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SPIN RESONANCE STRENGTH CALCULATION THROUGH SINGLE PARTICLE TRACKING FOR RHIC*

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Abstract

The strengths of spin resonances for the polarized-proton operation in the Relativistic Heavy Ion Collider are currently calculated with the code DEPOL, which numerically integrates through the ring based on an analytical approximate formula. In this article, we test a new way to calculate the spin resonance strengths by performing Fouier transformation to the actual transverse magnetic fields seen by a single particle traveling through the ring. Comparison of calculated spin resonance strengths is made between this method and DEPOL.

INTRODUCTION

The Relativistic Heavy Ion Collider (RHIC) at Brookhaven National Laboratory is the only high energy collider in the world to collide polarized protons with the particle energy up to 255 GeV. Polarization is the average value of the projection of the spins of particles in a bunch on the average spin direction. To maintain a high polarization during the beam transfer from the injectors to RHIC and on the acceleration and at store of RHIC, we need to have a good knowledge of all the spin resonance strengths on the way.

The strengths of spin resonances are normally numerically calculated based on analytical approximate formula. For an example, DEPOL [1] numerically integrates through the ring based on known magnetic fields and linear Twiss parameters. However, DEPOL itself does not calculate the linear optics. The linear optics parameters have been imported from other codes such as MADX. To improve the accuracy of resonance strength calculation, sometime we may need to split each magnet into several slices.

SimTrack [2] is a compact c++ library for beam optics calculation and particle tracking based on symplectic integration. It has been extensively used for dynamic aperture calculation with beam-beam interaction in RHIC. Recently we implemented proton spin tracking into this code [3]. Since SimTrack tracks particles in steps through each magnet, the particle coordinates and the magnetic fields the particles feel are all transparent to the users. Therefore it is possible to calculate the spin resonance strength through particle tracking.

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SPIN RESONANCES

Thomas-BMT equation describes the particle's spin motion in the presence of magnetic and electric fields,

$$\frac{d\mathbf{S}}{dt} = \mathbf{S} \times \mathbf{\Omega}, \tag{1}$$

$$\mathbf{\Omega} = \frac{e}{\gamma m}\left[(1+G\gamma)\mathbf{B}_{\perp} + (1+G)\mathbf{B}_{||}\right]. \tag{2}$$

$\mathbf{S}$ is the 3-dimensional spin vector in the particle's frame. $\mathbf{B}_{\perp}$ and $\mathbf{B}_{||}$ are the magnetic fields perpendicular and parallel to the particle velocity. It is convenient to use the path length s of the reference particle as the independent variable, then Eq. (1) turns to

$$\frac{d\mathbf{S}}{ds} = \mathbf{S} \times \mathbf{F}, \tag{3}$$

$$\mathbf{F} = \frac{\sqrt{(1+\frac{x}{\rho})^2 + x'^2 + y'^2}}{1+\delta}\frac{\mathbf{\Omega}}{(B\rho)_0}, \tag{4}$$

where $(B\rho)_0$ is the magnetic rigidity for the reference particle, ρ is the curvature for the reference particle on which the local coordinate system is built.

To study the polarization loss for a single particle when it crosses a single spin resonance, we would like to use the spinor equations where the bending angle θ is used as the independent variable. Then Eq. (3) is re-written as

$$\frac{d\mathbf{S}}{d\theta} = \mathbf{n} \times \mathbf{S}, \tag{5}$$

where $\mathbf{n} = -G\gamma\hat{y} - F_x\hat{x} - F_s\hat{s}$, $G\gamma$ is the spin tune for the reference particle. Defining $\xi = F_x - iF_s$ and expanding it in Fourier series, we have

$$\xi = \sum_K \epsilon_K e^{-iK\theta}. \tag{6}$$

Here ϵ_K is the strength of spin resonance. There are two important types of spin resonances: imperfection and intrinsic resonances. For imperfection resonances, $G\gamma = K$, K is an integer. For intrinsic resonances, $G\gamma = K \pm \mu_y$, μ_y is the fractional vertical tune.

According to Eq. (4) and Eq. (6), we have [4]

$$\epsilon_k = \frac{1}{2\pi}\oint\left[(1+G\gamma)\frac{B_x}{(B\rho)_0} - i(1+G)\frac{B_{||}}{(B\rho)_0}\right]e^{iK\theta}ds. \tag{7}$$

where B_x and $B_{||}$ are the projections of the magnetic fields seen by the particle. In Ref. [1, 4], ϵ_k can be presented with particle's coordinates as $\epsilon_k =$

$$-\frac{1}{2\pi}\oint\left[1+G\gamma)(\rho y'' + iy') - i\rho(1+G)(\frac{y}{\rho})'\right]e^{iK\theta}d\theta. \tag{8}$$

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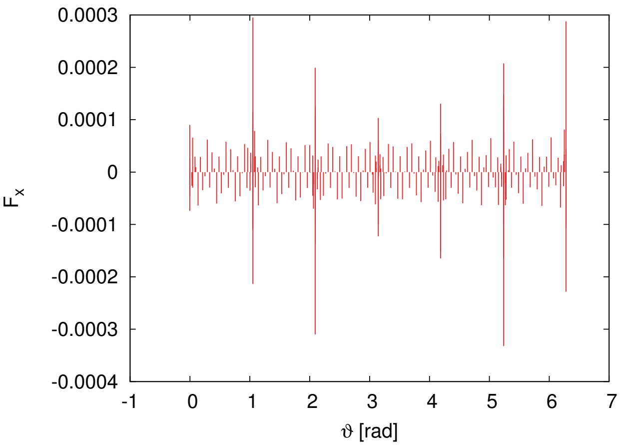

Figure 1: An example of $(1+G\gamma)\frac{B_x}{(B\rho)_0}$ seen by a test particle with $G\gamma = 62.33$.

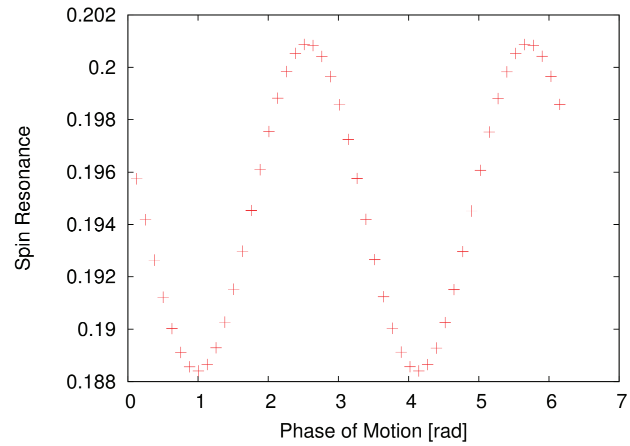

Figure 2: Calculated spin resonance strength versus the initial betatron motion phase.

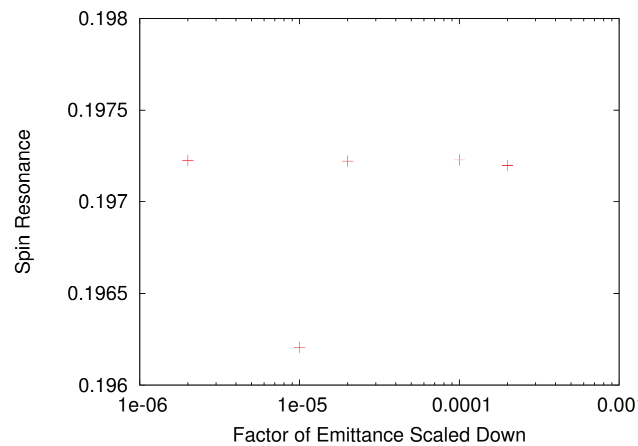

Figure 3: Calculated spin resonance strength versus the initial betatron motion emittance.

Eq. (8) is used for spin resonance strength calculation in DEPOL. In this article, we will directly use Eq. (7) to calculate it since the magnetic fields seen by the test particle are transparent during particle tracking.

SPIN RESONANCE STRENGHTH CALCULATION

In this section we present the procedure to calculate the spin resonance strength through a single particle tracking. The lattice for the 2015 RHIC 100 GeV polarized proton run is used. The lattice tunes are set to (29.68, 30.67) . We will track one particle one turn and record the magnetic fields it feels in each magnet. For convention, the spin resonance strength always refers to that with the normalized beam transverse emittance 10 π mm.mrad. However, for the particle tracking, we would like to scale down the emittance to avoid large polarization loss during particle tracking. In the following , we use 10^{-4} π mm.mrad for the initial emittance. The test particle is on momentum and only has non-zero initial vertical coordinates.

SimTrack is used to track the test particle element-by-element through the ring. At each integration step of each magnet, we calculate the direction of particle velocity and project the magnetic fields onto it. The projections of these fields onto the local reference coordinate frame are recorded at each integration step.

As an example, Fig. 1 shows $(1+G\gamma)\frac{B_x}{(B\rho)_0}$ seen by the particle along the ring. The horizontal axis is the accumulated bending angle θ. In this example, we track a particle with $G\gamma = 62.33$. $G\gamma = 62.33$ is the strongest spin resonance on the RHIC energy ramp.

Looking into ξ at each magnets, we found that its main contribution is from quadrupoles. The contributions from bending magnets is three orders of magnitude smaller than that in quadrupoles. The contribution from sextupole magnets is even smaller. And the term $(1+G)\frac{B_{||}}{(B\rho)_0}$ is much smaller than $(1+G\gamma)\frac{B_x}{(B\rho)_0}$ since the most magnetic fields are in the transverse plane.

Knowing $(1+G\gamma)\frac{B_x}{(B\rho)_0}$ and $(1+G)\frac{B_{||}}{(B\rho)_0}$, we can calculate the resonance strength with Eq. (7). The final spin resonance is scaled up by a factor of $\sqrt{10^5}$. By doing in this way, the calculated spin resonance strenth through particle tracking is 0.1957. The value from DEPOL is 0.1917. The difference between them is less than 2%.

In the above calculation, we set the initial vertical betatron phase to be zero. Figure 2 shows the calculated spin resonance versus the initial betatron phase. Here we sample 50 initial betatron phases with the same distance step between 0 to 2π. From Fig.2, the difference in the calculated spin resonance strengths with different initial betatron phases is less than 3%

Figure 3 shows the calculated spin resonance strength versus initial betatron motion emittance. The resonance strength scales with the betatron motion amplitude or the square root of emittance. We used different initial emittances between 2×10^{-3} π mm.mrad and 2×10^{-5} π mm.mrad. From Fig. 3, the variation in the calculated

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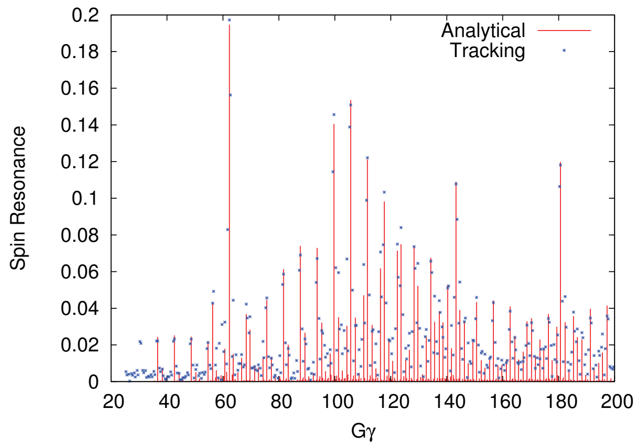

Figure 4: Comparison of $|\epsilon_K|$ between the new method and DEPOL.

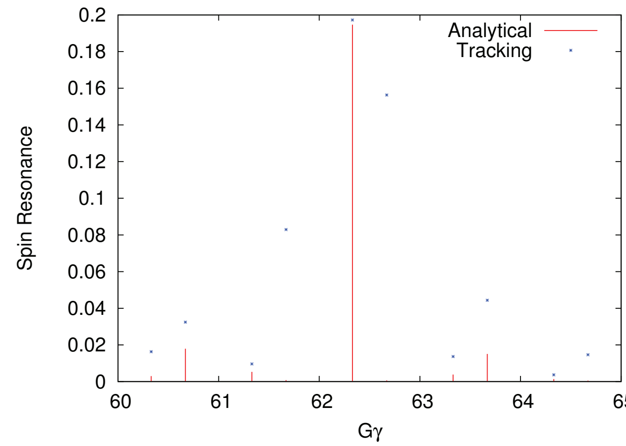

Figure 5: Comparison of $|\epsilon_K|$ between the new method and DEPOL around the area $G\gamma = 62.33$.

spin resonance through tracking is less than 0.5%.

For a single proton spin resonance crossing, the final vertical spin projection is given by Froissart-Stora Formula,

$$S_y = 2e^{-\frac{\pi|\epsilon_K|^2}{2\alpha}} - 1, \tag{9}$$

where $\alpha = dG\gamma/d\theta$ is the resonance crossing speed. With the initial and final spin vectors from particle tracking, we derive the spin resonance at $G\gamma$= 62.33 about 0.192, which is very close to the values calculated with the new method and DEPOL.

ON THE RHIC RAMP

In this section we calculate the strengths of all intrinsic spin resonances on the RHIC energy ramp. For simplicity, in this simulation study, the lattice designed for $\gamma = 35$ is used for the whole ramp. There are changes in the actual lattices at different energies on the RHIC energy ramp. However, we found that the difference in the calculated resonance strengths is small between these two cases: using the same lattice for whole ramp and using different lattices at different energies.

The injection and store energies for the 2015 RHIC polarized proton run are $\gamma = 25$ and $\gamma = 106$ respectively. Considering both the sum and difference intrinsic spin resonances, there are totally about 200 resonances on the RHIC energy ramp. The distance between the adjacent spin resonances is about 500 MeV. The sum resonance happens when $G\gamma = P - \mu_y$ and the difference resonance happens when $G\gamma = P + \mu_y$.

Figure 4 shows the spin resonance strengths on the RHIC energy ramp. The horizontal axis is $G\gamma$. The red lines are the spin resonance strengths calculated with DEPOL. The blue dots are the spin resonance strengths calculated through particle tracking. The strongest spin resonance happens at $G\gamma = 62.33$. Its strength is about 0.2.

From Fig. 4, at all strong spin resonances on the energy ramp of RHIC, the resonance strengths calculated through particle tracking agree very well with the analytical calculations with DEPOL. However, we notice that the particle tracking method gives a relative large resonance strength for resonances on both sides of a strong resonance.

For an example, for the area close to the strong resonance $G\gamma = 62.33$, Figure 5 shows that the resonance strengths calculated at $G\gamma = 61.67$ and $G\gamma = 62.67$ are much larger than that from DEPOL. These two spin resonances are adjacent to the strong spin resonance at $G\gamma = 62.33$.

One possible explanation is that the particle in the tracking method may sample the adjacent strong spin resonance. As we know, the stronger the resonance is, the wider its stop-band is. For an example, particles with $G\gamma = 61.67$ or 72.67 in tracking may sample the adjacent strong resonance at $G\gamma = 62.33$. However, further detailed studies are needed. Similar attempt was done for AGS with Fresnel integrals approximation for weak spin resonances [5].

SUMMARY

In the article, we tested a new way to calculate the spin resonance strength through a single particle tracking with the magnetic fields the particle feels. At strong spin resonances, the resonance strengths from this method agree well with that from analytical calculation with DEPOL. However, this method gives a relatively larger resonance strengths for the adjacent resonances to a strong one. The reason is under study.

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