INSTABILITY THRESHOLDS AND TUNE SHIFT ESTIMATION FOR SIRIUS

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Abstract

In this work we present the evaluation of longitudinal and transverse instability thresholds as well as tune shifts for Sirius using time and frequency domain codes that are being developed in-house and take into account various effects on the beam instability, such as bunch by bunch feedback system, quadrupolar impedances from undulator chambers and tune spreads.

INTRODUCTION

This contribution is a continuation of the work presented at IPAC14 [1], where we detailed the construction of the impedance budget for Sirius and used a frequency domain code based on the solution of the Linearized Vlasov Equation for equally spaced Gaussian bunches [2, 3] to calculate thresholds for single and coupled bunch instabilities. After analyzing the results we concluded that: a) Due to the strength of the coupled bunch instability induced by the chamber wall, a transverse bunch by bunch feedback system will operate from start for safety; b) With a small positive absolute chromaticity (maximum of 1.5 in both planes) it is possible to get a strong damping of the chamber wall instability. This motivated us to include this value in our dynamic aperture and lifetime optimizations [4]; c) The thresholds for intra-bunch instabilities in the three planes are relatively low and can compromise some operation modes.

The last item motivated us to implement a single bunch tracking code to better characterize the intra-bunch dynamics. Even though the chromaticity and the interplay between coupled and single bunch instabilities were taken into account in the frequency domain code, other important effects such as potential well distortion, third harmonic cavity, tune shift with amplitude, quadrupolar impedances and the bunch by bunch feedback were not considered. The simultaneous action of these forces changes significantly the behavior of each bunch, which can increase or decrease the thresholds.

The effects described above also influence the multi-bunch dynamics and consequently the coupled bunch motion. However, they are generally of a stabilizing nature due to the introduction of tune spreads. For this reason we decided to focus on the single bunch dynamics first. The implementation of a multi-bunch tracking code will be performed later.

SINGLE BUNCH TRACKING CODE

The single bunch tracking code is presently implemented using MATLAB® but an implementation using C/C++ is planned to gain speed in the simulations.

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Haissinski Equation Solver

The code is organized in two parts. First an equilibrium distribution in four dimensions is generated. In the transverse plane an exponential distribution of emittances and a uniform distribution of phases are used. In the longitudinal plane there are two options: a) generate energy deviations with a Gaussian distribution with the input value of the energy spread and longitudinal positions following the distribution determined by the input potential well (which can be arbitrary); b) solve the Haissinski equation, taking into account the input potential well and wakes to get the new equilibrium distribution to generate the particle's longitudinal positions and the new potential well.

The Haissinski equation solver may also change the value of the energy spread, because its algorithm first tries to solve the equation with the input value. If convergence is not achieved, it increases the value of the energy spread by a small amount and iterates again. This procedure is repeated until an equilibrium state is found [5, 6]. The new value of energy spread is then used to generate the energy deviation distribution.

Macro Particles Tracking

The second part of the code is the tracking itself. The one turn map is approximated by the following set of equations:

$$
\begin{aligned}
J &= \frac{(x - \eta_x \delta)^2}{\beta_x} + (x' - \eta'_x \delta)^2 \beta_x \\
\phi &= 2\pi \left(\nu_0 + \xi \delta + A J \right) \\
\begin{pmatrix} x \\ x' \end{pmatrix} &= \begin{pmatrix} \cos\phi & \beta_x \sin\phi \\ -\frac{1}{\beta_x} \sin\phi & \cos\phi \end{pmatrix} \begin{pmatrix} x - \eta_x \delta \\ x' - \eta'_x \delta \end{pmatrix} + \begin{pmatrix} \eta_x \\ \eta'_x \end{pmatrix} \delta \\
\tau &= \tau - T_0 \alpha \delta \\
\delta &= \delta + \frac{V(\tau)}{E_0} + K_{W_L}(\tau) \\
x' &= x' + K_{W_T}(x, \tau) + K_F(\langle x \rangle)
\end{aligned}
\tag{1}
$$

where J is the transverse action, A is the tune shift with the action coefficient, τ is the relative position of the particle ahead of the synchronous particle and K_{W_T}, K_{W_L} and K_F are the collective kicks generated by the transverse and longitudinal wakes and the feedback system, respectively. The other terms have the usual interpretation. When the longitudinal dynamics is solved with the Haissinski equation solver and the effect of the longitudinal wakes is considered in the longitudinal potential used in the tracking, K_{W_L} is zeroed to be consistent.

The wakes are generated from the Fourier Transform of the impedances, multiplied by the betatron function at the point where the kick is given, summed and passed to the tracking code as a table for interpolation. However,

for resonators, a special treatment is applied to reduce the simulation time to be, in the worst case, on the order of $O(Np \ln p)$ instead of $O(Np^2)$, where N is the number of turns and p is the number of particles. Actually a similar improvement can be achieved for any wake of the form: $W(\tau_i - \tau_j) = \sum_{k=1}^{M} F_k(\tau_i) G_k(\tau_j)$ where F_k and G_k are arbitrary functions and τ_i and τ_j are the longitudinal positions of the test and source particle. In this case, the simulation time is on the order of $O(MNp \ln p)$.

Bunch By Bunch Feedback

The model of the bunch by bunch feedback system can be summarized by the following equations:

$$
\begin{aligned}
\vec{n} &= (1, 2, \ldots, n) \\
\vec{F} &= \cos(\omega \vec{n} + \phi) \sin(2\pi \vec{n}/(2n))/(\vec{n}\pi) \\
K_i &= G\sqrt{\frac{\beta_B}{\beta_x}} \langle \vec{x} \rangle \, (i - (n + 1 - \vec{n}) - D) \cdot \vec{F} \quad (2) \\
K_{Fi} &= \sqrt{\frac{\beta_K}{\beta_x}} \operatorname{sign}(K_i) \min(|K_i|, K_{\max})
\end{aligned}
$$

where i is the turn number, $\langle \rangle$ denotes averaging over the bunch, ω, ϕ, n, G and $K_{\max}$ are adjustable parameters of the system, D is the total latency of the system and β_B and β_K are the betatron functions at the beam pickup and kicker, respectively. The frequency and phase of the filter $\vec{F}$ are generally chosen to select the betatron movement of the beam centroid and apply a kick 90° out of phase. Figure 1a shows that when this condition is matched and the feedback is successfully controlling the instability, its kick is also detuned by 180° in relation to the average wake's kick. Figure 1b shows the bunch spectrum, where we see that the shift of mode 0 is controlled and the coupling does not happen. However bunch oscillations still exist, which requires a high precision movement detection for machines such as Sirius, with very low emittance.

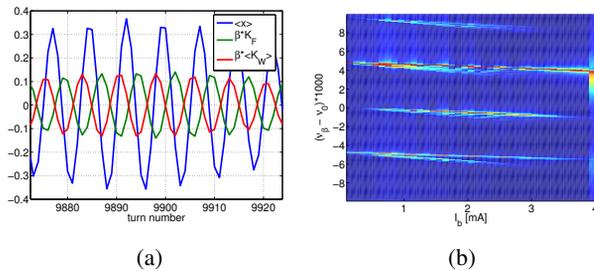(a)   (b)

Figure 1: a) Average beam position, Feedback kick and average wake field kick as a function of the turn number. b) Electron bunch spectrum as a function of current.

SINGLE BUNCH DYNAMICS

To understand the behavior of the bunch as a function of the various forces driving its movement, we studied some of them separately. For all the calculations in this section,

we used Sirius phase 1 parameters [1], and broad band resonators as wakes. The definition of the resonators is described in [1], but here a value of $Z_\parallel/n = 0.3\,\Omega$ instead of $0.2\,\Omega$ is used.

Quadrupolar Impedance

The quadrupolar wake introduces an incoherent tune shift with current which adds/opposes the coherent tune shift of mode 0 for the vertical/horizontal plane. Nevertheless, all modes shift the same way and even though this effect is a source of incoherence, it is not obvious *a priori* whether it increases the threshold of the mode coupling. Figure 2a shows that this is indeed what happens and the effect is similar in both planes, an indication that the tune shift of the modes is not what generates the damping, but the incoherence introduced by the wake. However, one difference emerges between the two planes when the detuning and driving wakes have equal shunt impedances. The cancellation of the kicks in the horizontal plane makes the dynamics very complicated, with modes coupling and decoupling several times as the current is increased.

At finite chromaticities the sign of the detuning plays a role in the head-tail modes, enhancing the mode 0 instability when $\xi < 0$ and damping mode -1 when $\xi > 0$ in the vertical plane, and damping both modes in the horizontal plane, as shown in Figure 2b. Further studies are necessary to better understand this behavior.

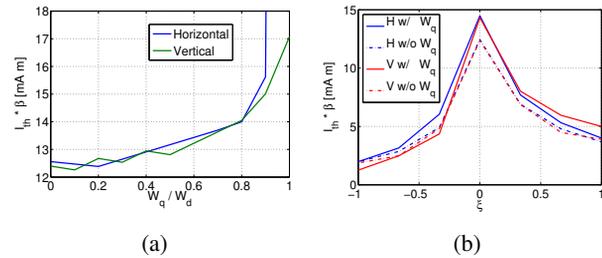(a)   (b)

Figure 2: a) Current thresholds for TMCI as a function of the quadrupolar wake strength. b) Current thresholds for head-tail modes as a function of chromaticity with and without quadrupolar wake.

Potential Well Distortion

The longitudinal potential well distortion is a very difficult effect to be properly introduced in frequency domain codes, because besides the change in the bunch length, other aspects of the longitudinal dynamics are affected as well, such as tunes and possibly the energy spread. Figure 3a shows the growth rates of the transverse instabilities with and without potential well distortion and Figure 3b shows the longitudinal distribution. At zero chromaticity the threshold is increased in the vertical plane because of the longitudinal bunch shortening. In the horizontal plane the contrary happens, because above the microwave threshold the bunch elongates abruptly, in such a way that the thresholds for the three instabilities happen almost at the same current. When

the chromaticity is positive its damping effect is enhanced by the higher energy spread and higher bunch length generated by the microwave regime.
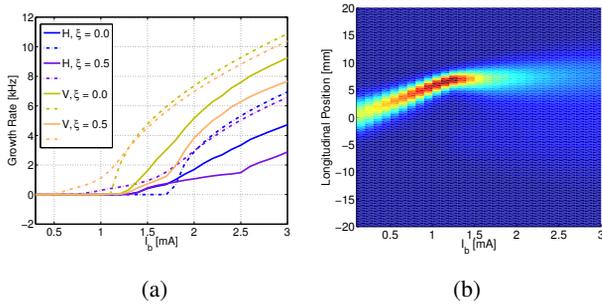

(a)   (b)

Figure 3: a) Current thresholds as a function of current with (full lines) and without (dashed lines) potential well distortion. b) Longitudinal distribution as a function of current and longitudinal position. Notice the onset of the turbulent bunch lengthening at ≈1.3 mA.

Tune Shift With Amplitude

The tune shift with amplitude introduces tune spread and is therefore a stabilizing force. However, this effect is not significant in the case of very small emittances. We see in Figure 4a that the tune shift with amplitude does not change the growth rate before saturation and, in Figure 4b, that the tune spread needed to damp the instability is around 10^{-2}. Thus this effect will not contribute to the stabilization of the beam, because Sirius tune shift with amplitude coefficient is $\approx 10^{-5}$ (nm.rad)$^{-1}$.
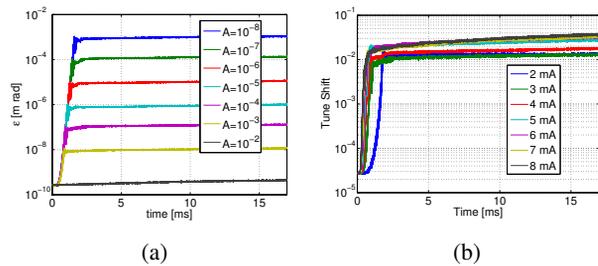

(a)   (b)

Figure 4: a) Beam emittance as a function of time and A for $I_b = 3$mA. The unit for A is (nm.rad)$^{-1}$. b) Tune spread introduced by the tune shift with amplitude as a function of time and the bunch current.

ESTIMATES FOR SIRIUS

To estimate the threshold instabilities for Sirius, we used the impedance budget described in [1] with a few changes. The broad band impedance for phase 1 will be the same as the one of the last section. Also, the impedance model for Injection Kicker was updated and for the Pulsed Multipole Magnet [7] was added. The initial values of emittance and energy spread used in the codes already take into account the effect of IBS and IDs of the respective operating phase of the ring.

Figure 5 show the longitudinal dynamics of phase 2, where we note that even with the Landau cavity we still have the microwave instability. For the transverse plane, we noted a good agreement between tracking and the frequency domain codes when the results of the longitudinal dynamics are considered in both codes. For this reason and because with the frequency domain code we can also estimate the effect of the long range wall impedance, we used this approach to estimate the instabilities threshold for Sirius. Figures 6a and 6b show the results for phase 1 and 2, respectively, where we note that the thresholds for the most unstable coupled bunch mode are much smaller than the modes not influenced by the long range wall wake and, given the operation current per bunch for uniform filling, we are still unstable even for positive chromaticities.
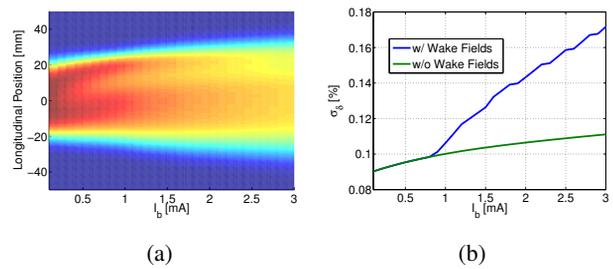

(a)   (b)

Figure 5: a) Longitudinal distribution as a function of current and longitudinal position for phase 2. b) Increase of the energy spread due to the microwave instability.

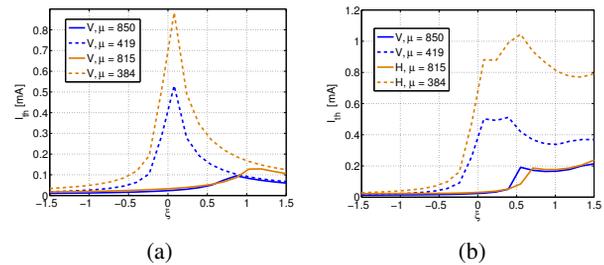(a)   (b)

Figure 6: Most unstable (full lines) and least unstable (dashed lines) coupled bunch modes as a function of chromaticity for a) phase 1 and b) phase 2.

CONCLUSIONS

With the single bunch tracking code and Haissinski equation we were able to calculate the effect of several factors important to the dynamics of the single bunch in Sirius. However a more detailed study of the effect of the bunch by bunch feedback is needed. Another important result of this work was the agreement between the frequency domain and time domain approaches when the longitudinal dynamics is taken into account properly. The continuation of the work involves the improvement of the tracking code and more detailed simulations.

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