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INITIAL ANALYSIS OF THE 4D TRANSFER MAP IN THE EMMA NON-SCALING FIXED FIELD ALTERNATING GRADIENT ACCELERATOR*

C.S.Edmonds†, A.Wolski, University of Liverpool, UK/Cockcroft Institute, UK
D. Kelliher, S. Machida, ASTeC, STFC Rutherford Appleton Laboratory, Didcot, UK
B.D.Muratori, ASTeC, Sci-Tech Daresbury, Warrington, UK/Cockcroft Institute, UK

Abstract

The EMMA non-scaling Fixed Field Alternating Gradient accelerator is a ring consisting of 42 quadrupole pairs [1]. The dipole fields that guide particles around the ring are produced by offsetting the quadrupoles from a reference axis. In the ideal case, first-order 4D transfer maps will describe the progression of particles in transverse phase space. This contribution describes the use of experimental data to calculate the 4D transfer map for EMMA at several different momenta, and makes comparisons with the maps produced through simulation.

ESTIMATION OF TWISS PARAMETERS

Measurements made in EMMA with one of the beam position monitors (BPM) show a betatron oscillation that decays within tens of turns [2]. The cause of the observed decay is decoherence. All of the particles within a bunch that is injected with some transverse offset from the closed orbit position can initially be described as being at the same phase of betatron oscillation. The chromaticity of the lattice coupled with momentum distribution of the bunch leads to the particles having a range of phase advances per turn. As the particles move out of phase with one another, the mean location of particles within the bunch (as is measured by a BPM) will tend towards the closed orbit.

The rapid decoherence of a particle bunch within EMMA adds difficulty in measuring the Twiss parameters. An approach developed for estimating the Twiss paramaters first involves applying a rotation to the measured phase space variables at the n^{th} turn, such that

$$g_n = e^{i\Psi_n} f_n, \tag{1}$$

where Ψ_n is the mean total phase advance of particles within a bunch since turn zero, and f_n are the normalised phase space coordinates,

$$f_n = \hat{y}_n + i\hat{p}_{y,n},$$

where $\hat{y}_n$ and $\hat{p}_{y,n}$ are found as

$$\hat{y}_n = \frac{y_n}{\sqrt{\beta_y}},$$

$$\hat{p}_{y,n} = p_{y,n}\sqrt{\beta_y} + \frac{\alpha_y y_n}{\sqrt{\beta_y}}. \tag{2}$$

The coordinates y_n and $p_{y,n}$ of the bunch centroid can be expressed in a way that takes into account the momentum distribution of a bunch:

$$y_n = \sqrt{2\beta_y J_y}\int_{-1/2\xi_y}^{1/2\xi_y} \cos(\Psi_n + \psi_n(\delta) + \phi_0)\Phi(\delta)d\delta,$$

$$p_{y,n} = -\sqrt{\frac{2J_y}{\beta_y}}\int_{-1/2\xi_y}^{1/2\xi_y} (\alpha_y \cos(\Psi_n + \psi_n(\delta) + \phi_0) + \sin(\Psi_n + \psi_n(\delta) + \phi_0))\Phi(\delta)d\delta,$$

with J_y being the action of the bunch (assumed the same for all particles in the bunch), ξ_y the chromaticity of the lattice, $\psi_n(\delta)$ being the difference in total phase advance between a particle at the mean momentum (P_0) and a particle with a fractional offset of momentum from the mean ($\delta = \Delta P/P_0$). ϕ_0 is the initial phase of the betatron oscillation, and $\Phi(\delta)$ is the momentum distribution of the bunch.

By taking the ratio of the imaginary to real parts of g_n,

$$\frac{\Im(g_n)}{\Re(g_n)} = -\frac{\int_{-1/2\xi_y}^{1/2\xi_y} \sin(\psi_n(\delta) + \phi_0)\Phi(\delta)d\delta}{\int_{-1/2\xi_y}^{1/2\xi_y} \cos(\psi_n(\delta) + \phi_0)\Phi(\delta)d\delta}$$

and given that

$$\int_{-1/2\xi_y}^{1/2\xi_y} \sin(\psi_n(\delta))\Phi(\delta)d\delta = 0,$$

when the momentum distribution, $\Phi(\delta)$, is symmetric or the total number of turns is small, then the ratio of imaginary to real parts of g_n should be constant with respect to turn number and given by

$$\frac{\Im(g_n)}{\Re(g_n)} = -\tan(\phi_0). \tag{3}$$

Experimentally, two BPM's separated by only a drift space can be used to obtain y_n and $p_{y,n}$ of a bunch centroid and applying the NAFF [3] correlator to the measurements from a single BPM can be used to find the modal phase advance per turn, μ_y, with good precision using several tens of data points. For calculating the normalised phase space coordinates of Eq. 2, values of α_y and β_y found through simulation can be used as initial estimates. If the simulated Twiss parameters match the experimental parameters, then applying Eq. 3 will return a value that is independent of n, otherwise, a fitting routine that seeks to minimise the standard deviation of g_n over a number of turns and has α_y and β_y as free parameters can be used to locate experimental estimates of the Twiss parameters at the locations of the BPMs.

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† chris.edmonds@cockcroft.ac.uk

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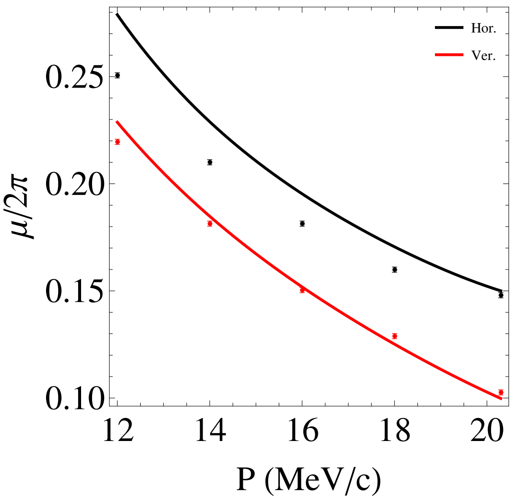

Figure 1: Experimentally measured betatron tune represented by points, whilst solid lines give tunes found by tracking through a fieldmap. The source of the difference between experimental measurement and simulation for the horizontal tune is still to identified.

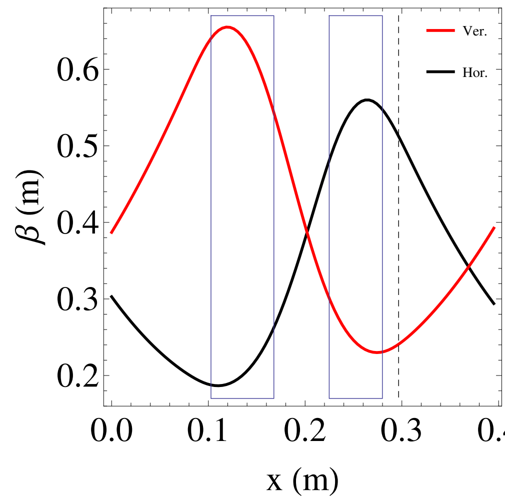

Figure 2: Horizontal (black) and vertical (red) beta function with respect to longitudinal position in an EMMA cell for an electron of momentum 15 MeV/c. The blue rectangles mark the position of the defocusing (left) and focusing (right) quadrupoles, whilst the black dashed line marks the position of the reference BPM.

4D MAP CALCULATION

A 2D transfer map may describe the linear progression of a particle's phase space coordinates along a transverse axis following one revolution in the accelerator. The map can be written in matrix form as:

$$M = \begin{pmatrix} \cos\mu_y + \alpha_y \sin\mu_y & \beta_y \sin\mu_y \\ -\gamma_y \sin\mu_y & \cos\mu_y - \alpha_y \sin\mu_y \end{pmatrix}.$$

In the absence of coupling between transverse planes, the Twiss parameter and tune measurements alone can be used to calculate the 4D transfer map. The EMMA lattice, which consists of 84 quadrupole magnets that are positioned in defocusing/focusing pairs around a 16 m circumference, has transverse magnet alignment errors (from survey) of 25-50 μm and is anticipated to have magnet rotation errors of a standard deviation up to 100 μrad. The rotational alignment errors will give non-zero values in the coupling terms of the 4D map. Applying the methods described in the previous section allows for a 4D transfer map without coupling terms to be determined.

For the purpose of comparison with experimental results, a field map model of an EMMA cell has been computed using the Opera design suite [4]. The variation of the Twiss parameters along the length of the cell and the cell tunes have been calculated using the Zgoubi tracking code [5]. Figure 1 shows that field map tracking yields vertical tunes that are consistent with existing experimental measurements [1], however differences between the simulated and experimental horizontal tune are still to be explained. Marked on a plot of β vs. longitudinal cell position (Fig. 2) is the position of a reference BPM at which the Twiss parameters will be measured experimentally, BPMs at this location are separated from a second BPM by only a drift space in 17 of EMMA's 42 cells. Two BPMs configured as such allow for the transverse momentum, p_y, of a bunch centroid to be calculated as

$$p_{y,n} \approx \frac{y_n^{(2)} - y_n^{(1)}}{L},$$

where $y_n^{(1)}$ and $y_n^{(2)}$ are the transverse positions of the bunch centroid at BPMs 1 and 2 respectively and L is the length of drift space separating the BPMs.

The momentum dependence of the Twiss parameters at the location of the reference BPM has been found through simulation, and is shown in Figs. 4 & 5.

PRELIMINARY INVESTIGATION OF COUPLING

Rotational misalignment of quadrupoles should be the dominant source of coupling between transverse axis in the EMMA accelerator. Analysis of the betatron frequencies found within BPM data combined with simulation studies may give insight into the magnitude of the rotational errors. As an example, experimental data taken for 17.6 MeV/c particles is considered. During measurements, the bunch centroid was oscillating with amplitudes of approximately 4 mm and 2 mm along the vertical and horizontal axis respectively at the position of the reference BPM. Figure 3 is a plot of the intensity of the NAFF correlator vs. betatron tune; it can be seen that a second (smaller) peak exists in the horizontal frequency spectrum at a frequency corresponding to that of the vertical tune. Analysis of data at 5 further BPMs located at the same position with respect to the quadrupole magnets but in different cells produced the same result, implying that misalignment of the BPM was not the main contributing factor. Preliminary simulations

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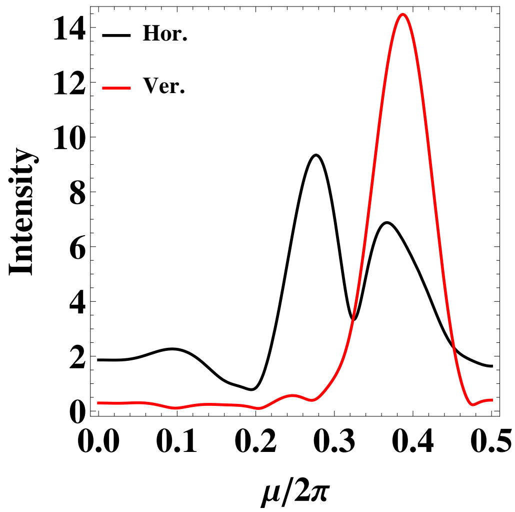

Figure 3: Intensity of the NAFF correlator vs. betatron tune. The second peak in the horizontal plot is potentially indicative of coupling between the transverse axes. The second peak being present for horizontal and not vertical data may be explained by the larger amplitude of the vertical oscillation; this has been reproduced by simulation, however further analysis is required.

carried out using a hard edge representation of EMMA indicate that in order to produce coupling to the extent of that suggested by Fig. 3, then rotational alignment errors with a standard deviation at least an order of magnitude higher than the anticipated 100 μrad are required.

RESULTS & CONCLUSIONS

Calculation of the Twiss parameters based upon experimental data has been carried out for four momenta, with a number of injection cycles analysed for each momentum in order to obtain a statistical error on the results. Figures 4 and 5 show the values of the Twiss parameters gained experimentally and their associated errors as well as the values for the Twiss parameters found through simulation. These preliminary results show that the measured Twiss parameters are consistent with those found by tracking through a field map, whilst comparison of the measured betatron tune with the field map values indicate that further work is required to understand differences in horizontal tune.

More work is also required to better understand coupling in EMMA; Figure 3 and simulation suggest that coupling coefficients in the 4D map will be larger than first expected.

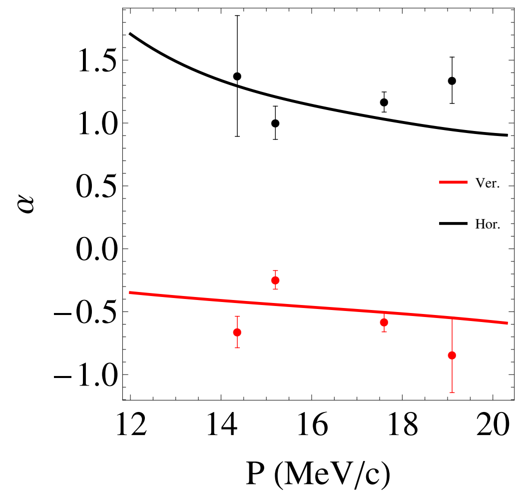

Figure 4: Twiss parameter α against particle momentum. Solid line gives the relationship obtained by tracking particles through a field map model, points are from experimental measurement in EMMA.

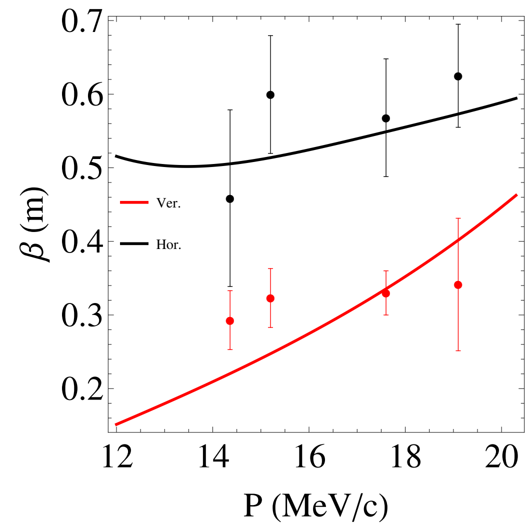

Figure 5: Twiss parameter β against particle momentum. Solid line gives the relationship obtained by tracking particles through a field map model, points are from experimental measurement in EMMA.

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