CORE-HALO LIMIT AS AN INDICATOR OF HIGH INTENSITY BEAM INTERNAL DYNAMICS

P.A.P. Nghiem*, M. Valette, N. Chauvin, N. Pichoff, D. Uriot

CEA/DSM/IRFU, Centre de Saclay, 91191 Gif-sur-Yvette cedex, France

Abstract

The dynamics of high-intensity beams is mainly governed by their internal space charge forces. These forces induce emittance growth and halo generation. They contribute to shape the beam density profile. As a consequence, a careful analysis of this profile can help revealing the internal dynamics of the beam. This paper recalls the precise core-halo limit determination proposed earlier, then studies its behaviour through a wide range of beam profiles and finally shows its relevance as an indicator of the limit separating the two specific space charge field regimes of the core and the halo.

INTRODUCTION

In high intensity accelerators, the outermost part of the beam, although tenuous, is often the object of great attention. Indeed, the halo is the main contributor to particle losses downstream, which must be maintained under specified levels. There is nevertheless no consensus on a definition of the halo [1]. Yet there is a real need of precisely and quantitatively defining halo. Simulations of halo generation and halo growth all need to know how much halo has been generated or how much it has grown. For halo measurement or halo cleaning there is a need to know what to measure and what to clean.

In this paper, we first recall the existing definitions of halo and study their relevance through a great variety of different density profiles. Then we point out the particular relevance of the definition we proposed earlier by showing that it is a good indicator of the beam internal dynamics.

EXISTING DEFINITIONS

Halo definitions often consist in comparing beam characteristics somewhere "far from" and "close to" beam centre. Are considered the ratio of particles contained in n times rms emittance to those contained in rms emittance, or the ratio of emittance containing x% of particles to rms emittance, with generally n>3 and x> 90%. But to be relevant, n or x must be adjusted to each particular density profile, according to visual inspection in most cases. This makes difficult studies of halo evolution that comes with beam shape evolution.

To our knowledge, there exist only two definitions able to be applied to any density profile.

The first one, widely used, stated that halo can be characterised by the parameter h, defined as ratio of the fourth to the square of the second invariant moment [2]. This corresponds to the profile kurtosis or "peakedness". Thus, the higher the kurtosis is, the larger the halo would

_____
* phu-anh-phi.nghiem@cea.fr

be. This definition, valid for a 1D projection, was later extended to a 2D phase space [3].

The second definition started from considering the extreme case of a uniform core surrounded by a tenuous halo, where the core-halo limit is indisputably at the foot of the discontinuity in density ρ, i.e. where there is an infinite change in the slope of ρ. For a more general case of continuously varying density, it is proposed that the core-halo limit is where there is the biggest change in slope, that is where the second derivative ρ" is maximal [4, 5]. Once this limit precisely known, it is then possible to define the parameters PHS and PHP, which are respectively the Percentage of Halo Size and Percentage of Halo Particles.

The relevance of the parameters h, PHS and PHP can be studied through their behaviour for a great variety of different density profiles.

FOR DIFFERENT DENSITY PROFILES

For this study, without lack of generality, it is enough to consider the cases of cylindrically symmetric beam where the density profile depends only on the radius, ρ(r). For the sake of comparison, we state that the beam external limit is where density decreases down to 10^{-6} of its maximum.

Let us consider first the large family of density profiles described by the Generalized Gaussian functions:

$$\rho(r) = \rho_0 e^{-\left|\frac{r}{\alpha\sqrt{2}}\right|^{\beta}}. \tag{1}$$

For given parameters ρ_0 and α (both fixed to 1 here), different profiles can be described by varying continuously β from ∞ (uniform) to 2 (Gaussian), and even ≤ 1 (sharply convex). The first six graphs of Fig. 1 shows some typical examples of such profiles ρ together with its second derivative ρ" of which the location of maximum marks the core-halo limit (black curves). The first graph of Fig. 2 gives the corresponding h, PHS, PHP. The regular decrease of all these parameters would let us think that h on one side and PHS, PHP on the other are compatible for this type of density profile [6]. The h parameter would combine the two information of halo size and halo particles, while PHS and PHP can analyse them separately. However, for more particular profiles like triangular or parabolic ones (two last graphs of Fig. 1), the kurtosis of the first profile is three times bigger than that of the second, while PHS=PHP=0 for both profiles, meaning no halo, as expected for profiles decreasing abruptly to zero.

The cases of a core and a halo described by the sum of two Gaussians can also be examined:

5: Beam Dynamics and EM Fields

D08 - High Intensity Linear Accelerators - Space Charge, Halos

$$\rho(r) = \rho_1 e^{-\left(\frac{r}{\sigma_1\sqrt{2}}\right)^2} + \rho_2 e^{-\left(\frac{r}{\sigma_2\sqrt{2}}\right)^2} \qquad (2)$$

where $\rho_2 \leq \rho_1$ and $\sigma_2 \geq \sigma_1$. Second and third graphs of Fig. 2 show the evolution of h, PHS and PHP for a main beam ($\rho_1 = \sigma_1 = 1$) with resp. halos with bigger and bigger lengths ($\rho_2 = 0.01$, $\sigma_2 = 1$ to 10) then bigger and bigger amplitudes ($\rho_2 = 0.01$ to 1, $\sigma_2 = 3$). In all the cases (density graphs are not shown here), the core-halo limit changes in accordance with visual inspection and PHS, PHP increase as expected. Their variation can even be calculated analytically. The h parameter is compatible with them for $\sigma_2 < 8$ in the first density type and only for $\rho_2 < 0.04$ in the second type. The relation between kurtosis and halo is not clear in case of particle distributions composed of a same core and different halos.

The family of hollow beams can also be studied (graphs are not shown here). Their density profiles can be represented by subtracting two generalized Gaussian functions like Equation (1). The last graph of Fig. 1 shows the evolution of the different halo parameters for the examples of $\alpha_1 = 1$, $\beta_1 = 5$, $\rho_1 = 1$ for the main beam and $\alpha_2 = 0.2$, $\beta_2 = 3$, $\rho_2 = 0$ to 1 for the hollow. Such distributions with hollow in the centre with constant size and growing amplitudes correspond to kurtosis decreasing linearly. But if we consider that the halo is only in the external part, its extension is always the same, independently of the central hollow importance. That is why the core-halo limit is always located at exactly the same position. Therefore PHS is strictly constant and PHP increases with the hollow amplitude.

The above studies of a wide variety of density profiles show that the core-halo limit defined as the location of ρ'' maximum, with the associated PHS and PHP, behave exactly as expected and in total accordance with visual inspection. But like most of the attempts to define halo, it relies on the analysis of only the particle distribution profile. The much more important question is to see if the halo so defined reflects somewhat the internal dynamics of the beam or not.

BEAM INTERNAL DYNAMICS

In high intensity accelerators, the beam internal dynamics is governed by the space charge self field. The latter is given, according to the Gauss' law in our case of cylindrically symmetric beam, by its radial component

$$E_r(r) = \frac{1}{\varepsilon_0 r} \int_0^r \rho(x)x\,dx \qquad (3)$$

where ε_0 is the vacuum dielectric permittivity. In order to study the possible link between halo and beam internal dynamics, we propose to consider the E_r profile and search to answer the following three questions:
1) Does the space charge field profile exhibit two distinct zones, one internal and one external?
2) If yes, is it possible to define a clear limit between the two zones?
3) If yes, where is the position of this limit compared to that of the core-halo limit?

Equation (3) shows that in case of a uniform beam, E_r is strictly linear inside the beam and proportional to $1/r$ outside (see first graph of Fig. 1, red solid curve). It also shows that in general cases, there will always be one internal region containing a part enough close to the centre where E_r is linear and an external region containing a part enough far from the centre where E_r is proportional to $1/r$ (all graphs in Fig. 1, red solid curves).

For defining a precise limit between these two regions, we propose to use the same principle as earlier for the core-halo limit: since for the uniform density the internal-external limit is indisputably at the foot of the discontinuity of the first derivative E_r' where there is an infinite change in slope, this limit can be defined, in case of continuously varying E_r', as the location where there is the biggest change in slope, i.e. where E_r''' is maximum.

Despite ρ'' and E_r''' are not equal, the locations of their maxima almost coincide, within less than 2%, for the very wide variety of density profiles studied above (all graphs in Fig. 1, dotted curves in black and red). The same for the family of curves resulting from sums of two Gaussians or Generalised Gaussians (graphs not shown here). This behaviour remains true for the two extreme cases:
- For density profiles decreasing abruptly to zero like parabolic or triangular profiles, the particle distribution can be considered as composed by uniquely core and no halo. E_r in the entire external part is strictly proportional to $1/r$.
- For very peaked density profiles described by Generalised Gaussians with $\beta \leq 1$, the two maximum peaks perfectly coincide at $r = 0$ meaning that there is only halo and no core. Indeed, for these profiles, the part where E_r is strictly proportional to r is reduced to $r = 0$.

More generally, a flatter core or flatter halo indicates a larger part of the core where the space charge field is linear or a larger part of the halo where it is proportional to $1/r$.

It can be therefore summarised that examining the core and halo parts of the density profile is enough to know the main features of its internal field profile, thus its internal dynamics governed by this field profile.

Notice also that the so defined core-halo limit can constitute the missing bridge between the two different approaches often used for studying halo, namely the core-particle and self-consistent approaches. It is coherent with the view of two different beam regions of the core-particle methods, the halo particles experiencing space charge field induced by the core, and can help to precisely quantify the halo in case of continuous distributions studied by self-consistent methods. Thanks to this, it should be possible to check the coherence between the two approaches.

CONCLUSIONS

High intensity beams feature important internal space charge forces that can induce emittance growth via halo formation. The core-halo limit defined as the location of

density second derivative maximum allows to precisely identify the halo part. It corresponds well to a visual inspection of the density profile and varies as expected for the very wide range of different profiles studied. In addition, this core-halo limit reveals the internal dynamics of the beam which is submitted to two different regimes of space charge forces. The circle is complete: space charge forces induce halo that can be characterized according to properties of space charge forces themselves.

An extension of this definition to the 2D phase space can also allow to define the emittance and Twiss parameters of the core and the halo separately [7].

REFERENCES

[1] J. Wei et al., proceedings of HALO'03 workshop, vol. 693, AIP (2003)

[2] T. P. Wangler & K. R. Crandall, Proc. of XX International Linac Conference (2000)

[3] C.K. Allen & T. P. Wangler, Phys. Rev. Spec. Topics - Accelerators and Beams, vol. 5, 124202 (2002)

[4] P.AP. Nghiem et al., Appl. Phys. Lett. 104, 074109 (2014).

[5] P.A.P. Nghiem et al., Proc. of IPAC'14, THPME003, Dresden, Germany (2014).

[6] N. Pichoff et al., Proc. of IPAC'14, THPME057, Dresden, Germany (2014).

[7] M. Valette & P.A.P. Nghiem, Proc. of IPAC'15, MOPWA010, Richmond, VA, USA (2015).fa
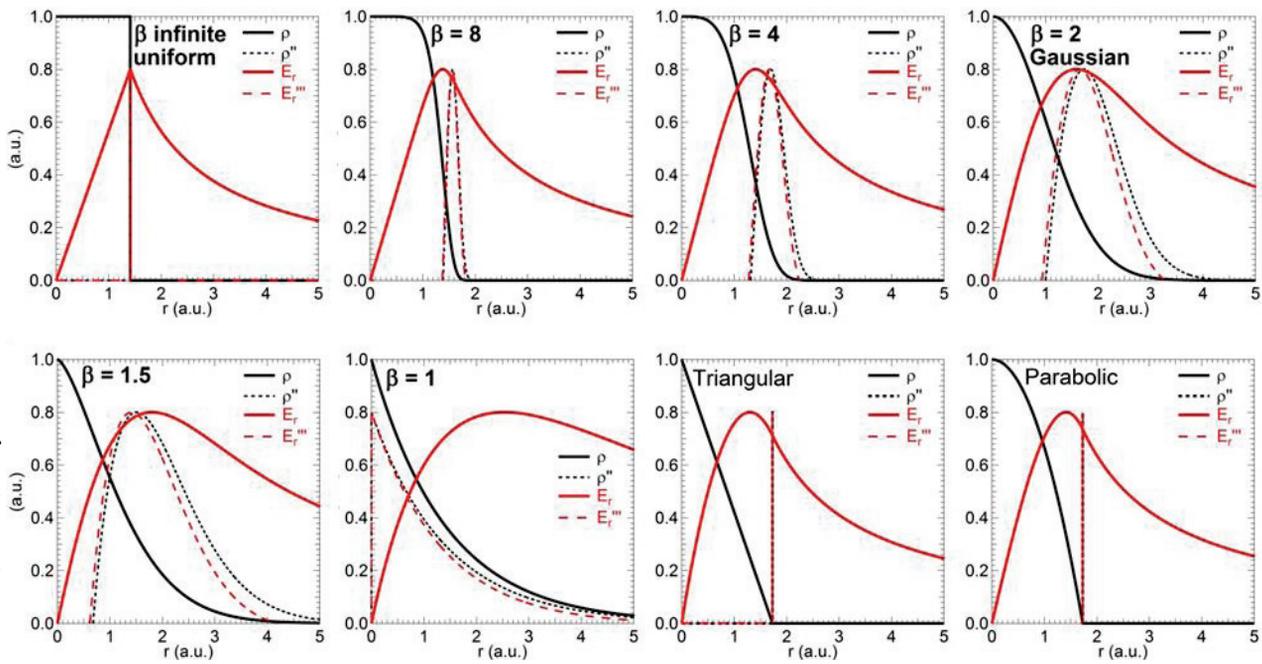
Figure 1: Density ρ, space charge field E_r and their derivatives ρ'', E_r''' as functions of radius r for Generalized Gaussian distributions (first six graphs $\beta = \infty$ down to 1) and for Triangular (7th graph), Parabolic (8th graph) distributions. The density maximum is normalised to 1 and the others to 0.8.
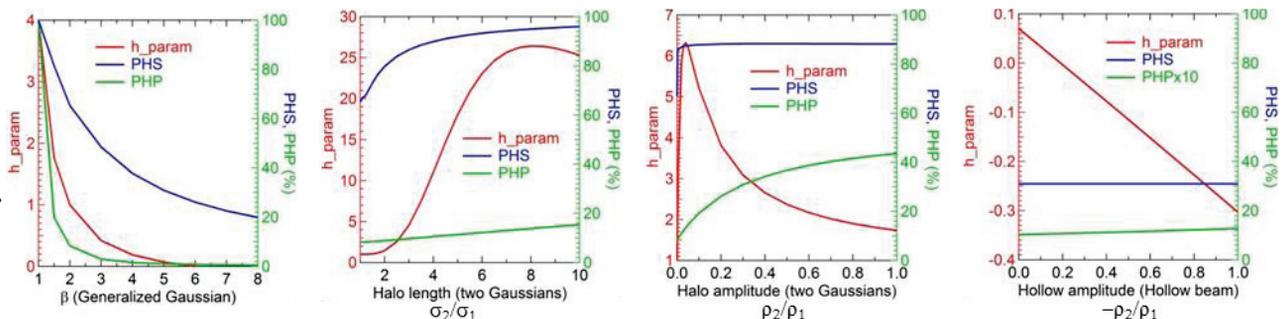
Figure 2: PHS, PHP as functions of Generalized Gaussian's β (1st graph), then of Halo length, Halo amplitude in case of two Gaussians (2nd and 3rd graph) and finally of Hollow amplitude in case of hollow beam (4th graph).