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SHAPE OPTIMIZATION FOR THE ESRF II MAGNETS

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Abstract

Magnets are a keystone of the ESRF upgrade programme. The specifications of the magnets of the ESRF II lattice are stringent: high gradients, extended Good Field Region (GFR) and vertical gaps large enough for the X-ray beam ports. The magnet design approach is presented here. Shape optimization of the magnet poles is systematically used. The magnet design is treated as an ill-posed, non linear, constrained problem. Iterative algorithms have been developed; the algorithms converge in less than 10 iterations, leading to very short computation time. This design method has been applied to high gradient quadrupole magnets. The shape optimization leads to original pole profiles.

INTRODUCTION

A major upgrade of the ESRF storage ring is under study. A lattice based on a 7-bent achromat is being developed [1]. This new lattice relies on sophisticated magnet designs: high gradient quadrupoles, with gradients up to 90 T/m, dipoles with longitudinal gradient, combined dipole quadrupoles, etc. In this context, it was necessary to go beyond the classical hyperbolic pole based magnets, and to look further in details. This paper will focus on the design of the quadrupole magnets, and more specifically to the optimization of their pole shape. The complete design of a quadrupole magnet is out of the scope of this article.

Computer-aided magnet optimization was introduced by Halbach in 1967 [2]. Optimization methods have been applied to the two dimensional design of the accelerator magnets [3] and MRI magnets [4] since the 80's. Hyperbolic poles with smooth optimized shims have been used in some of the 3rd generation light sources [5]; a good review of mathematical optimization techniques for magnet design has been written by Russenschuck [6]. User friendly optimization toolboxes are now included in some commercially available magnet design codes.

The magnet design method developed at the ESRF combines the Radia software [7], which is a boundary integral method for field computations, with a nonlinear descent method and a simple parameterization of the pole shapes; details are given in the next sections. The main advantage of this method is its short computation time. On top of that, some operations in the optimization routine can be easily parallelized, leading to extremely time efficient pole shape optimization.

POLE SHAPE OPTIMIZATION

Simulation Tools

A few blocks are needed for pole shape optimization: a magnetic model of the object under design, allowing magnetic field computations, a set of parameters for this model, and a set of optimization functions. The parameters and the optimization functions have been described in Mathematica language. The Radia magnet simulation software is used for the field computation. It is interfaced with Mathematica. The Radia software has been developed in the ESRF Insertion Device lab and has been largely used for undulator, wiggler and multipole magnet design [8-11]. This software does not rely on a Finite Element Method as most of the field computation codes do, but on a Boundary Integral Method. The magnets are described as a set of field sources, *i.e.* magnetization or currents. In the present version of Radia, elementary magnetized objects have a uniform magnetization and are modeled as equivalent magnetic charges. The magnetic field and its integral along a straight line can be computed for each field source, at any point, using analytical formulae. The Radia software is particularly time efficient for integrated field computations, since the integrated field is computed directly from each field source, and not as the sum of the fields computed at a large number of points. Moreover, a good estimation of the integrated field can be obtained with a small number of longitudinal elements. This makes it a good candidate for 3D accelerator magnet optimization.

Regularized Descent Method

Let us assume that the magnet has been partially optimized, *i.e.* its main dimensions, bore radius, coils, outer pole shape (excluding the "hyperbolic" part), etc. are set. The problem then is to find a convenient shape for the pole, leading to the best field quality at some specified working points. As the optimization procedure starts with a "not so bad" magnet, it is assumed that finding the local minimum of a cost function is sufficient. This assumption is of prior importance because it allows us to restrict our study to descent methods and to exclude time consuming metaheuristics such as simulated annealing or genetic algorithms.

The first step towards an optimization method is to define a cost function. Harmonic field expansions are an efficient way to express the field strengths and the field errors with a small number of coefficients. It is a common use to define a complex 2D magnetic field $B = B_Y + iB_X$ where B_X is the transverse component and B_Y the vertical component of the field. In 3D magnetic modeling, B_X and B_Y are integrated other the longitudinal direction. For this 2D field and outside the iron material, the Maxwell equations are equivalent to the Cauchy-Riemann equations and the complex field B is an analytic function

$$B = B_N \sum_{n=1}^{\infty} \left(b_n + i\, a_n\right)\left(z/\rho_0\right)^{n-1} \qquad (1)$$

where $z = x + iy$, ρ_0 is the reference radius, and $(a_N, b_N) = (0,1)$ for the main multipole; the multipole strength is B_N / ρ_0^{N-1}. The a_n and b_n coefficients can be determined from the Fourier expansion of the field along a circle; they are commonly referred to as circular multipole coefficients. Even if circular multipoles are widely used by magnet designers, there exist alternative definitions of field multipoles. In particular, elliptic multipoles [12] are of interest for field optimization over ellipic Good Field Region (GFR) which may be encountered in dipole or combined function magnet design. If one restricts the summation in Eq. (1) to its first terms, one obtains

$$\mathbf{B} = \mathbf{TC}$$

where $\mathbf{B}$ is the vector which contains the field computed at z_m, with $1 \le m \le M$, $\mathbf{C}$ contains the N first complex multipole coefficients, and $\mathbf{T}$ is a $M \times N$ matrix. The T_{mn} coefficients depends on the points z_m, on the field components computed (*i.e.* transverse and vertical, or radial and tangential, etc.) and of the type of multipole expansion used (mainly circular or elliptic). If the field is known at points z_m, the multipole coefficients are obtained from the pseudoinverse $\mathbf{T}^+$ of the matrix $\mathbf{T}$. (This matrix formalism was originally developed for magnetic measurements, see the reference [13] for details.)

Let $\mathbf{C_0}$ be a target multipole vector: for a quadrupole and with circular multipoles, one gets $\mathbf{C_0} = (0, B_2, 0 \ldots 0)$. An error vector can be defined as $\boldsymbol{\varepsilon} = \mathbf{C} - \mathbf{C_0} = \mathbf{T}^+\mathbf{B} - \mathbf{C_0}$. The norm ε of this vector is a good cost function for magnet optimization.

Without constraints, pole shape optimization algorithms tend to enlarge the poles outside acceptable values; this may cause pole gap closure or other issues. A simple way to avoid problems of this kind is to implement a barrier function which has a negligible value for pole gap above a user defined limit, and a large value in the other case. One just has to stack this value in the error vector $\boldsymbol{\varepsilon}$.

This optimization problem is non-linear: the field variation at a given point doesn't scale with the variation of the pole shape. An intuitive approach consists of linearizing the problem (*i.e.* computing the Jacobi matrix), and to iterate a few times. This corresponds to the so-called Gauss-Newton algorithm.

The cost function ε^2 is minimized with a modified Gauss-Newton algorithm. First, let ζ_n be a generic magnet parameter, and $\boldsymbol{\zeta}$ the vector of parameters to optimize (a simple and efficient parameterization of the pole profile will be suggested in the coming paragraphs). The coefficients of the Jacobi matrix $\mathbf{J}$ are $J_{mn} = \partial\varepsilon_m / \partial\zeta_n$. The k^{th} iteration of the Gauss-Newton algorithm is

$$\begin{aligned} \boldsymbol{\zeta}_{\mathbf{k+1}} &= \boldsymbol{\zeta}_{\mathbf{k}} - \left(\mathbf{J}_{\mathbf{k}}^{\mathbf{T}}\mathbf{J}_{\mathbf{k}}\right)^{-1}\mathbf{J}_{\mathbf{k}}^{\mathbf{T}}\boldsymbol{\varepsilon}_{\mathbf{k}} \\ &= \boldsymbol{\zeta}_{\mathbf{k}} - \mathbf{J}_{\mathbf{k}}^{+}\boldsymbol{\varepsilon}_{\mathbf{k}} \end{aligned} \tag{2}$$

where $\mathbf{J}_{\mathbf{k}}^{+}$ is the pseudoinverse of the Jacobi matrix at iteration *k*.

If one of the parameters is modified, one can assume that the field variations along the boundary of the GFR will be smooth: a spike on the pole profile translates to a "bump" on the field amplitude. It makes the Jacobi matrix ill-conditioned, and the Gauss-Newton algorithm diverges rapidly. The Singular Value Decomposition is a powerful tool to overcome this problem. The matrix $\mathbf{J}$ can be written as

$$\begin{aligned} \mathbf{J} &= \mathbf{U\Sigma V^T} \\ &= \sum_i \mathbf{u_i}\sigma_i\mathbf{v_i}^{\mathbf{T}} \end{aligned}$$

where $\sigma_1 \ge \sigma_2 \ge \ldots \ge 0$ are the singular values of $\mathbf{J}$, $\mathbf{u_i}^{\mathbf{T}}\mathbf{u_j} = \delta_{ij}$ and $\mathbf{v_i}^{\mathbf{T}}\mathbf{v_j} = \delta_{ij}$. A simple way to compute a friendly, *regularized* pseudoinverse of $\mathbf{J}$ is to truncate its SVD: $\mathbf{J}^+ = \mathbf{V\Sigma^+U^T}$ where $\mathbf{\Sigma}^+$ is a diagonal matrix with diagonal elements $1/\sigma_i$ if $\sigma_i/\sigma_1 > r$, 0 elsewhere. In practice, good pole shape optimization results are obtained with $10^{-3} < r < 10^{-2}$. Equation (2) becomes

$$\boldsymbol{\zeta}_{\mathbf{k+1}} = \boldsymbol{\zeta}_{\mathbf{k}} - \mathbf{J}_{\mathbf{k}}^{+\mathbf{r}}\boldsymbol{\varepsilon}_{\mathbf{k}},$$

where $\mathbf{J}^{+\mathbf{r}}$ denotes the regularized pseudoinverse of $\mathbf{J}$. (The Tichonov regularization is another classical approach. In that case, the $\mathbf{\Sigma}^+$ diagonal elements are multiplied by $\sigma_i^2/\left(\sigma_i^2 + \lambda^2\right)$, where λ is a regularization parameter which constrains the norm of the solution. The widely-used Levenberg-Marquard algorithm relies on Tichonov regularization. See for instance reference [14].)

Pole Profile Parameters

A natural way to parameterize the pole profile is to introduce deviations from the "standard" hyperbolic profile: $X_k = x_k + \xi_k$ and $Y_k = y_k + \psi_k$, where $1 \le k \le K$ and $y_k = \rho^2/(2x_k)$. These deviations can be expressed as a sum of smooth functions, *e.g.* Legendre polynomials:

$$\begin{aligned} \xi_k &= \sum_{l=1}^{L}\alpha_l P_l\left(2\frac{k-1}{K-1} - 1\right) \\ \psi_k &= \sum_{l=1}^{L}\beta_l P_l\left(2\frac{k-1}{K-1} - 1\right). \end{aligned}$$

where α_l and β_l are the pole shape parameters and the P_l are the Legendre polynomials. These expressions for the pole deviations have some advantages. The number of parameters to optimize does not depend on the discretization of the profile, but on the polynomial order of the pole deviations, *i.e.* on the smoothness of the poles. The pole smoothness is handled by the maximum polynomial order L. Moreover, it is observed that this parameterization leads to a better conditioning of the Jacobi matrix.

QUADRUPOLE MAGNETS

Context

The high gradient quadrupoles designed for the ESRF II lattice have a nominal gradient of almost 90 T/m, with gradient errors $\Delta G/G \leq 10^{-3}$ within a 7 mm radius GFR. The minimum pole gap was set to 11 mm, leaving space for the vacuum chambers and the X-ray ports. The magnet length is 500 mm and the material is low carbon steel AISI 1006.

As the tuning range of these magnets is ±5 %, the field was optimized at only one excitation.

The field quality criterion is defined from the circular integrated field harmonics, as described above. The magnet is modelled in 3D with the Radia software. The end effects have an impact on the integrated multipoles, and this effect is corrected by the pole shape, all along the magnet. The optimization of the pole ends is out of the scope of this paper; it is clear that the conventional pole chamfering does not work with non-hyperbolic (and possibly non-convex) pole profiles.

Optimization Results

Optimized quadrupole profiles have been computed in two cases: with a weak constraint on the pole gap (vertical gap above 9 mm) and with a stronger constraint (gap above 11 mm). The pole profiles are shown in figure 1.

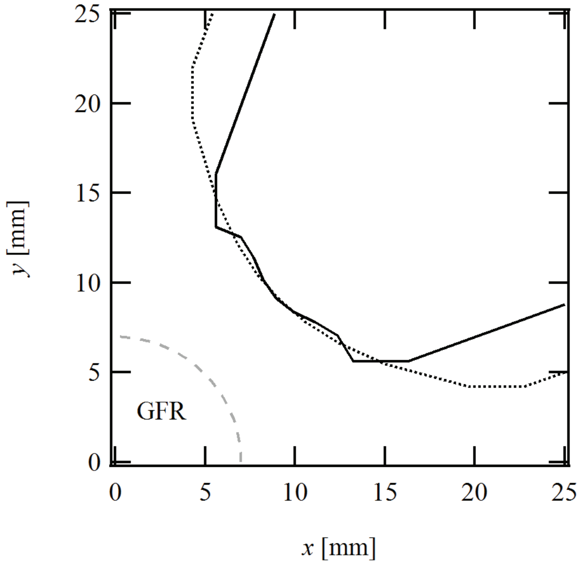

Figure 1: Optimized pole shape. Solid line: 11 mm gap between poles. Dotted line: 8 mm gap between poles. Dashed line: Good Field Region. For the 11 mm gap profile, the main systematic multipoles are $b_6 = -0.8810^{-4}$ and $b_{10} = 0.2410^{-4}$ at 7 mm. The gradient (integrated gradient normalized by magnetic length) is $G = 89.3$ T/m.

The computations have been done on the ESRF computer cluster with 10 CPUs. A very good field quality was obtained with 5 iterations and 10 parameters ($L = 5$ for horizontal and vertical displacements). With a reasonable magnet discretization, the computation time was 15 minutes. The computation time would be more or less 150 minutes on a single CPU, which is still short for a 3D optimization. The optimized pole profile has been cross-checked by simulations with an increased number of longitudinal elements. Table 1 shows that the errors are stable with finer longitudinal segmentations.

Table 1: Computation Time and Norm of the Multipole Errors at 7 mm, for an Increasing Number of Longitudinal Elements

Number of longitudinal elements	**Computation time**	$\left(\sum_{n=1}^{20} b_n^{2}\right)^{1/2}$
12	1 min 59 s	$0.82\ 10^{-4}$
20	4 min 41 s	$1.02\ 10^{-4}$
40	54 min 24 s	$1.10\ 10^{-4}$

CONCLUSION

Pole shape optimization is not a new idea. Yet, advances in field computations and optimization algorithms, combined with the development of computer clusters, open the way to systematic 3D optimization. The development of very fast, user friendly 3D models and optimizers changes the design approach. It is now possible for a designer to be field quality carefree, and to check almost immediately how pole shaping decreases the higher order multipoles.

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