MODELLING OF A SHORT-PERIOD SUPERCONDUCTING UNDULATOR

B.J.A. Shepherd[#], J.A. Clarke, ASTeC, STFC Daresbury Laboratory, Warrington, UK

E.C. Longhi, Diamond Light Source, Oxfordshire, UK

V. Bayliss, T.W. Bradshaw, STFC Rutherford Appleton Laboratory, Oxfordshire, UK

Abstract

STFC, in collaboration with Diamond Light Source, are designing and building a 15.5 mm period, 1.26 T superconducting undulator. This paper describes the modelling of the undulator, using Radia and Opera. Extensive numerical modelling has been carried out to simulate the effect of manufacturing tolerances on the quality of the magnetic field, in order to meet the demanding overall 3° rms phase error specification.

INTRODUCTION

STFC and Diamond Light Source are collaborating to design and build a superconducting undulator for the UK [1]. The undulator will be installed on Diamond, the UK's 3 GeV synchrotron light source. The main proposed parameters are listed in Table 1. Superconducting undulators have the potential to give higher on-axis peak fields than any other technology, but potentially have very challenging requirements for field quality.

Table 1: Main Undulator Parameters

Parameter	Value	Units
Magnet Length	2.0	m
Period	15.5	mm
Peak Field on-axis	1.266	T
Undulator K Parameter	1.83	
Required RMS Phase Error	<3	°
Magnet Gap	7.4	mm

The low phase error requirement will enable the undulator to generate high-brightness photon beams from 7 keV and above. However, this requirement implies extremely tight tolerances on the manufacturing of the formers and the winding of the coils. This paper describes the work that has been done to estimate the values of these tolerances, and whether it is possible to meet the phase error specification.

The undulator is made from fourteen formers, each of which are made from solid blocks of steel with machined grooves, into which the superconducting coils are wound (in eleven layers of six turns each). The grooves are filled with a polyester insulator (Isopon) before re-machining. The coils are wound into the grooves, and finally compressed to be flush with the pole surface.

_____

#ben.shepherd@stfc.ac.uk

MAGNET MODELLING

To determine the magnet geometry, models were built in Radia [2] and Opera-3D [3]. These models demonstrated the capability to build an undulator that met the requirements for peak field and K parameter.

Modelling of manufacturing errors is more demanding. Small changes in the position or size of poles or coils, integrated over the 2 m length of the undulator, can give rise to large trajectory errors, large phase errors, and hence a significant degradation in the quality of the output photon beam. An accurate model including realistic errors on each part of the device must have a large number of elements and must dispense with symmetry. This leads to very long computation times to solve such models. By their nature, manufacturing errors are random and therefore it is necessary to simulate a large number of undulators with random errors to enable good statistics to be gathered.

An alternative method was used to simulate errors, which enabled many simulations of random undulators to be run in a very short time. The numerical method is similar to that used by Bahrdt and Ivanyushenkov [4], but many more different types of errors are considered here.

ERROR SIGNATURES

Radia was used to generate a short (32 period) model of the undulator with no errors and no symmetries. Following that, a model containing a single error was created. The difference between the two field maps gives us the *error signature*:

$$\delta B(z) = B_{err}(z) - B_{ideal}(z) \qquad (1)$$

The shape of the error signature depends on the type of error under consideration – some are single-peaked, some double-peaked and some have a more complex structure. The types of errors considered were:

- **Groove position errors**. Each groove is machined with respect to a single datum, so tolerances cannot stack up. An error in the groove position affects the width of the neighbouring poles.
- **Groove width errors**. An error in the width of the machined groove in the steel former, affecting the width of the neighbouring poles, but not the eventual width of the coil.
- **Coil width errors**. An error in the width of the machined groove in the Isopon former. A larger groove allows more space for the coil stack, reducing the effective current density.

- **Groove height errors**. This can be in the steel or Isopon former, and again has the effect of altering the size of the coil stack and the current density.
- **Former alignment**. The separate formers must be aligned longitudinally and transversely on the supporting I-beam, ensuring the peak field and period is uniform along the undulator.
- **Pole height**. This refers to the machined flatness of the pole face along the length of the former.
- **Gaps between formers**. The plan is to design in small gaps between the formers to allow them to be correctly aligned. The periodicity will be maintained, but peak heights will be affected in several locations.

A schematic of part of the undulator, showing the first four of the errors listed here, is shown in Fig. 1.
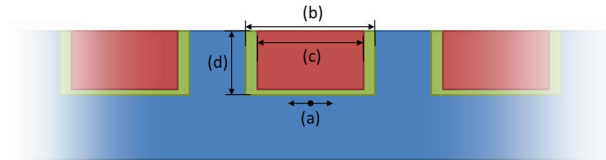

Figure 1: Errors considered in this model: (a) groove position error; (b) groove width error; (c) coil width error; (d) coil height error. The steel former is shown in blue, with green for the Isopon insulator and red for the coils.

NUMERICAL MODELLING

For small errors, the amplitude of the signature is a linear function of the size of the error. A function can then be fitted to this signature to approximate its shape; for instance, a Gaussian function is a reasonable approximation for single-peaked error signatures produced by a groove position error:

$$\delta B(z) = q\, \delta z \exp - \left(\frac{z - z_0 + \delta z}{\lambda/4} \right)^2 \qquad (2)$$

where λ is the undulator period, z is the longitudinal coordinate along the undulator, z_0 is the nominal groove position, δz is the error in the groove position, and q is the sensitivity coefficient relating groove position error to field error amplitude. Double-peaked errors are approximated by the derivative of a Gaussian:

$$\delta B(z) = q\, \delta z\, (z - z_0 + \delta z) \exp - \left(\frac{z - z_0 + \delta z}{\tau} \right)^2 \qquad (3)$$

where τ is the width, typically 4-6mm depending on the type of error. Figure 2 shows an illustration of both of these error signatures.

Figure 2: Comparison of single-peaked (left) and double-peaked (right) error signatures.

Table 2: Types of errors modelled using the numerical technique. For each case, the shape of the error signature is described. The ± figure in the last column refers to the full width (3σ) of the error distribution.

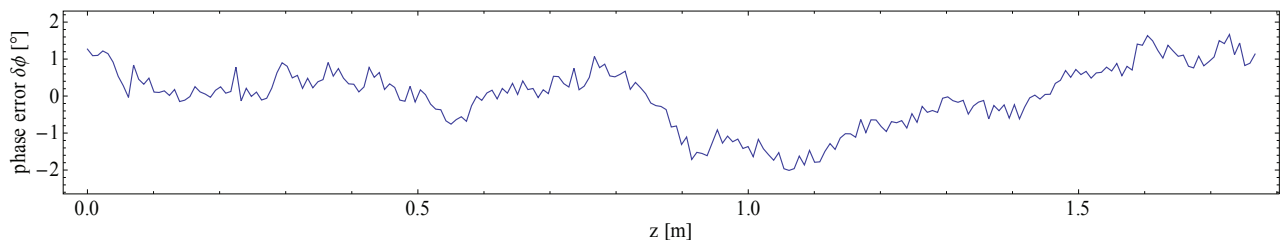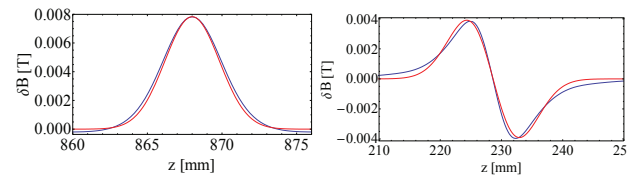| Error type	Signature shape	Sensitivity coefficient q	Amount for 1° phase error
Groove position	Single peak	0.313 T/mm	± 10 µm
Groove width	Double peak	0.025 T/mm²	± 40 µm
Coil width	Double peak	0.011 T/mm²	± 100 µm
Isopon base thickness	Double peak	0.015 T/mm²	± 40 µm
Former vertical alignment	Sinusoid (in phase)	0.2 T/mm	± 20 µm
Former angular alignment	Tapered sinusoid	36.9 T/rad	± 10 µrad
Former longitudinal alignment	Sinusoid (90° out of phase)	0.2 T/mm	± 80 µm
Pole height	Single peak	0.077 T/mm	± 10 µm
Gap between formers	Single peak	0.14 T/mm	Max 50 µm

Figure 3: Instantaneous phase error calculated for one undulator with random errors. The RMS is about 0.7°.

Radia was used to find the shape and sensitivity coefficient for several different types of error. Following that, a numerical simulation of the undulator was constructed using *Mathematica*, consisting of a sinusoidal field profile with errors added to it. The errors were determined at random from a Gaussian distribution centred about zero with a particular width σ. This field map with random errors can then be used to calculate the trajectory of a 3 GeV electron, and hence the RMS phase error of the undulator, using the following equations:

$$x'(z) = \frac{e}{\gamma mc} \int_0^z B \, dz \qquad (4)$$

$$\varphi(z) = \frac{2\pi}{\lambda} \left(\frac{z}{2\gamma^2} - \frac{1}{2} \int_0^z x'^2 \, dz \right) \qquad (5)$$

where x' is the angle the trajectory makes with the longitudinal axis, e, m and γ are the charge, mass and Lorentz factor of the electron, c is the speed of light, and φ is the phase of the electron beam relative to the photon beam. A straight line is fitted to the function $\varphi(z)$, and the undulator's RMS phase error is defined as the RMS deviation from the linear fit, evaluated at the poles [5].

$$\sigma_\varphi \equiv \sqrt{\langle \delta\varphi^2 \rangle_{z=n\lambda/2}} \qquad (6)$$

Figure 3 shows the phase error calculated for one undulator with errors. The RMS phase error is about 0.7°.

For a given width of error distribution, this process was repeated for many different random undulators to get good statistics for the expected values of phase error. The process was repeated using several different error distribution widths, and for different types of error. The results are summarised in Table 2. For each error type, the error required for a 1° phase error is estimated. It is assumed that the phase error arising from each source will add in quadrature to produce a total. So a 1° contribution from eight sources gives a total of 2.8°, which would be within Diamond's 3° specification.

The results show that some of the tolerances must be extremely tight in order to meet the specification. In particular, there is a very tight tolerance on the positions of the machined grooves. The field error generated by a small groove position error is a single-peaked distribution, centred about the groove where the field passes through zero. The phase is varying most rapidly at this point, so any small change here has a large effect.

ACCURACY OF SIMULATION

Several factors influence the accuracy of the predicted effect of these tolerances on the field quality of the undulator:

- **Accuracy of error signatures**. The error signatures were based on a Radia field map as described above. Some cross-checks were done using Opera-3D, and the results were found to agree reasonably well.
- **Longitudinal extent of error signatures.** An error in the centre of the undulator may have a larger range than one near one of the ends.
- **Linearity of error signatures**. Within the range of errors discussed here, the amplitude of field errors was exactly linear with error size.
- **Randomness of errors**. These are mostly machining errors, which typically arise from tool wear. This is a systematic component, which has not been taken into account here – all errors are random in the model. Systematic phase error has a different effect on the radiation output, as shown by Walker [6].
- **Combinations of errors**. The effect of many errors combined may in fact be less than that expected from a single error, as some may compensate for others.

As a check on the numerical model, an Opera-2D model was generated with random pole height errors and a smaller number of periods. The results from this model – extrapolated to 128 periods, assuming a $\sqrt{n}$ scaling – give a larger acceptable tolerance than the numerical method, about 20 μm compared to 10 μm for a 1° phase error. This indicates that the method of combining errors may be somewhat conservative. Further tests using combinations of errors in an Opera model may be useful.

CONCLUSION

In place of full 3D simulations of undulators with errors, a numerical method has been used to estimate the field errors produced in a superconducting undulator by errors in the manufacturing, winding and alignment. The allowable tolerances for various dimensions has been estimated in view of the aim to build a short-period undulator with a phase error of less than 3°. The values of these tolerances obtained using this method are likely to be lower than those needed in reality. This method provides a rapid qualitative comparison of the effect of various types of error, and gives a good idea of which tolerances can be relaxed and which must be tightened.

REFERENCES

[1] J.A. Clarke et al, "Status Of The UK Superconducting Planar Undulator Project", IPAC'13, Shanghai, May 2013, p. 2259 (2013).

[2] Radia magnet simulation software, http://www.esrf.eu/Accelerators/Groups/InsertionDevices/Software/Radia

[3] Opera simulation software, www.vectorfields.com

[4] J. Bahrdt and Y. Ivanyushenkov, "Effects Of Geometrical Errors on the Field Quality in a Planar Superconducting Undulator", IPAC'12, New Orleans, p. 708 (2012).

[5] R.P. Walker, "Interference effects in undulator and wiggler radiation sources", Nuclear Instruments and Methods A **335**, p. 328 (1993).

[6] R.P. Walker, "Phase errors and their effect on undulator radiation properties", Phys. Rev. ST Accel. Beams **16**, 010704 (2013).