ISBN: 978-3-95450-168-7 doi:10.18429/JACoW-IPAC2015-MOPMN030

PROTON SPIN TRACKING WITH SYMPLECTIC INTEGRATION OF ORBIT MOTION*

Y. Luo, Y. Dutheil, H. Huang, F. Meot, V. Ranjbar
Brookhaven National Laboratory, Upton, NY USA

Abstract

Symplectic integration had been adopted for orbital motion tracking in code SimTrack. SimTrack has been extensively used for dynamic aperture calculation with beam-beam interaction for the Relativistic Heavy Ion Collider (RHIC). Recently proton spin tracking has been implemented on top of symplectic orbital motion in this code. In this article, we will explain the implementation of spin motion based on Thomas-BMT equation, and the benchmarking with other spin tracking codes currently used for RHIC. Examples to calculate spin closed orbit and spin tunes are presented too.

INTRODUCTION

The Relativistic Heavy Ion Collider (RHIC) is capable of colliding heavy ions and polarized protons. In the polarized proton operation, the Figure of Merit (FOM) is LP^2 and LP^4 for the single and double spin programs. Here L is luminosity and P is polarization. Polarization is the average value of the projection of the spins of particles in a bunch on the average spin direction. Spin is a 3-D vector associated with the particle's magnetic moment. To improve the FOM in the polarized proton operation, many efforts had been made to understand through numeric simulations the mechanisms of polarization loss in the RHIC's injector-Alternating Gradient Synchrotron (AGS) , and during during injection, energy acceleration, and physics store in RHIC.

Currently there are two main codes for spin simulation calculations at Brookhaven National Laboratory: one is SPINK [1] developed by A. Luccio, and the other is Zgoubi [2] developed by F. Meot. Since the spin motion is linked to the magnetic fields the particles feel on their orbits, therefore we need an accurate modeling of orbit motion. SPINK does not have its own orbit motion tracker. It uses the first and second order matrices generated with MADX. Zgoubi is a ray-searching code, which tracks particle's orbit motion directly based on Lorentz equation. The particles position, velocity, and the magnetic fields are all expanded to 5th and higher order. Orbit motion tracking in both codes is not symplectical. For a long term tracking, especially for proton accelerators like RHIC, symplecticity is crucial to avoid the unphysical results from numeric simulations.

SimTrack [3] is a compact c++ library for linear optics calculation and particle tracking for circular accelerators. It adopts the 4th order symplectic integration to transfer particles through magnets. Its optics transport in magnets had been benchmarked with Tracy-II. Its 6-D symplectic beam-beam interaction was benchmarked with SixTrack. Since 2009, SimTrack had been extensively used for particle tracking with beam-beam interaction for RHIC. For an example, simulation results from SimTrack showed that the non-luminous particle loss in the gold-gold ion collision is due to limited off-momentum dynamic aperture. SimTrack is also used to simulate head-on beam-beam compensation with electron lenses in RHIC.

In this article we first introduce symplectic integration of orbit motion and its application in SimTrack. Then we implement proton spin tracking based on Thomas-BMT equation. Examples of benchmarking of spin tracking with Zgoubi are presented. In the end, we show some examples to calculate the spin closed orbit, spin tune, and to simulate a single spin resonance crossing.

SYMPLETIC ORBIT TRACKING

Symplectic integration is one kind of numeric integrations to solve differential equations. Different from other methods, symplectic integration will preserve the symplectic condition of particle motion governed by a Hamiltian. R. Ruth invented the forth order symplectic integrator for a splitable Hamitonian [4],

$$H(p,q) = H_1(p) + H_2(q). \tag{1}$$

Here p, q are the canonical positions and momenta. In each step of integration, the solution of particle orbit motions is given by

$$\begin{aligned} e^{-:H:} = {} & e^{:-c_1LH_1/2:}e^{:-d_1LH_2:}e^{:-c_2LH_1/2:}e^{:-d_2LH_2:} \\ & e^{:-c_2LH_1/2:}e^{:-d_1LH_2:}e^{:-c_1LH_1/2:} + O(L^5), \end{aligned} \tag{2}$$

where $c_{1,2}$ and $d_{1,2}$ are coefficients given in Ref. [4].

Symplectic integration has been used with an expanded Hamiltonian in many codes, where H_1 represents a drift and H_2 a thin magnetic kick. The expanded Hamiltonian is a good approximation for high energy accelerators where the particle's velocity is very close to the speed of light. Eq. (2) also can be used for solvable Hamiltonians H_1 and H_2, where H_1 normally represents an ideal bending maget and H_2 a thin magnetic kick.

In SimTrack, both expanded and exact Hamiltonians are implemented. With the expanded Hamiltonian, the computing time will be shorter than that with the exact Hamiltonian. Expanded Hamiltonian is normally used for particle tracking in RHIC at store energies. However, for a low

* This work was supported by Brookhaven Science Associates, LLC under Contract No. DE-AC02-98CH10886 with the U.S. Department of Energy.

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energy accelerator like AGS with the proton energy from 2.5 GeV to 25 GeV, the exact Hamiltonian has to be used. Otherwise, the longitudinal motion will not be accurately modeled.

SPIN TRACKING IMPLEMENTATION

Here we start with the famous Thomas-BMT equation [5],

$$\frac{d\mathbf{S}}{dt} = \mathbf{S} \times \mathbf{\Omega}, \tag{3}$$

$$\mathbf{\Omega} = \frac{e}{\gamma m}\left[(1+G\gamma)\mathbf{B}_{\perp} + (1+G)\mathbf{B}_{||}\right]. \tag{4}$$

$\mathbf{S}$ is the spin vector in the particle's frame. $\mathbf{B}_{\perp}$ and $\mathbf{B}_{||}$ are the magnetic fields perpendicular and parallel to the particle velocity. $\mathbf{B}$ and t refer to the laboratory frame. In Eq. (4) we ignored the electric field.

We normally track particles element by element in the laboratory frame. The path length s of the reference particle is preferred as the independent variable. Therefore, we re-write Eq. (4) as

$$\frac{d\mathbf{S}}{ds} = \mathbf{S} \times \mathbf{F}, \tag{5}$$

$$\mathbf{F} = \frac{\sqrt{(1+\frac{x}{\rho})^2 + x'^2 + y'^2}}{1+\delta}\frac{\mathbf{\Omega}}{(B\rho)_0}, \tag{6}$$

where $(B\rho)_0$ is the magnetic rigidity for the reference particle, ρ is the curvature radius for the reference particle on which the local coordinate system is built.

In SimTrack, we integrate the particle motion step-by-step through each magnet. For each integration step, we evaluate $\mathbf{B}_{\perp}$ and $\mathbf{B}_{||}$ at the middle of this slice. Knowing the magnetic field, we model the spin motion of Eq. (5) as a rotation motion in the 3-d space. The rotation axis is given by $\mathbf{F}$ and the absolute rotation angle is $|\mathbf{F}|$. Keep in mind that spin rotates clockwise with the axis. Therefore, at each integration,

$$\mathbf{S}_2 = \mathbf{R}\mathbf{S}_1, \tag{7}$$

$\mathbf{R}$ is a 3-d rotation matrix. Considering $|\mathbf{R}| = 1$, we always have $|\mathbf{R}_2| = |\mathbf{R}_1|$.

BENCHMARKING WITH ZGOUBI

We extensively benchmarked single proton spin tracking between SimTrack and Zgoubi. Lattices for both RHIC and AGS are used. We compared spin vector evolution through a single element, such as a sector bend, a quadrupole, and a sextupole, etc. The results from both codes agreed very well. Figure 1 shows an example of spin tracking through a single sextupole, with the initial spin pointing to s direction. The initial coordinates are $(x, x', y, y', -cdt, dp/p_0)$ are (0.01, 0.001, 0.01, 0.001, 0,0). The sextupole length is 1.5 m long with $K_2 = 2\text{m}^{-2}$. The vertical axis is the spin component in the x direction which is plotted in log scale to see the difference between these two codes.

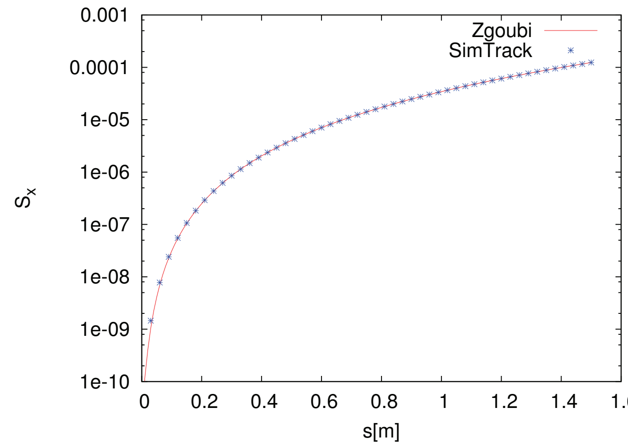

Figure 1: Comparison of spin vector through a single sextupole between SimTrack and Zgoubi. Initial spin points to s direction. $G\gamma = 44.5$.

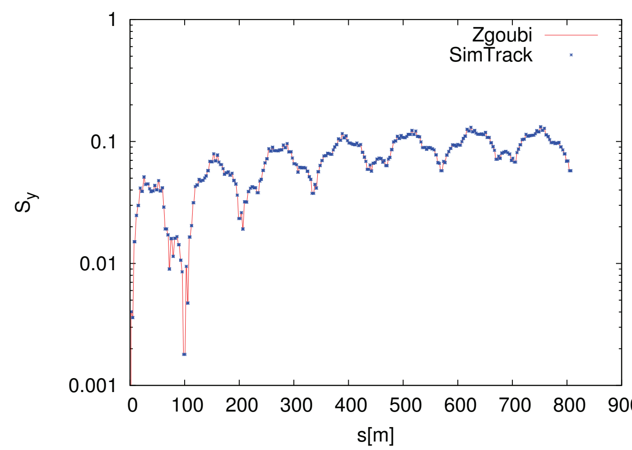

Figure 2: Comparison of spin vectors in one-turn AGS spin tracking between SimTrack and Zgoubi. Initial spin points to s direction. $G\gamma = 44.5$.

The AGS lattice was used for the one-turn and multi-turn spin tracking benchmarking between SimTrack and Zgoubi. AGS consists of combined function magnets which includes bending, quadrupole, and sextupole components. For an accurate modeling of the particle orbit and spin motions, the length of each integration step in AGS can be as short as 1 mm. Figure 2 shows an example of spin tracking for one turn in AGS. The initial coordinates are $(10^{-3}, 10^{-4}, 10^{-3}, 10^{-4}, 0, 0)$. The initial spin is in the s direction. The difference in S_y between SimTrack and Zgoubi is less than 0.1% after one turn. Snakes in AGS are not included in this comparison.

OTHER EXAMPLES

The loss of a single proton's vertical spin component during crossing of a single spin resonance is given by Froissart-Stora formula. Assuming the initial spin is pointing to the vertical direction, the final polarization after res-

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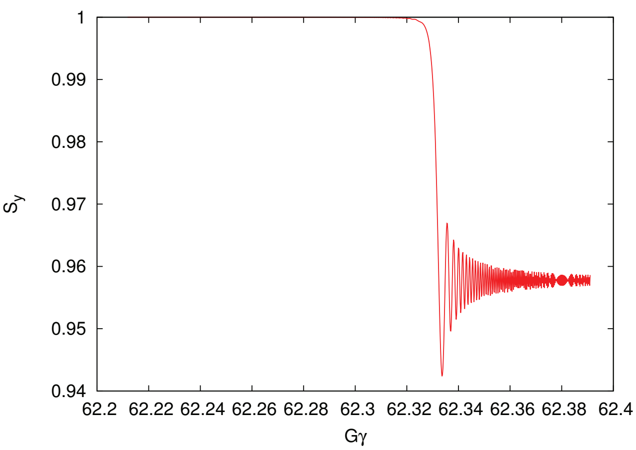

Figure 3: An example of spin vector evolution through a spin resonance crossing. $G\gamma = 62.33$ is crossed.

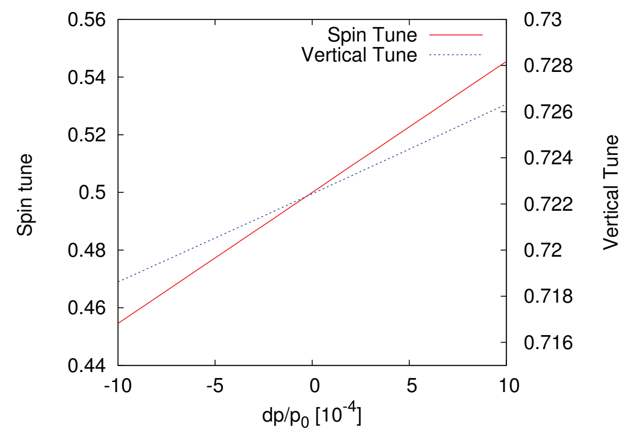

Figure 4: An example of spin tune versus relative momentum deviation dp/p_0.

onance crossing is given by

$$S_y = 2e^{-\frac{\pi|\epsilon|^2}{2\alpha}} - 1, \tag{8}$$

where $\alpha = dG\gamma/d\theta$ is the crossing speed, θ is the bending angle, $d\theta = ds/\rho$. $|\epsilon|$ is the strength of spin resonance.

Fig. 3 show the spin vector S_y across an intrinsic spin resonance crossing in RHIC. For this test, the vertical fractional tune is 0.67 and $\alpha = 2.7343 \times 10^{-6}$. The spin resonance at $G\gamma$= 62.33 is crossed. Since the spin resonance strength is so strong, here we intentionally reduced the initial particle emittance by a factor of 100000 from the normalized emittance 10 π mm.mrad. With Eq. (8), together with the initial and final spin components in vertical direction from particle tracking, we can derive the spin resonance strength at $G\gamma$= 62.33 about 0.192, which is close to its analytical estimate.

With accurate proton spin tracking, we can calculate the spin closed orbit. Spin closed orbit is the direction at one point in the ring that the spin vector repeats itself after one-turn tracking. Knowing the spin closed orbit, we are able to calculate the spin tune either from particle tracking or through linear matrix transportation. Here we launch a single particle with its initial spin direction perpendicular to the spin closed orbit.

As an example, Figure 4 shows the spin tunes and vertical tunes for particles with different relative momentum deviation dp/p_0. The RHIC lattice is used and $G\gamma = 60.5$. The spin closed orbit for off-momentum particles points vertical direction. In this study, snake and rotator magnets are not included.

Back to AGS, Figure 5 shows the vertical spin component in a 10^5 turn tracking. Again snake magnets are not included here. The particle is accelerated from injection to extracion. The energies at injection and extraction are 2.5 GeV and 25 GeV respectively. Particles cross the transition at $\gamma = 15$. During the acceleration, adiabatic damping in the particle emittance and momentum deviation are observed.

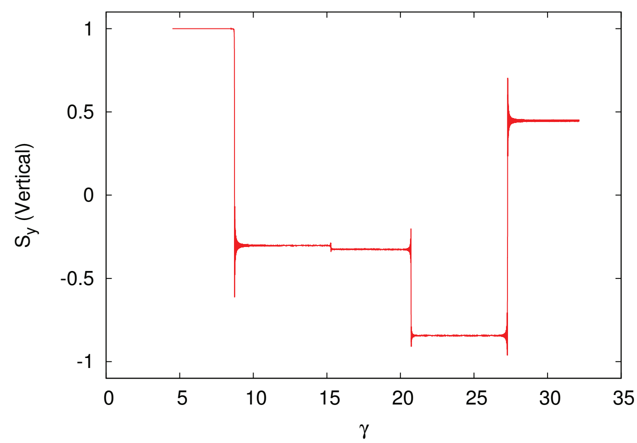

Figure 5: An example of spin vector evolution as a function of particle energy in AGS.

SUMMARY

In the article, we report implementation of proton spin tracking on top of symplectic orbit motion in SimTrack. We explained sympletic integration of orbit motion and its implementation in SimTrack and how to include spin tracking on top of it. Benchmarking of spin tracking between SimTrack and Zgoubi, and some application examples with AGS and RHIC lattices are presented.

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