IMPROVEMENTS IN THE OPTICS MEASUREMENT RESOLUTION FOR THE LHC

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Abstract

Optics measurement algorithms which are based on the measurement of beam position monitor (BPM) turn-by-turn data are currently being improved in preparation for the commissioning of the LHC at higher energy. The turn-by-turn data of one BPM may be used more than once, but the implied correlations were not considered in the final error bar. In this paper the error propagation including correlations is studied for the statistical part of the uncertainty. The confidence level of the measurement is investigated analytically and with simulations.

INTRODUCTION

BPMs are used to measure the turn-by-turn data of betatron oscillations, which are excited by an AC dipole [1]. The phase of this oscillation can be derived by a harmonic analysis of the turn-by-turn data at every BPM position using a modified version of SUSSIX [2, 3]. With the phase advance and transfer matrix in between three BPMs the β-function can be calculated at the positions of the three BPMs [4]. The
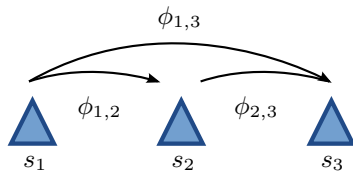

Figure 1: The phase advances $\phi_{i,j}$ in between three positions s_i are needed to derive the β-functions at those positions.

β-function at the positions s_i can be obtained with Eq. (1) where $\phi_{i,j}$ is the phase advance and $M_{mn(i,j)}$ are the transfer matrix elements from s_i to s_j, cf. Fig. 1. ϵ_{ijk} is the Levi-Civita symbol which allows for a compact notation of the three cases of deriving the Twiss parameters at the different BPMs. No summation over equal indices is implied.

$$\beta_i = \frac{\epsilon_{ijk} \cot(\phi_{i,j}) + \epsilon_{ikj} \cot(\phi_{i,k})}{\epsilon_{ijk} \dfrac{M_{11(i,j)}}{M_{12(i,j)}} + \epsilon_{ikj} \dfrac{M_{11(i,k)}}{M_{12(i,k)}}} \quad (1)$$

The usual measurement procedure in the 2012 run was to record at least three times the BPM turn-by-turn data for 2000 turns, while an oscillation is excited on the beam [5]. The phase advances are then averaged among the measurement files.

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Uncertainty of the Measured β-function

If Eq. (1) is used to derive the β-function, two phase advances between BPMs are used ($\phi_{i,j}$, $\phi_{i,k}$) in which the BPM (i) appears twice. This introduces a correlation which must be regarded in the error propagation. Furthermore the β-function at one position is calculated by combining three β-functions that are obtained from using different BPM combinations, which increases the contribution of correlations, because the same BPMs might be used more often. The error of the measured phase advance can be derived from the standard deviation

$$\sigma_{\phi_{i,j}} = t(n) \sqrt{\frac{1}{n-1} \sum_{k=1}^{n} \left(\overline{\phi_{i,j}} - \phi_{i,j,(k)} \right)^2} \quad (2)$$

where $t(n)$ is the t value correction from the Student t distribution, which compensates the underestimation of the uncertainty for a small sample size. During the LHC Run I the error was calculated from a normal standard deviation without the t correction and by dividing the sum by n instead of (n-1). This has been changed since the mean value of the phase advance is also obtained from the measurements, and there are only (n-1) degrees of freedom left for the calculation of the standard deviation. Table 1 shows $t(n)$ for different number of measurements, which shows that this correction is needed since due to limits in the beam time, the amount of measurements is always limited. The corre-

Table 1: Values for the t Correction for a Confidence Interval of 68.3%

Number of measurements	$t(n)$
2	1.84
3	1.32
4	1.20
5	1.15
10	1.06

lation between two phase advances which have one BPM in common, $\phi_{i,j}$ and $\phi_{i,k}$, depends on the uncertainty of the single phase ϕ_i at the common BPM. The error of the single phase ϕ_i is not known, because it cannot be compared among the measurement files since its value is arbitrary and may vary. However simulations show that the uncertainty of the phase measurement depends on the β-function at this position, $\sigma_\phi \sim \beta^{-\frac{1}{2}}$ cf. Fig. 2. Therefore the error of the single phase can be approximated by

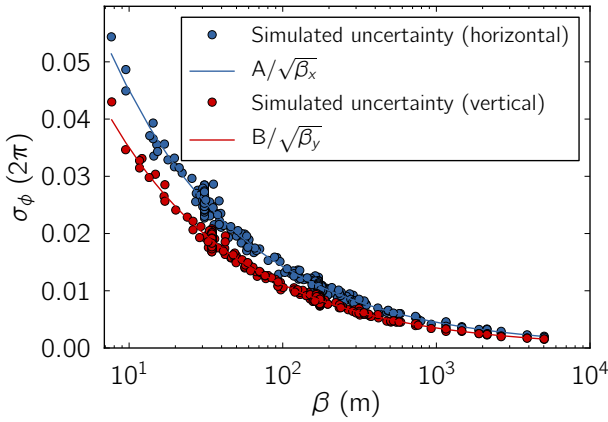$$\sigma_{\phi_i} = \sigma_{\phi_{i,j}} \left(1 + \frac{\beta_i}{\beta_j} \right)^{-\frac{1}{2}}. \quad (3)$$

Figure 2: Simulated single phase uncertainties depending on the β-function. The error has been derived from the variation of the phase when a Gaussian noise of 300 μm was added to the BPM turn-by-turn data which was obtained from tracking with MAD-X [6].

The correlation between two phase advances is then

$$\rho(\phi_{i,j}, \phi_{i,k}) = \frac{\sigma_{\phi_i}^2}{\sigma_{\phi_{i,j}}^2 \sigma_{\phi_{i,k}}^2}. \tag{4}$$

Let the phase at the probed BPM be ϕ_1, all other phase advances can be calculated with respect to this BPM. The elements of the correlation matrix for the different phase advances $\phi_{1,2}$ to $\phi_{1,n}$ are defined by

$$C_{i-1,j-1} = \frac{\partial \phi_{1,i}}{\partial \phi_1} \frac{\partial \phi_{1,j}}{\partial \phi_1} \rho(\phi_{1,i}, \phi_{1,j}) \sigma_{\phi_{1,i}}^2 \sigma_{\phi_{1,j}}^2, \tag{5}$$

which is $\sigma_{\phi_{1,i}}^2$ on the diagonal axis and $\sigma_{\phi_1}^2$ elsewhere. Using the transformation matrix

$$T = \begin{pmatrix} \frac{\partial \beta_1}{\partial \phi_{1,2}} & \cdots & \frac{\partial \beta_3}{\partial \phi_{1,2}} \\ \vdots & \ddots & \vdots \\ \frac{\partial \beta_1}{\partial \phi_{1,n}} & \cdots & \frac{\partial \beta_3}{\partial \phi_{1,n}} \end{pmatrix}, \tag{6}$$

the correlation matrix for the phases can be transformed to a correlation matrix for the three β-functions which are calculated from using different BPM combinations,

$$V = T^T C T. \tag{7}$$

The final β-function is then a weighted average of the three β_i

$$\beta = \sum_{i=1}^{3} w_i \beta_i \tag{8}$$

where the weights can be calculated from the inverse correlation matrix

$$w_i = \frac{\sum_{k=1}^{3} V_{ik}^{-1}}{\sum_{k=1}^{3} \sum_{j=1}^{3} V_{jk}^{-1}} \tag{9}$$

This equation replaces the simple average introduced in [7]. The uncertainty for this measurement is

$$\sigma_\beta^2 = \sum_{k=1}^{3} \sum_{j=1}^{3} w_j w_k V_{jk} \tag{10}$$

Simulation of the Uncertainties

In order to determine the requirements on the number of measurements for a reasonable error bar, simulations of the optics measurement have been performed. These simulations are furthermore a test of the correct implementation of the equations in the optics analysis code. Particles were tracked for 2000 turns using MAD-X, while at the beginning a kick with an amplitude of 1 mm was applied to the particle. The oscillations of the orbit at the BPM positions were recorded and afterwards a Gaussian noise of 300 μm was added. This has been done to create 500 sets of BPM turn-by-turn data, which corresponds to 500 measurements.

Since in contrast to a real measurement, in this simulation the phase at each BPM is comparable, it is possible to derive the uncertainty of the phase for each BPM position from its variation. As the uncertainties of the single phases and also of the phase advances are known, they were used directly in Eq. (4) to create the correlation matrix. The afore described error propagation was applied and the β-function derived according to Eq. (8), with its uncertainty according to Eq. (10).

The distribution of the β-function in these 500 data sets has been fitted to a Gaussian for each BPM. The value of the σ from the fit was then compared to calculated uncertainties of the β-function, cf. Fig. 3. The calculated values of the
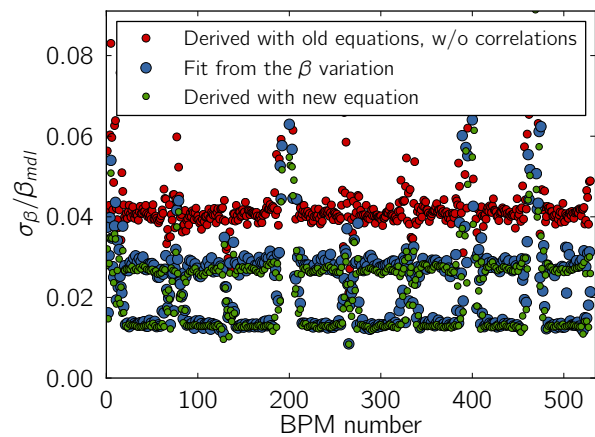

Figure 3: Relative uncertainty of the β-function derived in the error propagation compared to a fit of the variation of calculated β-functions.

uncertainty fit well to the expected value from the variations of the β-function, which is not the case for the old equations for the error calculation. In this plot one can furthermore see that most of the points are located at two levels. This is due to the fact that the BPMs in the arcs, which are most of the

05 Beam Dynamics and Electromagnetic Fields

D01 Beam Optics - Lattices, Correction Schemes, Transport

BPMs, are alternating between two β values, and the larger β-function can be measured with a higher relative precision.

Uncertainty of the Error Bar

The study of the uncertainty of the error bar gives an important insight in the accuracy of the measurement method and will give a recommendation of how many measurements need to be taken for a reliable result. Simulated turn-by-turn data with the same noise level as before were used for this analysis. Several measurement files were now used together for one analysis, which means that the error of the phase advance is now to be calculated from Eq. (2) by using the standard deviation of the phase advances from the different measurements. This was done for the range of using two to ten measurement files together, and repeated for the 500 measurement files. The deviations of the calculated error bar of the β-function to the uncertainty that is calculated from the known phase uncertainty is fitted with a Gaussian distribution. The σ of this fit is shown in Fig. 4 as a distribution for all BPMs. This plot shows that the precision of the error
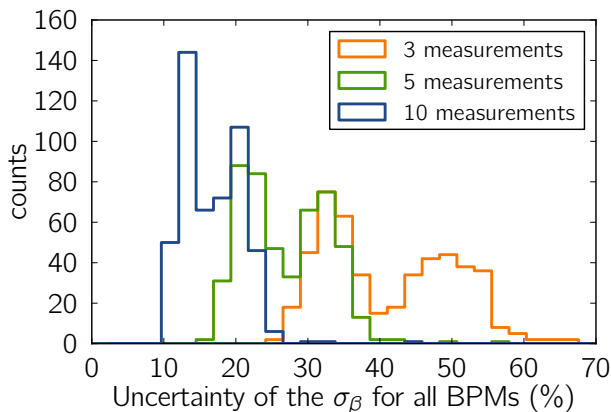

Figure 4: The distribution shows for all BPMs the width of the deviation of the derived uncertainty of the β-function compared to the calculated error bar from the known phase uncertainty.

bar is poor if only three measurements are used, reaching an uncertainty of up to 60%. Significantly more precise is the error bar when using five measurements, where its uncertainty varies from 20-35%. This number further decreases when more measurements are used, for ten measurements the uncertainty is only at 10-22%.

The mean value of fitting the deviation of the error bar to the real uncertainty with a Gaussian distribution shows if either the error bar is biased towards smaller or larger values, which is shown in Fig. 5. One can see in this plot that for three measurements the distribution is not centered around zero, but at a positive value, which means that there is tendency to overestimate the error bar. Also the width of the distribution is rather large when using less than five measurements. This also shows that the t value correction is useful, as without it the error bars were biased to underestimate the real error.
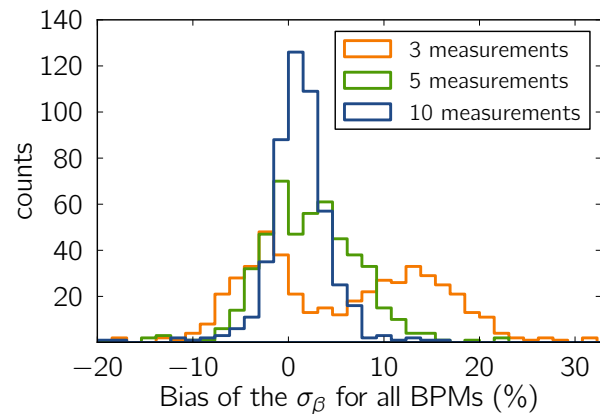

Figure 5: The distribution shows for all BPMs the mean of the deviation of the derived uncertainty of the β-function compared to the calculated error bar from the known phase uncertainty.

CONCLUSION

An improved method for deriving the β-function from BPM turn-by-turn data has been presented which makes use of the individual uncertainties of the measured phases at each BPM. It also takes correlations into account when the phase at one BPM is used more often, if the β-function at one location is derived by using different BPM combinations. This will allow for a more precise measurement of the β-function. The presented error bar is based on the statistical uncertainty of the measured phase. Systematic errors arising from model uncertainties will have to be added to the correlation matrix, which is not shown here. The study of the statistical error bar shows that it is recommended to take at least five measurements for the analysis.

ACKNOWLEDGMENT

REFERENCES

[1] J. Serrano and M. Cattin, "The LHC AC Dipole system: an introduction", CERN-BE-Note-2010-014 (CO) (2010).

[2] R. Bartolini and F. Schmidt, "SUSSIX: A Computer Code for Frequency Analysis of Non–Linear Betatron Motion", CERN SL/Note 98-017 (AP) (1998).

[3] T. Bach and R. Tomás, "Improvements for Optics Measurement and Corrections Software", CERN-ACC-NOTE-2013-0010 TECH (2013).

[4] P. Castro-Garcia, "Luminosity and beta function measurement at the electron-positron collider ring LEP", PhD thesis, 1996.

[5] R. Tomás et al., "Record low beta beating in the LHC", Phys. Rev. ST Accel. Beams **16**, 091003 (2013).

[6] MAD-X, http://cern.ch/mad

[7] R. Tomás et al., "First β-beating measurement and optics analysis for the CERN Large Hadron Collider", Phys. Rev. ST Accel. Beams **12**, 081002 (2009).