ISBN: 978-3-95450-168-7 doi:10.18429/JACoW-IPAC2015-MOPJE033

COUPLED ORBIT RESPONSE COEFFICIENTS WITH CONSTANT REVOLUTION TIME

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Abstract

We calculate orbit response coefficients for arbitrarily coupled lattice which keep the orbit length constant as is needed to maintain synchronicity with a radio-frequency system.

INTRODUCTION

In a circular accelerator the orbit correction system and diagnostic tools such as CALIF [1] or LOCO [2] rely on the knowledge of how steering magnets affect the beam position measured at beam position monitors (BPM). Historically the so-called response coefficients C^{ij} or response matrix elements between steering magnet labeled j and BPM i can be expressed in terms of TWISS parameters or transfer matrices by

$$C^{ij} = R^{ij}(1 - R^{jj})^{-1} \tag{1}$$

where R^{ij} is the transfer matrix from location j to location i and R^{jj} is the full-turn matrix starting at location j. Equation 1 does not constrain the revolution time, despite the fact that for example a horizontal steering magnet with kick angle θ^j lengthens the circumference by $D^j\theta^j$, where D^j is the dispersion at the steering magnet [3, 4]. This can be accommodated in the response coefficient by an additional term to Eq. 1 containing the dispersion at the location of the steering magnet and the BPM

$$C^{ij}_{12} = \left[R^{ij}\left(1 - R^{jj}\right)^{-1}\right]_{12} - \frac{D^i D^j}{\eta C} \tag{2}$$

with the ring circumference C and the the phase slip factor η.

In this note we generalize this expression to the fully coupled case.

COUPLED CASE

The six-dimensional full-turn matrix R^{jj} starting at location j maps the orbit vector $\vec{x} = (x, x', y, y', \tau, \delta)$ onto itself after one turn. If there is a perturbation vector $\vec{v}$ present at location j the requirement for a periodic solution reads

$$\vec{x} = R^{jj}\vec{x} + \vec{v}\,. \tag{3}$$

Note that the requirement to have equal entries in the arrival time and energy location, constrains the revolution time to be constant. The solution to this equation is given by $\vec{x} = (1 - R^{jj})^{-1}\vec{v}$ and the top left 4×4 part of $(1 - R^{jj})^{-1}$ is the response matrix which constrains the revolution time. This is well-known and used for example in the accelerator toolbox [5] to determine the closed orbit.

In order to find a generalized version of Eq. 2 and express the response coefficients in terms of the dispersions we need to split Eq. 3 in a transverse and longitudinal part. To achieve this we write the matrix R^{jj} in terms of its 2×2 sub-matrices

$$R^{jj} = \begin{pmatrix} A & B & C \\ D & E & F \\ G & H & I \end{pmatrix} \tag{4}$$

and split the equation into a transverse and a longitudinal part

$$\begin{aligned} |x\rangle &= \hat{R}\,|x\rangle + \begin{pmatrix} C \\ F \end{pmatrix}\begin{pmatrix} \tau \\ \delta \end{pmatrix} + |v\rangle \\ \begin{pmatrix} \tau \\ \delta \end{pmatrix} &= (G, H)\,|x\rangle + I\begin{pmatrix} \tau \\ \delta \end{pmatrix}. \end{aligned} \tag{5}$$

here $|x\rangle$ denotes the transverse part of the closed orbit vector $\vec{x}$. Solving the second, longitudinal equation for τ and δ, inserting in the first equation and collecting terms we arrive at

$$\left[1 - \hat{R} - \begin{pmatrix} C \\ F \end{pmatrix}(1 - I)^{-1}(G, H)\right]|x\rangle = |v\rangle \tag{6}$$

The matrix in the square brackets is the inverse of the response matrix c^{jj}. To express it in terms of the dispersion we first need to identify the dispersions in a coupled ring. It is given by the periodic orbit vector subjected to the perturbation defined by the $R_{i,6}$ matrix elements, namely

$$|D\rangle = \hat{R}\,|D\rangle + \begin{pmatrix} R_{16} \\ R_{26} \\ R_{36} \\ R_{46} \end{pmatrix}. \tag{7}$$

Consequently we introduce the dispersion-like quantities with a tilde on top

$$\begin{pmatrix} \tilde{C} \\ \tilde{F} \end{pmatrix} = (1 - R)^{-1}\begin{pmatrix} C \\ F \end{pmatrix}. \tag{8}$$

and note that the right column of $\tilde{C}$ and $\tilde{F}$ contains the dispersions. This leads to

$$\left[1 - \begin{pmatrix} \tilde{C} \\ \tilde{F} \end{pmatrix}(1 - I)^{-1}(G, H)\right]|x\rangle = (1 - R)^{-1}|v\rangle \tag{9}$$

and the special form of the matrix in the square brackets permits us to explicitly calculate its inverse with the result

$$|x\rangle = \left[1 + \begin{pmatrix} \tilde{C} \\ \tilde{F} \end{pmatrix} Q\,(1 - I)^{-1}(G, H)\right](1 - R)^{-1}|v\rangle \tag{10}$$

where Q is given by

$$Q = \left[1 - (1 - I)^{-1}\left(G\tilde{C} + H\tilde{F}\right)\right]^{-1}. \tag{11}$$

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We can exploit the symplecticity of the transfer matrix and express the matrices G and H in terms of C and F by

$$G = \frac{1}{\det I}\left[ISC^t SA + ISF^t SD\right] \qquad (12)$$
$$H = \frac{1}{\det I}\left[ISC^t SB + ISF^t SE\right]$$

with the symplectic metric S. Extensive further algebra eventually leads to

$$|x\rangle = \Big[(1-R)^{-1} - \begin{pmatrix} \tilde{C} \\ \tilde{F} \end{pmatrix} Q\,(1-I)^{-1}\,\frac{IS}{\det I}(\tilde{C}^t S, \tilde{F}^t S)\Big]\,|v\rangle \qquad (13)$$

where $\tilde{C}$ and $\tilde{F}$ are dispersion like quantities and Q is given by Eq. 11.

PHYSICAL QUANTITIES

In order to connect the expression in Eq. 13 to the commonly known equation 2 we need to assume that the matrix I is given by unity on the diagonal and an R_{56} which renders $(1-I)$ non-invertible. Closer analysis reveals that in that case we can substitute

$$(1-I)^{-1} \rightarrow \begin{pmatrix} 0 & 0 \\ -1/R_{56} & 0 \end{pmatrix} \qquad (14)$$

wherever $(1-I)^{-1}$ appears. Evaluating Q and substituting the dispersions $|D\rangle$ into $\tilde{C}$ and $\tilde{F}$ in Eq. 13 we can finally write the response matrix from location j to j as

$$\tilde{C}^{jj} = (1-\hat{R})^{-1} - \frac{1}{\eta C}\begin{pmatrix} D_x \\ D'_x \\ D_y \\ D'_y \end{pmatrix}\left(-D'_x, D_x, -D'_y, D_y\right) \qquad (15)$$

with the definition of the phase slip factor η and circumference C

$$-\eta C = R_{56} + R_{51}D_x + R_{52}D'_x + R_{53}D_y + R_{54}D'_y\ . \qquad (16)$$

The response at the BPM i is trivially computed by left multiplying $\tilde{C}^{jj}$ with the 4×4 transfer matrix $\hat{R}_{ij}$ from the steering magnet to the BPM.

CONCLUSIONS

We calculate the orbit response coefficients for arbitrarily coupled storage rings in case the revolution time is constrained and exciting a steering magnet causes a variation of the orbit length. The computations only involve manipulations of 6×6 transfer matrices that are usually available from beam optics codes. This makes the method easy to implement numerically. Expressing the additional term in terms of the coupled dispersion we arrived in Eq. 15 at a generalized version of the well-known result from Eq. 2.

Discussions with Xiaobiao Huang, SSRL about the internals of the accelerator toolbox are gratefully acknowledged.

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