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EFFECT ON BEAM DYNAMICS FROM WAKEFIELDS IN TRAVELLING WAVE STRUCTURE EXCITED BY BUNCH TRAIN

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Abstract

Electron bunch train technology is used to excited coherent high power RF radiation in travelling wave (TW) structures. This article concentrates on the analytical expression of wakefields excited by bunch train in TW structures and the effects of wakefields on beam dynamics. We focus on the first monopole mode and the first dipole mode wakefields. The long range wake function has a linear decrease which agrees well with the ABCi simulations. Taking example of the 11.7 GHz wakefields structure at the Argonne Wakefield Accelerator (AWA) facility, with 1.3 GHz interval drive electron bunch train, we have done the beam dynamics simulation with a point to point (P2P) code. Results shows the effects of wakefields on the energy distribution and the transverse instability for each sub-bunch.

INTRODUCTUION

Electron bunch train is used as drive beam to resonantly excited wakefields in travelling wave (TW) structure. In the two beam accelerator scheme in linear accelerators [1], driven beam is usually consists of *ps* sub-bunches, and the interval between bunches is the RF periods. The beam dynamics of the drive beam is affected by both the self-wake of each bunch and the long range wake from the head bunches. A lot of tracking codes can be applied to study the self-wake effect by means of a convolution integral or sum, based on the mesh technology, such as in ASTRA [2]. For the case of long range wake field effect caused by the head bunches, since the bunch interval is much longer than the bunch length, sub-bunches are always taken as point particles in order to reduce mesh and computation cost [3].

When concern about the motion of each particles in the drive beam, due to bunch instability such as beam break up (BBU) issue, we have to calculate the interaction between the wakefields and all the particles, thus a point to point (P2P) beam dynamics simulation code is desired. This paper starts from the analysis expression of the first monopole and the first dipole mode wake fields in TW structure. Taking the 11.7 GHz X-band TW PETS as an example, with drive bunch train of 1.3 GHz interval at the Argonne Wakefields Accelerator (AWA) facility, the P2P simulations show the longitudinal and transverse wake effects on each sub-bunch.

WAKEFIELDS IN TRAVELLING WAVE STRUCTURE

The Gradient of a Single Mode Wakefield

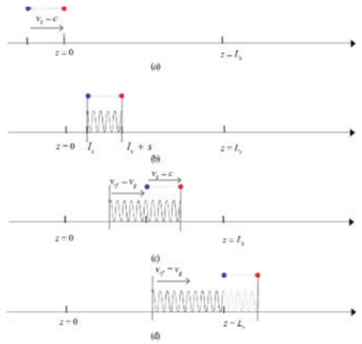


Figure 1: Single mode wakefields in TW structure.

When a drive particle with charge Q_d (red dot shown in Fig. 1) passing through a PETS of length L_s, wakefields is generated. Ignoring the attenuation, the electric field has constant gradient along Z direction for a single mode wake [4]. Once we know the parameters of the structure at certain mode, including the R over Q, relative group velocity β_g, and wake frequency ω, we can figure out the gradient. The gradient of the first monopole mode (TM_{01}) wake field is:

$$E^{//}_{w01}(\mathrm{s}) = 2Q_d \cdot \kappa_{01} \cdot \cos(\frac{\omega_{01} s}{c}) \cdot H(s) \cdot Boolean(\mathrm{s} \le L^{01}_{w}) \qquad (1)$$

Here k_{01} is the loss factor of TM_{01} mode, H(s) is the Heavenside step function and L^{01}_{W} is the duration of the wake field. The $Boolean(\mathrm{s} \le L^{01}_{W})$ in Eq. 1 declares the finite duration of wakefields in finite TW structure. For clarity we will omit the *Boolean* word in the following discussion. The duration of the wake is:

$$L^{01}_{w}(\mathrm{s}) = L_S(1/\beta^{01}_{g} - 1) \qquad (2)$$

Loss factor in Eq. 1 is defined with the group velocity modification in TW structure

$$\kappa_{01} = \frac{\omega_{01}}{4}\left(\frac{R}{Q}\right)_{01}\frac{1}{(1-\beta^{01}_{g})}$$

The loss factor of the structure and the drive particle charge determine the amplitude of the wakefield as a constant of $2Q_d \cdot k_{01}$.

Linear Decrease of the Wake Function

Wake function is used to describe the integrated wake field effect along the whole structure. The definition is

$$w_{//}(\mathrm{s}) = -\frac{1}{Q_d}\int_0^L dz \cdot [E^{//}_{W}(\mathrm{s,z,t})]_{t=(z+s)/\mathrm{c}} \qquad (3)$$

We introduce another test particle (blue dot shown in Fig.1) right behind the drive particle, the distance between two particles is s. Both particles travel at speed of light. Drive particle excites wakefields as shown in Fig. 1(a). Once generated, the RF pulse (wakefields) travels at group velocity in TW structure. The tail of RF pulse is generated when drive particle is at position z=0, and the head of RF pulse is always newly generated. The test particle will catch up with the tail of the RF pulse at position $z = l_c$ as shown in Fig. 1(b), here, the time of the drive particle travels is equal to that of the RF pulse tail propagates in the structure $(l_c + s)/c = l_c/v_g$, which gives:

$$l_c = \frac{s}{1/\beta_g - 1}$$

Then the test particle will 'experience over' the RF pulse because the travel velocity differs between the particles and the RF pulse. Substitute the constant wake field of TM_{01} mode and the expression of duration L_W^{01} in Eq. 2, the wake function is

$$w_{//}(s) = -\frac{1}{Q_d}\int_{l_c}^{L_s} dz \cdot E_{w01}^{//} = \frac{E_{w01}^{//} L_s}{Q_d} \cdot (1 - s / L_w^{01}) \qquad (4)$$

which shows the linear decrease of wake function verses the distance s between drive particle and witness particle.

Wake Potential from ABCi

It's hard to get the single particle wake function in simulation. Wakefield simulation code like ABCi gives the wake potential of electron beam. For a bunch with a normalized line change density distribution $\rho(z)$, the wake potential is a convolution of the single particle wake function and the distribution $\rho(z)$:

$$W_z(s) = \int_{-\infty}^{s} w(s - z)\rho(z)\,dz$$

If we only consider a single mode with a given wave number $k = \omega/c$, by introducing the form factor at the corresponding frequency $F(k, \rho)$, which is the Fourier transformation of beam distribution, we can get the analytical expression of the wake potential as $W_z(s) = w(s) \cdot F(k, \rho)$. Therefore wake potential of TM_{01} mode is:

$$W_z^{01}(s) = -2\kappa_{01} L_s \cdot \cos(k_{01} s) \cdot (1 - \frac{s}{L_w^{01}}) \cdot (s \le L_w^{01}) \cdot F(k_{01}, \sigma_z) \qquad (5)$$

For a Gaussian bunch with rms. length σ_z, the form factor is $F(k, \sigma_z) = \exp\left(-\frac{(k \cdot \sigma_z)^2}{2}\right)$. We can also get the similar formula for the first dipole TM_{11} mode by introducing the transverse wake function as defined in Ref. [5].

$$W_z^{11}(s) = -\frac{2\kappa_{11}}{ka^2} \cdot L_s \cdot \sin(k_{11} s) \cdot (1 - \frac{s}{L_w^{11}}) \cdot (s \le L_w^{11}) \cdot F(k_{11}, \sigma_z) \qquad (6)$$

The loss factor of TM_{11} mode is $\kappa_{11} = \frac{\omega_{11}}{4}\left(\frac{R}{Q}\right)_{\perp}^{11} \frac{1}{1 - \beta_g^{11}}$, where $k = \frac{\omega_{11}}{c}$ is the wave number and a is the radius of the structure. The unit of transverse wake function W_x^{11} is V/m/C/m.

We compare our analytical analysis Eq. 5 and Eq. 6 with the simulation results from ABCi for the 30 cm long, 35-cell metallic periodic structure of 11.7 GHz X-band PETS at AWA. In simulation, the rms. bunch length of the drive beam is 1.5 mm. As shown in Fig. 2, the analysis and the simulation agree well both for the TM_{01} and TM_{11} mode, which demonstrates the linear decrease of the wake function in the finite TW PETS. The 1.3 GHz bunch train positions is marked with black dots, which means the wake field generated from head bunch will affect the following 4~5 sub-bunches. The number of sub-bunches affected is decided by the wake duration L_W^{mn} of this structure.

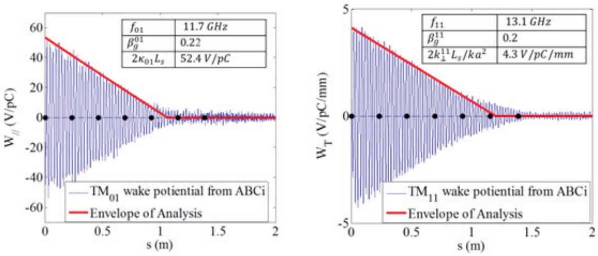


Figure 2: Comparison of long range wake potential from ABCi simulation with the envelope of analysis (left: TM_{01} mode, right: TM_{11} mode). The black dots shows the position of sub-bunches in 1.3 GHz bunch train.

Superposition of The Wake Functions

Wake function is defined for single particle as in Eq. 3, we will refer to the 'wake function for bunch' in the following discussion, which really means the wake function for particles in that bunch. For the TM_{01} mode wakefields from Fig. 2, if the bunch distribution keeps the same, we can get the wake function for bunch train simply by superposing the wake functions at the responding z position. As shown in Fig. 3, we reposition the wake function for each bunch at $s = 0$ for comparison. For bunch 1 (first bunch in the train), wake function is just the self-wake function as shown in the red curve in Fig. 3, for the following bunch n ($n > 1$), the wake function is the superposition of wake functions from the $n - 1$ head bunches at position $s = n \cdot z_b$ in Eq. 4 and the self-wake. Here z_b is the bunch interval. The wake function from head bunches are cosine waves with peaks at $\tilde{s} = 0$ for bunch n. Because of the linear decrease of the wake function, the superposition will saturate when head bunches number is larger than 5.

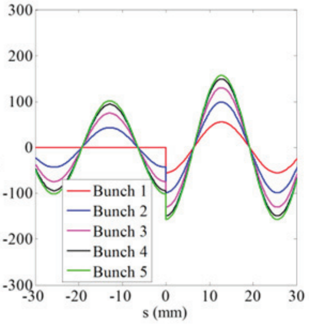


Figure 3: Superposition of monopole wake function.

Now consider the energy loss for each bunch, as shown in Fig. 3. If bunch 1 loss energy U_0, where $U_0 =$

$Q_d \int_0^\infty \rho(s) \cdot W(s) \cdot ds$, then bunch n ($2 \leq \mathrm{n} \leq 5$) will lose energy $\mathrm{U_n}$. Due to the symmetric of the cosine functions and the linear decrease of the wake potential $W = W(1 - \mathrm{n}\frac{z_b}{\mathrm{L_w}})$ at position $s = n \cdot z_b$, we can derive that,

$$U_n = U_0 + 2(n-1)U_0 - n(n-1)\frac{z_b}{L_w}U_0 \qquad (7)$$

The first term on the right side U_o is the energy loss due to self-wake, while the second term $2(n-1)U_o$ can be taken as the coherent enhancement radiation within the bunch train. The last term is energy of wakefields failed to be coherent super-posited since RF pulses keep travelling forwards in TW structure, which is the same reason for the linear decrease of the wake function.

BEAM DYNAMICS SIMULATION

Equations of Motion in Wakefields

The wakefields interaction with electron beam, both for self-wake and long range wake, can be analysed by point to point interaction, Equations of motion (EOM) affected by $\mathrm{TM_{01}}$ and $\mathrm{TM_{11}}$ mode are shown in Eq. 8.1- Eq. 8.4.

For a finite beam distribution (say, the bunch train distribution) with total number of particles N, the particle at position s_i (longitudinal) and x_i (horizontal) in the beam experiences both longitudinal and transverse wake field generated by all the particles ahead of it.

$$F_z = q_i \sum_{m=0}^{1} \sum_{n=1} \sum_{j=0}^{i-1} \left(x_i x_j\right)^m q_j A_{mn}^{//} \cos\left(\frac{\omega_{mn}}{c}(s_j - \mathrm{s}_i)\right) \qquad (8.1)$$

$$F_x = q_i \sum_{m=1} \sum_{n=1} \sum_{j=0}^{i-1} x_j q_j A_{mn}^{\perp} \sin\left(\frac{\omega_{mn}}{c}(s_j - \mathrm{s}_i)\right) \qquad (8.2)$$

here $A_{m1}^{//}$ and $A_{11}^{\perp}$ are the envelopes for each mode, which we can acquire from ABCi simulation or from analytical expression as discussed above, say, $A_{01}^{//} = 2k_{01}H(s_j - s_i)\left((s_j - s_i) < L_W^{01}\right)$. For the ith particle:

$$dP_{z,i} / dt = F_z \qquad (8.3)$$

$$dP_{x,i} / dt = F_x \qquad (8.4)$$

where $P_X = m\gamma\dot{x}$ and $P_z = m\gamma c$ are the longitudinal and transverse momentum respectively.

Monopole Wakefields Effect on Bunch Train

Substitute the wake function of the 11.7 GHz structure into the EOM, we perform P2P beam dynamics simulation for the AWA 1.3 GHz bunch train. We assume that the sub-bunches are identical. Single sub-bunch charge is 40 nC, initial horizontal rms beam size is 1 mm, average energy is 75 MeV, rms energy spread is 100 keV, and the rms bunch length is 1.5 mm. After passing through the 11.7 GHz PETS, each sub-bunch has different energy distribution as shown in Fig. 5. The downstream sub-bunches loss more energy, which agrees well with the analytical results in Fig. 2. The energy loss of bunch *n* from the statistic of the dynamics simulations also agrees with the calculation in Eq. 7 with $\mathrm{U_0} = 0.97\ MeV$.

Bunch Train Transverse Instability

In the P2P code, we assume that there is an initial Xoff = 1mm / 3mm for each 40 nC sub-bunch in the train before the PETS. The X-Z distribution is shown in Fig. 5, the blue dots stand for bunches right after the PETS, it shows the distribution keeps almost the same as the initial distribution, thus BBU instability matters little for single shot experiment with this short PETS. However, the transverse kick will convert to large position change after drifting a certain distance. As Fig.5 shown, the red dots are distributions of bunches after 1 meter's drift, and the green ones are after a 4 meter's drift. It shows a serious transverse instability issue for bunch train, especially for tail bunches, since the transverse wake frequency 13.1 GHz is quite close to the harmonic of bunch spacing 1.3 GHz. The tail bunches suffer stronger kick compared to head bunches. Also the head and the tail of each bunch experience kick of opposite direction, which makes it difficult to refocus the beam. If we want to reuse the beam in the downstream beam line, the transverse instability issue need to be carefully treated in the future staging experiments [6].

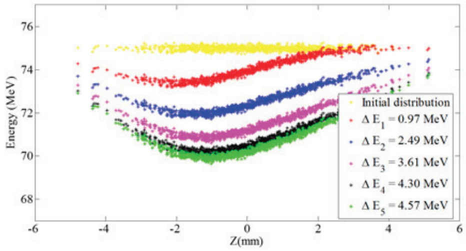
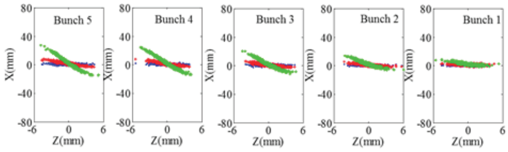
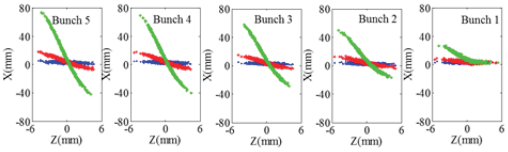


Figure 4: Longituinal phase space distribution and energy loss of each bunch due to the monopole wake effect.



(a) Xc_initial = 1 mm



(b) Xc_initial = 3 mm

Figure 5: X-Z distribution of each sub-bunch (blue:before PETS, red: after PETs and drift *1m*, green: after PETS and drift 4*m*). up: initial *xoff* = *1mm*, down: initial *xoff* = *3mm*.

CONCLUSION

We acquire the analytical expression of the wakefields excited by the bunch train in TW structure, both for the first monopole mode and the first dipole mode. Analysis shows that long range wake potential has a linear decrease, which agrees well with the ABCi simulations. Taking the 11.7 GHz PETS at the AWA facility as an example, with 1.3 GHz drive electron bunch train, we performed the beam dynamics simulation with a P2P code. The simulation results show the effects of wakefields on the energy distribution and transverse instability (BBU) for each sub-bunch. The BBU study with the P2P code will benefit the future staging work in AWA.

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