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RADIATION OF A BUNCH MOVING IN THE PRESENCE OF A BOUNDED PLANAR WIRE STRUCTURE*

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Abstract

Three-dimensional [1, 2] and planar [3] periodic structures can be used for non-destructive diagnostics of charged particle bunches. Here we consider a semi-infinite planar structure comprised of thin conducting parallel wires. If the period of the structure is much less than the typical wavelength of the electromagnetic field, then the structure's influence can be described with help of the averaged boundary conditions [4]. We study radiation of a charged particle bunch with small transversal size and arbitrary longitudinal one in two cases: (i) the bunch moves orthogonally to the grid at some distance from the edge (Fig. 1) and (ii) it moves along the edge of the grid (Fig. 2). The problems are solved analytically. In both cases the bunch generates a surface wave which contains the information about the size of the bunch. The shape of the surface waves is similar to the radiation generated in the presence of 3D periodical wire structures [1], however planar structure is simpler for use in accelerating system. Some typical numerical results for bunches of various shapes are given.

PASSING BY THE GRID

Firstly we consider a bunch moving orthogonally to the semi-infinite wire grid at some distance from the edge (Fig. 1). It is assumed that the bunch has an infinitesimal transverse size and an arbitrary constant longitudinal charge distribution profile $\eta(\zeta)$. Current density has form

$$\rho_q(\vec{r}, t) = \delta(x + a_0)\,\delta(y)\,\eta(z - vt)\,. \tag{1}$$

We solve the problem in a long-wave approximation (i.e. $\lambda \gg a$). Such approximation allows us using averaged boundary condition (ABC) technique [4], which was applied for consideration of analogous problems earlier [3, 5]. The ABC are used to set electromagnetic boundary conditions in semi-plane $z = 0$, $x > 0$ (i.e. occupied by wires).

Thus, we get the problem of diffraction of the incident field of the bunch on the semi-infinite screen with specific boundary conditions. Then we use Wiener-Hopf technique [6] in order to acquire a strict analytical solution. The field of radiation has two parts: volume radiation and the surface wave. The spectral angular density of volume radiation in spherical coordinates system has the following form

$$w_\omega^v = \left(1 - \sin^2\theta\cos^2\varphi\right) cR^2 k_0^2 |A_{x\omega}^v|^2, \tag{2}$$

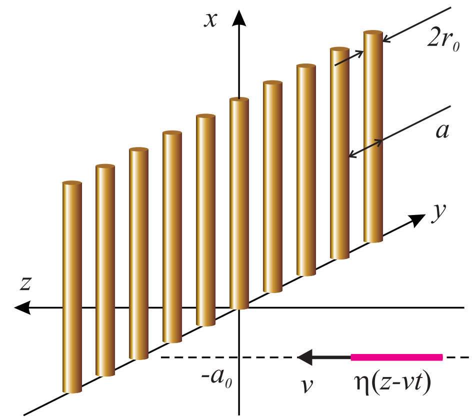

Figure 1: Bunch passing by the edge of a a grid structure.

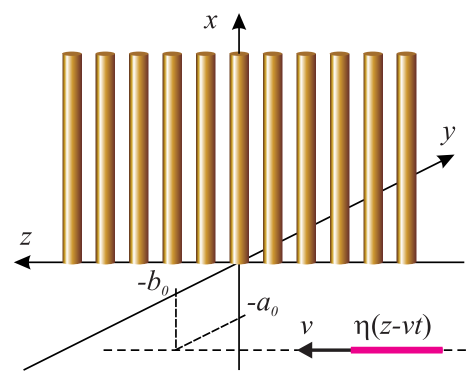

Figure 2: Bunch moving along the edge of a grid structure.

$$A_{x\omega}^v = \frac{\tilde{\eta}(\omega/v)}{\tilde{\kappa} c\beta} \frac{e^{ik_0 R}}{R} \frac{\left(k_0 + \hat{k}_{x0}\right)^{-1}}{\tilde{G}_+\left(\hat{k}_{x0}, k_0 \sin\theta\sin\varphi\right)} \times$$
$$\times \frac{\left(k_0 \sin\theta\cos\varphi - \hat{k}_{x0}\right)^{-1} (1 - \sin\theta\cos\varphi)^{-1}}{\tilde{G}_-\left(k_0\sin\theta\cos\varphi, k_0\sin\theta\sin\varphi\right)}, \tag{3}$$

$$\{x = R\sin\theta\cos\varphi,\ y = R\sin\theta\sin\varphi,\ z = R\cos\theta\},$$

where $k_0 = \omega/c$, $\hat{k}_{x0} = ik_0/\beta\sqrt{1 - \beta^2 + \beta^2\sin^2\theta\sin^2\varphi}$, $\tilde{\eta}(\xi)$ is a Fourier transform of bunch profile,

$$\tilde{G}_\pm\left(k_x, k_y\right) = \exp\left\{\ln\sqrt{\frac{\sin\sigma \pm \sin\tau}{1 \pm \sin\tau}} \pm \frac{1}{2\pi}\int_{\tau\pm\sigma\mp\pi}^{\tau\mp\sigma} \frac{tdt}{\sin t}\right\},$$

$$\tau = \arcsin\left(k_x/w_0(k_y)\right), \quad \sigma = \arccos\left(i/\left(\tilde{\kappa} w_0(k_y)\right)\right).$$

Here $-\pi < \mathrm{Re}\,\tau < \pi$, $\mathrm{sgn}\,\mathrm{Im}\,\tau = -\mathrm{sgn}\,\mathrm{Re}\,\tau$, $0 < \mathrm{Re}\,\sigma < \pi$, $w_0(k_y) = \sqrt{k_0^2 - k_y^2}$, $\tilde{\kappa} = \kappa/k_0 = \frac{a}{\pi}\ln\frac{a}{2\pi r_0}$.

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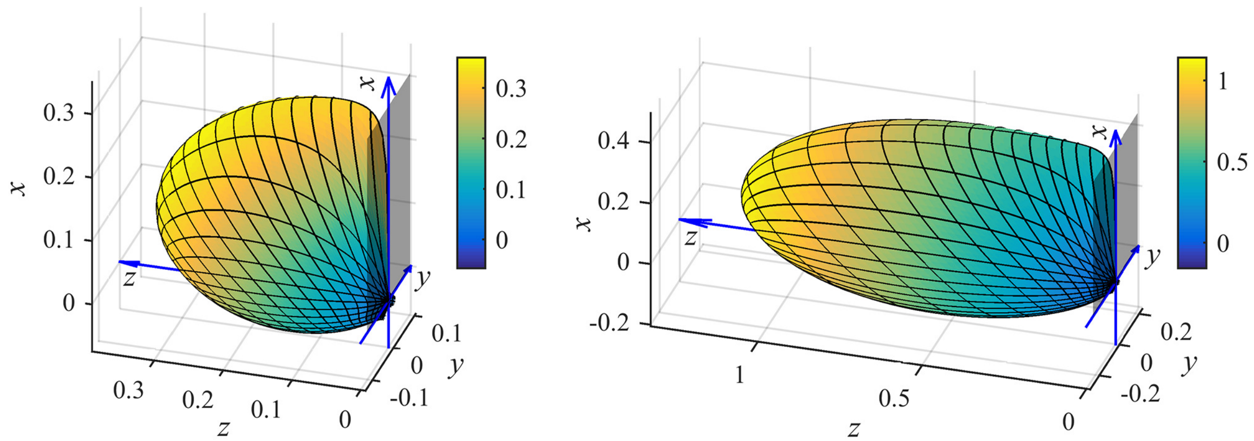

Figure 3: Spectral angular density of energy radiated by a charge moving orthogonally to semi-infinite wire grid at small distance from the edge. Parameters of the grid: wires are made of copper ($\sigma_e = 5.8 \times 10^7$ S/m), $a = 10$ mm, $r_0 = 1$ mm; frequency $\omega = \pi c/(5a)$, bunch velocity $\beta = 0.7$ for the left plot and $\beta = 0.9$ for the right plot. Radiation density is normalized on $4|\tilde{\eta}|^2/c$.

The radiation pattern of volume radiation in the case under consideration differs strongly from the case of infinite wire grid [3]. It is asymmetrical with respect to z-axis and non-zero in the direction of the charge motion. The maximum of this volume radiation lies in semi-plane $\varphi = 0$. Some numerical results are presented in Figure 3.

The surface wave represents a greater interest. In general, the non-zero components of electric field of this wave has form:

$$\begin{Bmatrix} E_y^s \\ E_z^s \end{Bmatrix} = -\frac{2i\beta}{\tilde{\kappa}} \int_0^\infty dk_y \int_{-\infty}^\infty dk_0 \begin{Bmatrix} \sin\left(k_y y\right) \\ \operatorname{sgn} z \cos\left(k_y y\right) \end{Bmatrix} \times \\ \times \frac{k_0 \tilde{\eta}\left(k_0/\beta\right) e^{i a_0 k_{x0} - |z| k_y + i k_0 (x - ct)}}{\left(\beta^2 k_y^2 + k_0^2\right) \tilde{G}_-\left(k_0, k_y\right) \tilde{G}_+\left(k_{x0}, k_y\right)}, \quad (4)$$

where $k_{x0} = \sqrt{k_0^2 - k_0^2/\beta^2 - k_y^2}$.

This wave propagates along the wires at the speed of light in vacuum without decaying because the losses in the wires are negligible. The wave does not change over time and propagates as a whole. In Fig. 4 some numerical results for the surface wave in the plane of the structure are given. It is apparent that the distance between the peak values of the electric field components and the energy flow density along the x-axis corresponds to the length of the bunch. The surface wave that is generated on the semi-infinite wire grid is asymmetrical, in contrast to the case of the infinite grid [3].

MOVEMENT ALONG THE GRID

Secondly we considered the same bunch moving along the edge of semi-infinite grid (Fig. 2), which generates the following charge density

$$\rho_q\left(\vec{r}, t\right) = \delta\left(x + a_0\right) \delta\left(y + b_0\right) \eta\left(z - vt\right). \quad (5)$$

Here we also use Wiener-Hopf method for acquiring analytical solution.

The volume radiation is absent in this case, the surface wave has the following expressions for non-zero components

$$\begin{Bmatrix} E_y^s \\ E_z^s \end{Bmatrix} = \frac{1}{\beta\tilde{\kappa}} \int_{-\infty}^\infty dk_x \int_{-\infty}^\infty dk_0 \begin{Bmatrix} i k_0 \operatorname{sgn} y \\ |k_0| \end{Bmatrix} \times \\ \times \frac{k_x \tilde{\eta}\left(k_0/\beta\right) e^{i k_0 (x - ct + z/\beta) - |k_0 y|/\beta + i k_x a_0 + i k_{y0} b_0}}{k_{y0}\left(k_x^2 - k_0^2\right) \tilde{G}_-\left(k_0, k_0/\beta\right) \tilde{G}_+\left(k_x, k_0/\beta\right)}, \quad (6)$$

$k_{y0} = \sqrt{k_0^2 - k_0^2/\beta^2 - k_x^2}$.

As before, this wave propagates along the wires with speed of light in vacuum and its shape doesn't change. The analysis of its structure allows determining the size of the bunch. The structure of the wave in some particular cases is shown in Fig. 5. The distance between the peak values of the electric field components and the energy flow density along the z-axis corresponds to the length of the bunch.

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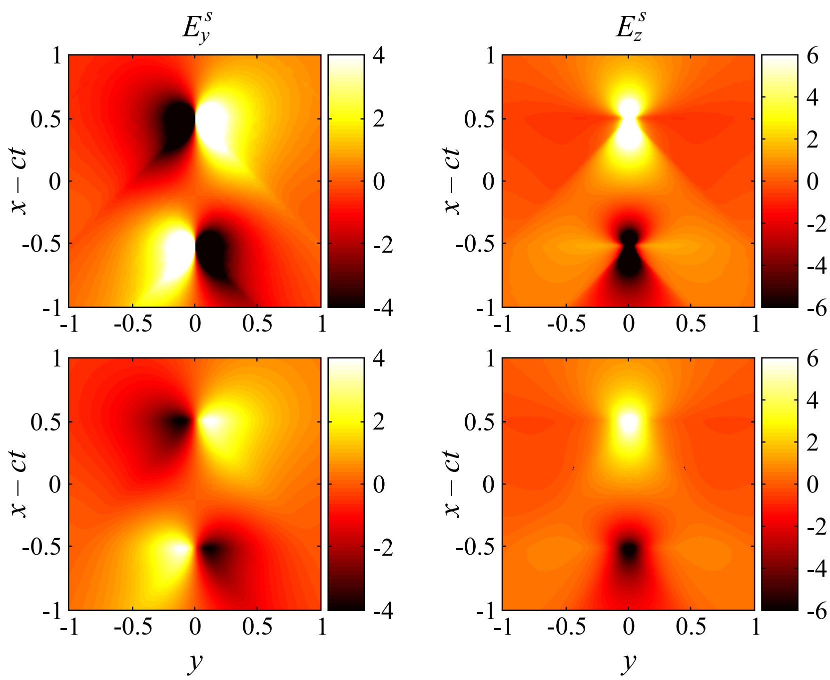

Figure 4: The field components and the energy flow density of the surface wave generated by “rectangular” bunch passing by the edge of semi-infinite wire grid (Fig. 1) in the plane $z = +0$. Wires are PEC, Gaussian units are used. The bunch length is 1 cm, the charge is $q = +1$ esu; $a = 0$ cm (the first row), $a_0 = 0.1$ cm (the second row), $\tilde{\kappa} = 0$; , $ct > \sigma$, $\beta = 1$.

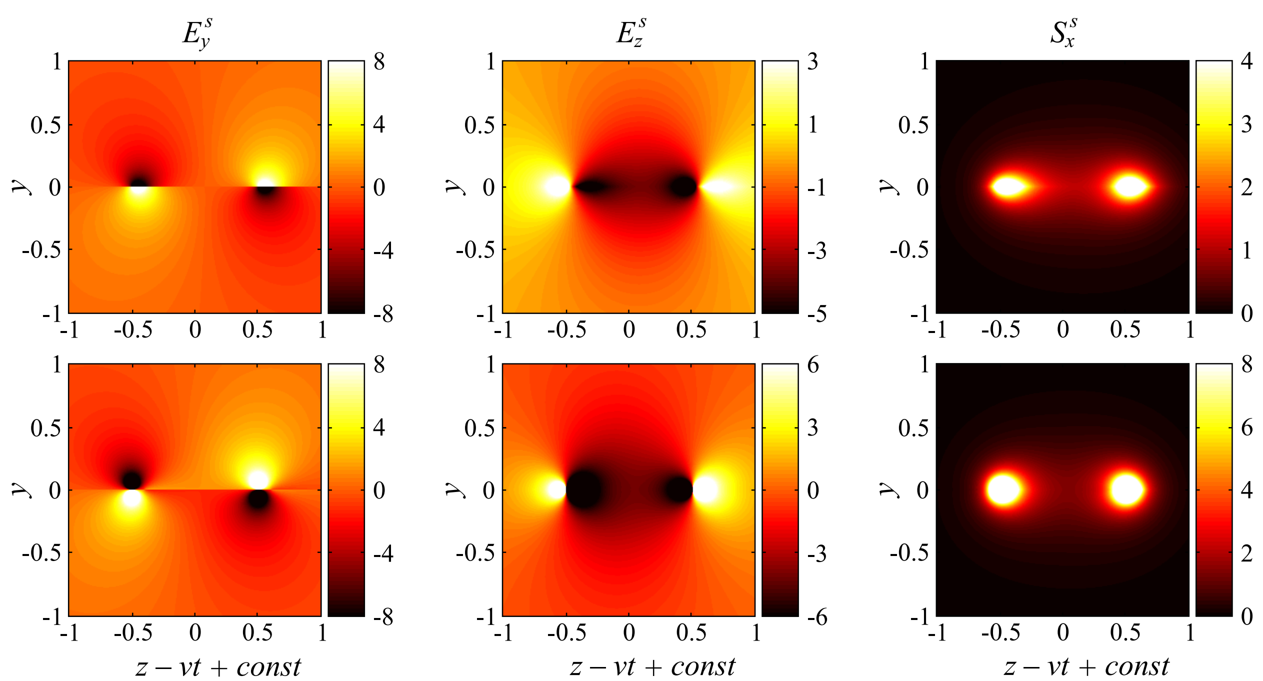

Figure 5: The field components and the energy flow density of the surface wave generated by rectangular moving along the edge of semi-infinite wire grid (Fig. 2) in the plane $x = x_0 = const > 0$, $\zeta = z - vt$. Wires are PEC, Gaussian units are used. Bunch length is 1 cm, the charge is $q = +1$ esu; $a_0 = 0$ cm and $b_0 = 0.1$ cm (the first row), $a_0 = 0.1$ cm and $b_0 = 0$ cm (the second row), $\tilde{\kappa} = 0$; $\beta = 1$.