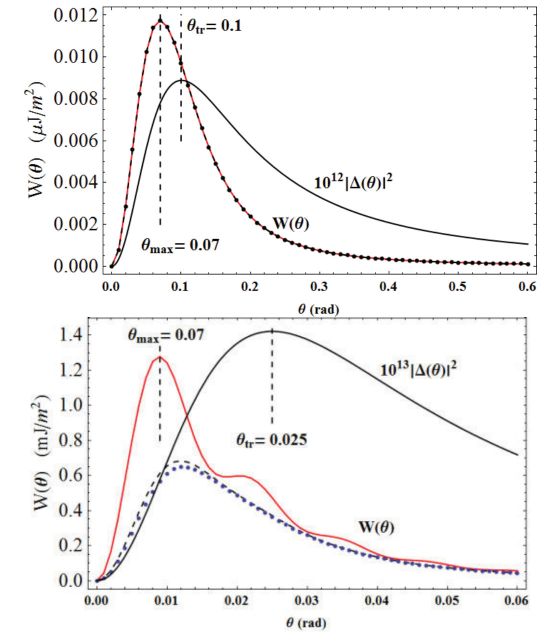

Figure 4: The angular distribution of the energy density incident on a photographic film at the particle energy: $\gamma = 10$ (top), $\gamma = 40$ (bottom); $\sigma = 5\cdot10^3\Omega^{-1}m^{-1}$ (red solid), $5\cdot10^2\Omega^{-1}m^{-1}$ (dash-dotted), $5\cdot10^4\Omega^{-1}m^{-1}$ (dotted); $a = 1cm, d = 1\mu m$.

CONCLUSION

Explicit expressions for the wakefield radiation from the open end of a two-layer waveguide for particles with arbitrary energy are obtained, the method for its decomposition into its spectral components is also proposed. The acceptable parameters of waveguides for experimental verification of the results on the accelerator complex AREAL [4] are obtained. The principal scheme of the experiment is suggested with predicted results.

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