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SIMPLE MODELS DESCRIBING THE TIME-EVOLUTION OF LUMINOSITY IN HADRON COLLIDERS

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Abstract

In recent years, several studies have been performed to describe the evolution of the losses in circular proton machines. Considerations based on single-particle, non-linear beam dynamics allowed building models that, albeit simple, proved to be in good agreement with measurements. These initial results have been generalised, thus opening the possibility to describe the luminosity evolution in a circular hadron collider. In this paper, the focus is on the derivation of scaling laws for the integrated luminosity, taking into account both burn off and additional pseudo-diffusive effects. The proposed models are applied to the analysis of the data collected during the LHC Run I and the outcome is discussed in detail.

LUMINOSITY EVOLUTION AND INVERSE LOGARITHM DECAY

The starting point is the expression of the luminosity

$$\mathcal{L} = \frac{\gamma_{\rm r} f_{\rm rev} k_{\rm b} n_1 n_2}{4\pi\epsilon^* \beta^*} F(\theta_{\rm c}, \sigma_{\rm z}, \sigma^*), \tag{1}$$

where $\gamma_{\rm r}$ is the relativistic γ-factor, $f_{\rm rev}$ the revolution frequency, $k_{\rm b}$ the number of colliding bunches, $n_{\rm i}$ the number of particles per bunch in each colliding beam, ϵ^* is the rms normalised transverse emittance, and β^* is the value of the beta-function at the collision point. The total beam intensity will be defined as $N_{\rm i} = k_{\rm b}\, n_{\rm i}$.

The factor F accounts for the reduction in volume overlap of the colliding bunches due to a crossing angle and is a function of the crossing angle $\theta_{\rm c}$, the transverse and longitudinal rms dimensions $\sigma^*, \sigma_{\rm z}$, respectively:

$$F(\theta_{\rm c}, \sigma_{\rm z}, \sigma^*) = \frac{1}{\sqrt{1+\left(\frac{\theta_{\rm c}\sigma_{\rm z}}{2\sigma^*}\right)^2}} . \tag{2}$$

To note that $\sigma^* = \sqrt{\beta^*\, \epsilon^*/(\beta_{\rm r}\, \gamma_{\rm r})}$ where $\beta_{\rm r}$ is the relativistic β-factor. Equation (1) is valid in the case of round beams ($\epsilon_x^* = \epsilon_y^* = \epsilon^*$) and round optics ($\beta_x^* = \beta_y^* = \beta^*$). For our scope, Eq. (1) will be recast in the following form:

$$\mathcal{L} = \Xi\, N_1\, N_2, \qquad \Xi = \frac{\gamma_{\rm r} f_{\rm rev}}{4\pi\epsilon^*\beta^*\, k_{\rm b}} F(\theta_{\rm c}, \sigma_{\rm z}, \sigma^*) \tag{3}$$

in which the dependence on the total intensity of the colliding beams is highlighted and the other factors are included in the Ξ term.

In general, only the emittances and the bunch intensities can change over time. Therefore, Eq. (1) is better interpreted as peak luminosity at the beginning of the fill, while $\mathcal{L}$ will be a function of time. When the burn off is the only relevant mechanism for a time-variation of the beam parameters, it is possible to estimate the time-evolution of the luminosity, starting from the following equation

$$N'(t) = -\sigma_{\rm int}\, n_{\rm c}\, \mathcal{L}(t) = -\sigma_{\rm int}\, n_{\rm c}\, \Xi\, N^2(t) \tag{4}$$

where $\sigma_{\rm int}$ represents the cross section for the interaction of the charged particles[1]. Here, $n_{\rm c}$ stands for the number of collision points.

For beams of same initial intensity the solution is given by

$$N(t) = \frac{N_{\rm i}}{1+\sigma_{\rm int}\, n_{\rm c}\, \Xi\, N_{\rm i}\, t}, \tag{5}$$

with $N_{\rm i}$ the initial beam intensity. Equation (5) implies

$$\mathcal{L}(t) = \frac{\Xi\, N_{\rm i}^2}{(1+\sigma_{\rm int}\, n_{\rm c}\, \Xi\, N_{\rm i}\, t)^2} . \tag{6}$$

In reality, the situation is much more complex. In the case of a hadron collider, e.g., beam-beam and IBS affect the beam parameters in such a way that the model (6) is not valid anymore.

Several approaches can be followed, for instance, in Refs. [3,4] phenomenological fit models were proposed and applied with success to the characterisation of luminosity evolution in the Tevatron. Alternatively, in Ref. [5] the luminosity evolution is studied starting from numerical simulations taking into account the relevant physical processes.

A different model has been proposed in [6]. The basis for such a model is the evolution of the dynamic aperture (DA) with time in a hadron collider. The analysis of single-particle tracking results showed that the time-evolution of the DA follows a simple law [7, 8], whose justification is not entirely phenomenological. Recently, this approach was successfully applied to the analysis of intensity evolution in hadron machines [9]. So far, however, the results were obtained for single-particle simulations or for conditions in a running machine that were not including any collective effect. To extend the proposed scaling law to luminosity evolution, it is necessary to show that it is valid also in the presence of beam-beam effects, which seems to be case in numerical simulations [10].

The proposed approach is a refinement of what presented in Ref. [6] and assumes that all possible pseudo-diffusive effects can be modelled by a scaling of the intensity with time as

$$N(\tau) = N_{\rm i}\left[1-\int_{D(\tau)}^{+\infty}\hat{\rho}(r)\, dr\right] = N_{\rm i}\left[1-e^{-\frac{D^2(\tau)}{2}}\right] . \tag{7}$$

[1] $\sigma_{\rm int}$ = 73.5 mb for 3.5 TeV and 76 mb for 4 TeV [1] for protons (representing the inelastic cross-section), while for ions at 3.5 TeV is 449.2 b [2] (representing hadronic interaction, electromagnetic dissociation, and bound free pair production phenomena).

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where

$$D(\tau) = D_\infty + \frac{b}{[\log \tau]^\kappa}. \qquad (8)$$

The parameters D_∞, b, κ are normally fitted to the experimental data and the variable τ represents the turn number and satisfies $\tau \in [1, +\infty[$. Some properties of the parameters as highlighted in Refs. [8, 9].

To use this scaling law for the analysis of the luminosity evolution we need to consider that: i) the proton burn off occurs mainly in the core of the beam distribution; ii) the diffusive processes are mainly affecting the tails of the beam distributions; iii) the characteristic times of the two processes are different; iv) the fit parameters in Eq. (8) might depend on the beam intensity. However, if one assumes that the overall intensity variation over one physics fill is not too large, it is possible to assume that the pseudo-diffusive effects are, to a good extent, constant. Under these assumptions, the intensity evolution is proposed to follow

$$N'_j(t) = -\sigma_{\rm int}\, n_{\rm c}\, \Xi\, N_1(t)\, N_2(t) - \hat{\mathcal{D}}_j(t) \qquad j = 1, 2\,. \quad (9)$$

The terms $\hat{\mathcal{D}}_i$ represent the intensity-independent pseudo-diffusive effects. A different time variable will be considered, namely

$$\tau = f_{\rm rev}\, t \qquad \text{giving} \qquad \frac{d}{d\,t} = f_{\rm rev}\, \frac{d}{d\,\tau}, \qquad (10)$$

In the following the derivative with respect to τ will be indicated by $\dot{}$, while $'$ will indicate the derivative with respect to t. Equation (9) becomes

$$\dot{N}_j(\tau) = -\varepsilon\, N_1(\tau)\, N_2(\tau) - \mathcal{D}_j(\tau) \qquad j = 1, 2\,. \qquad (11)$$

with $\varepsilon = \sigma_{\rm int}\, n_{\rm c}\, \Xi / f_{\rm rev}$ and $\mathcal{D}_j = \hat{\mathcal{D}}_j / f_{\rm rev}$. Typical values of ε are 1.1×10^{-24}, or 3.4×10^{-20} assuming the beam parameters during the 2011 physics run for protons and Lead ions, respectively. Therefore, about 3.1×10^4 protons or 52 Lead ions are removed from the bunches each turn, corresponding to 0.24 ppm and 0.47 ppm, respectively. With such a change of time-variable the luminosity definition should be replaced by $\mathcal{L} \to \mathcal{L}/f_{\rm rev}$ and the symbol L will be used in the rest of the paper for the re-scaled luminosity.

The explicit expression for $\mathcal{D}_j(\tau)$ can be found by noting that these functions are the solutions of

$$\dot{N}_j(\tau) = -\mathcal{D}_j(\tau) \qquad j = 1, 2 \qquad (12)$$

and that the explicit solution has been assumed to be of the form (7) [6, 9, 10]. Therefore, one obtains

$$\mathcal{D}_j(\tau) = -N_{{\rm i},j}\, D_j(\tau)\, \dot{D}_j(\tau)\, e^{-\frac{D_j^2(\tau)}{2}} \qquad j = 1, 2\,. \quad (13)$$

The solution of the homogeneous part of Eq. (11), indicated as $N^{\rm h}_{1,2}(\tau)$, can be of two types [11] and if the two beams have the same initial intensity one has

$$N^{\rm h}_j(\tau) = \frac{N_{\rm i}}{1 + \varepsilon\, N_{\rm i}\, (\tau - 1)} \qquad j = 1, 2 \qquad (14)$$

where $N_{\rm i} = N_{{\rm i},1} = N_{{\rm i},2}$ stands for the initial intensity.

The most general solution of Eq. (11) is obtained by adding to the general solution of the homogeneous equation a special solution of the inhomogeneous one. The latter can be found by assuming that such a solution can be developed as a power series of the small parameter ε (see Ref. [11] for more detail).

$$N^{\rm s}_j(\tau) = \sum_{m=0}^{+\infty} \varepsilon^m\, \mathcal{N}_{m,j}(\tau) \qquad j = 1, 2. \qquad (15)$$

INTEGRATED LUMINOSITY VS. FILL LENGTH

Assuming the simple case of equal intensities for both beams, it is possible to obtain

$$L_{\rm int}(\tau) = \int_1^\tau L(\tau)\, d\,\tau = \Xi\, N_{\rm i}^2\, \frac{\tau}{1 + \varepsilon\, N_{\rm i} \tau}, \qquad (16)$$

which can also be expressed as

$$L_{\rm norm}(\bar{\tau}) = \frac{L_{\rm int}(\bar{\tau})}{L_{\rm int}(+\infty)} = \frac{\bar{\tau}}{1 + \bar{\tau}}\,, \qquad (17)$$

with

$$L_{\rm int}(+\infty) = \frac{\Xi\, N_i}{\varepsilon} \qquad \text{and} \qquad \bar{\tau} = \varepsilon\, N_{\rm i}\, (\tau - 1)\,. \quad (18)$$

The very simple scaling law of $L_{\rm norm}(\bar{\tau})$ allows comparing experimental data from physics runs with different beam parameters, such as β^*, crossing angle, bunch intensity, and number of bunches. This treatment can be generalised to the case of different initial intensities [11].

To include pseudo-diffusive effects, the computations should be repeated using the solution based on the sum of components $N^{\rm h}_{1,2}(\tau)$ and $N^{\rm s}_{1,2}(\tau)$.

The cross terms depending on $N^{\rm h}_{1,2}$ and $N^{\rm s}_{1,2}$ contain a dependence on $\varepsilon\, N_i$ that allows developing them and retaining only the lowest order, i.e., the components of the functions $N^{\rm s}_{1,2}$ that do not depend on the small parameter

$$\begin{aligned} &L_{\rm norm}(\bar{\tau}, \tau) = L_{\rm norm}(\bar{\tau}) + \\ &+ \frac{\Xi}{L_{\rm int}(+\infty)} \int_1^\tau \left[N_1^h\, \mathcal{N}_{0,2} + N_2^h\, \mathcal{N}_{0,1} + \mathcal{N}_{0,1}\, \mathcal{N}_{0,2} \right] d\,\tau\,. \end{aligned} \qquad (19)$$

The final result is obtained by neglecting small terms of order εN_i and it reads

$$L_{\rm norm}(\bar{\tau}, \tau) = L^{\rm bo}_{\rm norm}(\bar{\tau})\, L^{\rm pd}(\tau), \qquad (20)$$

which shows that the pseudo-diffusive effects are a correction to the scaling obtained by including only the burn off. It is also clear that, under the simplifying assumption that the two beams behave similarly in terms of pseudo-diffusion, the term $L^{\rm pd}(\tau)$ reads

$$L^{\rm pd}(\tau) = \left[1 - e^{-\frac{D^2(\tau)}{2}} \right]^2. \qquad (21)$$

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ANALYSIS OF EXPERIMENTAL DATA

The models derived in the previous chapters will be applied to the analysis of the LHC performance data collected during Run I. Detailed information on this topics can be found in Refs. [12–15], while in Ref. [16] a preliminary analysis has been made, without focusing on the models to describe the luminosity and its time-evolution. Here, only the proton physics run will be considered and the data analysed can be found at [17]. As an example, the evolution of some key parameters is shown in Fig. 1 as a function of the fill number, which is an incremental integer number representing in a unique way the physics fill. The peak luminosity

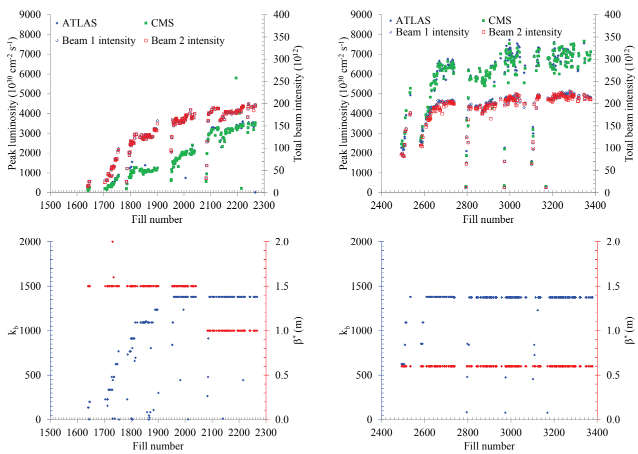

Figure 1: LHC performance during Run I. The evolution of peak luminosity and beam intensity (upper), and of β^* and k_b (lower) is plotted vs. the fill number. The data for 2011 (left) and 2012 (right) are shown.

$\mathcal{L}$ and the total beam intensities are shown in the upper row of Fig. 1, while in the lower row the evolution of β^* and k_b is also shown. Data for the 2011 and 2012 runs are shown in the left and right columns, respectively.

The data shown in Fig. 1 are also used in the following analysis of the luminosity evolution. Among the full data set available from [17], a selection has been considered including only the fills that resulted in successful physics runs, the so-called stable beams, of a total duration exceeding 10^3 s and featuring $N_{i,1,2} > 10^{13}$ p.

The difference in beam intensity at the beginning of a physics fill is at the level of few percent [16], thus justifying the assumption $N_{i,1} = N_{i,2}$.

The integrated luminosity delivered in a single physics fill against fill duration is shown in Fig. 2 for both 2011 (left) and 2012 (right) runs. The large spread observed for the 2011 data is simply due to the change of parameters, i.e., β^*, transverse beam emittance, beam intensity, occurred during the year, whereas the situation in terms of beam parameters has been much more stable during the 2012 run.

The filtered set of Run I data has been used to find a fitting pseudo-diffusive model based on Eqs. (20) and (21) and the results are shown in Fig. 3. Two models are represented in Fig. 3: one represents the scaling law (17) (contin-

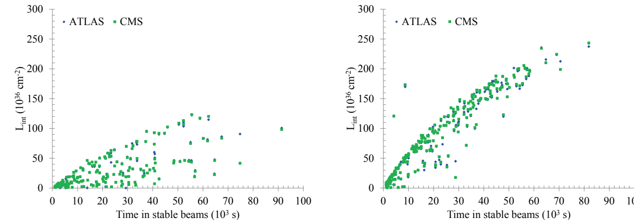

Figure 2: LHC performance during Run I. The integrated luminosity delivered in a single fill is shown as a function of the fill duration. The data for 2011 (left) and 2012 (right) are shown.

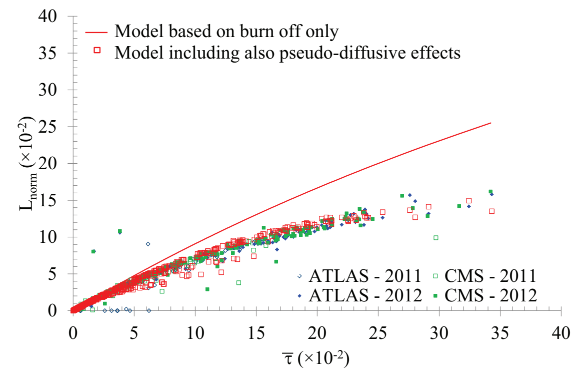

Figure 3: The normalised integrated luminosity defined from Eq. (17) plotted against the normalised time defined from Eq. (18) together with models based on burn off and pseudo-diffusive effect. The model has been fitted using the complete set of 2011 and 2012 data. A remarkable agreement between model and data is visible.

uous curve), while the second one represents the proposed scaling law (20) (open red markers). The first model departs from the real data away from the origin, where data and model feature the same derivative. The discrepancy is a clear sign of additional effects neglected in this simple model. On the other hand, a remarkable agreement is restored when the pseudo-diffusive effects are included in the model. In this case, even the behaviour of the data for large values of $\hat{\tau}$ is well-reproduced.

CONCLUSION

An effective model including pseudo-diffusive effects has been proposed for the evolution of the integrated luminosity of a circular collider. The model has been bench-marked against LHC data from Run I. As a next step, additional effects will be included in the proposed model. It is worth stressing that this approach based on simple analytic models is aimed at deriving scaling laws and, by no means, it should be considered as an alternative to numerical simulations based on detailed beam dynamics models.

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