THE DAMPING OF TRANSVERSE COHERENT INSTABILITIES BY HARMONIC CAVITIES

F. Cullinan*, R. Nagaoka, Synchrotron SOLEIL, Gif-sur-Yvette, France

G. Skripka, P. F. Tavares, MAX IV Laboratory, Lund, Sweden

Abstract

At nonzero chromaticity, the threshold current due to transverse coupled bunch instabilities in an electron storage ring is defined by intrabunch head-tail motion of higher than zeroth order. Multibunch tracking simulations predict that this threshold can be increased to several times its original value through the introduction of bunch lengthening harmonic cavities. One previously suggested explanation is the narrower spectra of the elongated bunches but reliable estimates for the threshold currents are not obtainable for anything other than rigid beam motion since the usual Sacherer formalism is not directly applicable to beams in a non-harmonic potential. A new scheme has been developed in which the decay time of a higher than zeroth order transverse head-tail mode may be estimated by taking into account the synchrotron tune spread generated by the harmonic cavity potential. This scheme is presented along with the results of numerical simulations performed in order to confirm the analytical predictions and justify the assumptions made. The extension of the scheme to more complex scenarios is also discussed.

INTRODUCTION

Many present and future light sources based on an electron storage ring make use of passive or active harmonic cavities (HCs) [1] [2] [3]. In most cases, these are used to lengthen the electron bunches, thus reducing the impact of intrabeam scattering and allowing for a higher beam current while maintaining the same small transverse emittance. HCs work by modifying the radio frequency (RF) potential in which the electron bunches are confined longitudinally. Since the potential is no longer quasi-harmonic, the synchrotron tune of a single particle is no longer approximately independent of its amplitude of synchrotron oscillation. The stronger amplitude dependence results in a large tune spread within a bunch and Landau damping of longitudinal instabilities [4].

The use of harmonic cavities has important consequences in the transverse plane, particularly regarding head-tail motion. A head-tail mode describes coherent motion within a bunch. At the lowest order ($m = 0$), this is a dipole oscillation of the whole bunch while at higher order, there is a change in betatron phase along the bunch so that there is some longitudinal structure to the transverse oscillation with nodes and antinodes. The effect of nonzero chromaticity is to add a further modulation of the transverse motion at the chromatic frequency, shifting its spectrum in the frequency domain. Head-tail motion is important for multibunch instabilities since the collective motion of multiple bunches

is strongly linked to the overall motion of each individual bunch.

A HC lengthened bunch corresponds to a narrower range of frequencies and so a small positive chromaticity is more effective in decoupling the zeroth order head-tail mode from the damaging impedance at negative frequency, so called head-tail damping. Frequency domain calculations therefore show a significant improvement in coupled bunch instability threshold currents for the $m = 0$ mode [5]. As the chromaticity is increased further, for a single RF system, the threshold current would then be limited by the presence of a first order head-tail mode which is not damped since its spectrum peaks at a negative frequency. However, because of the synchrotron tune spread from the HC-modified potential, the higher than zeroth order head tail modes will no longer remain coherent indefinitely as they would in a harmonic potential. Studies of this regime with HCs are therefore, at present, limited to multiple particle tracking and although these simulations show that an improvement in the current threshold is maintained [6], it can no longer be attributed solely to the lengthening of the bunch. This paper attempts to quantify the contribution of the synchrotron tune spread.

THEORY

The position of a particle in longitudinal phase space can be expressed using cartesian coordinates τ and δ where the former is the arrival time with respect to the synchronous particle ($\tau > 0$ refers to the head of the bunch) and the latter is the energy deviation normalised by the design energy. Using, as an approximation, a sinusoidal oscillation at synchrotron tune Q_s, the motion of a particle in longitudinal phase space can be expressed as

$$\tau = \hat{\tau} \cos(\omega_s(\hat{\tau})t + \psi_0) \tag{1}$$

$$\delta = -\frac{\dot{\tau}}{\alpha_c} = \frac{\omega_s(\hat{\tau})}{\alpha_c} \hat{\tau} \sin(\omega_s(\hat{\tau})t + \psi_0) \tag{2}$$

where $\hat{\tau}$ is the synchrotron amplitude, ψ_0 is the synchrotron phase at $t = 0$, α_c is the momentum compaction factor and $\omega_s = Q_s \omega_0$ is the angular synchrotron frequency for angular revolution frequency ω_0. ω_s has been expressed as a function of the amplitude $\hat{\tau}$ so that the equations can approximate nonharmonic potentials, ignoring the inevitable harmonics that these introduce.

Longitudinal phase space can be normalised and converted into convenient polar coordinates (r, θ) by multiplying the δ coordinate by $\alpha_c / \omega_s(r)$. A particle's synchrotron amplitude and phase can then be determined from its position in synchrotron phase space. Under the current approximation, $\hat{\tau} = r$ and $\psi_0 = \theta - \omega_s(\hat{\tau})t$ [7].

---
* francis.cullinan@synchrotron-soleil.fr

In a single, purely azimuthal head-tail mode of order m, the phase of a particle's betatron oscillation is dependent on the synchrotron phase and, when there is chromaticity, the synchrotron amplitude [7]. Taking one point (r, θ), the complex betatron coordinate, whose imaginary component is proportional to the trajectory angle, can be written as

$$y(r, \theta, t) = \hat{y} \exp\left[i\left(m(\theta - \omega_s(r)t) - \omega_\beta t + r\frac{\xi\omega_0}{\alpha_c}\cos\theta\right)\right] \quad (3)$$

where ω_β is the angular betatron frequency, $\hat{y}$ is the betatron amplitude and ξ is the chromaticity. Assuming all particles have the same betatron amplitude, the dipole moment of a bunch can then be calculated as

$$\langle y\rangle(t) = \frac{\int\int y(r, \theta, t)\rho(r, \theta)drd\theta}{\int\int \rho(r, \theta)drd\theta} \quad (4)$$

where ρ is the beam charge distribution in longitudinal phase space. Integrating over a sector of longitudinal phase space gives the coherent motion of one part of the bunch.

Radial Bunch Distributions in a Flat Potential

An important case for a double RF system is the flat potential condition where the tuning of the harmonic cavities is such that the first and second time derivatives of the potential are zero at the synchronous phase of the RF. When this condition is met, the bunch has a quartic distribution in time and is as long as possible without being double peaked or asymmetric. For small synchrotron amplitudes, the synchrotron tune is then proportional to the amplitude with the constant of proportionality k_s given by

$$k_s = Q_{s0}h\omega_0^2 \frac{\pi}{2K(1/\sqrt{2})}\sqrt{\frac{n^2-1}{6}} \quad (5)$$

where Q_{s0} is the synchrotron tune without the HC, K is the complete elliptic integral, h is the ring harmonic number and n is the RF harmonic of the HCs [8].

Equation 4 has been evaluated for a flat potential and for two approximate radial particle distributions $\rho(r)$. The first
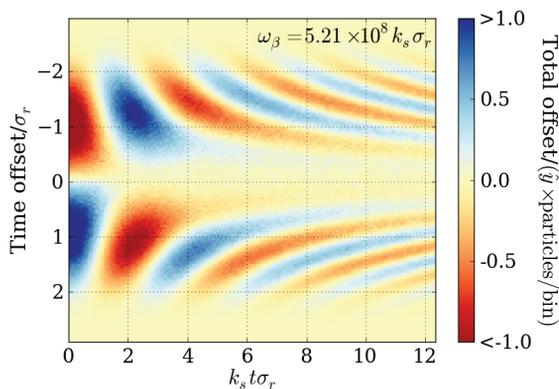

Figure 1: Decoherence of the $m = 1$ mode in a turn by turn simulation of a beam in a flat potential. The betatron oscillation has been undersampled for illustrative purposes.

is a uniform particle distribution given by

$$\rho_{\text{uniform}}(r) = \begin{cases} \frac{q}{4\pi\sigma_r^2} & \text{if } 0 \le r < 2\sigma_r \\ 0 & \text{if } r \ge 2\sigma_r \end{cases} \quad (6)$$

where q is the bunch charge and σ_r is the RMS bunch length. The second is a radial Gaussian distribution given by

$$\rho_{\text{Gauss}}(r) = \frac{q}{2\pi\sigma_r^2}\exp\left(-\frac{r^2}{2\sigma_r^2}\right). \quad (7)$$

The limits of the integration over the synchrotron phase in Eq. 4 were set to determine the average betatron coordinate of the particles in a sector of longitudinal phase space of angular spread $2\Delta\theta$ around synchrotron phase θ.

After integration of the denominator and integration of the numerator over r, Eq. 4 becomes

$$\langle y\rangle(\theta, \Delta\theta, t) = \int_{\theta-\Delta\theta}^{\theta+\Delta\theta}\frac{\hat{y}\exp(i[m\theta' - \omega_\beta t])}{2\Delta\theta}\times$$
$$F\left(mk_st\sigma_r - \frac{\xi\omega_0}{\alpha_c}\sigma_r\cos\theta'\right)d\theta' \quad (8)$$

where F is a function whose form depends on the distribution. In the case of the uniform distribution, it is given by

$$F_{\text{uniform}}(x) = \frac{e^{-2ix} - 1 + 2ixe^{-2ix}}{2x^2} \quad (9)$$

whereas for the Gaussian distribution, it is given by

$$F_{\text{Gauss}}(x) = \left[1 - \frac{x}{\sqrt{2}}D\left(\frac{x}{\sqrt{2}}\right) - i\sqrt{\frac{\pi}{2}}xe^{-\frac{x^2}{2}}\right] \quad (10)$$

where D is the Dawson function.

In the case where the chromaticity ξ is zero, the remaining integration over θ' can be evaluated analytically:

$$\langle y\rangle(\theta, \Delta\theta, t) = \frac{\hat{y}\exp(i[m\theta - \omega_\beta t])\sin(m\Delta\theta)}{m\Delta\theta}F(mk_st\sigma_r). \quad (11)$$

Equations 8 and 11 are similar for both distributions in a harmonic potential, i.e. where the synchrotron tune is independent of the particle amplitude. It is simply necessary to set $k_s = 0$ and to multiply the right hand side by $\exp(-im\omega_{s0}t)$ where $\omega_{s0} = Q_{s0}\omega_0$. For the case where the chromaticity is nonzero, numerical integration of Eq. 8 (and the harmonic potential equivalent) is possible for both distributions.

NUMERICAL MODEL

A numerical model was used to test the theory outlined in the previous section. Turn by turn transformations in longitudinal phase space were performed using the equations

$$\tau_{i+1} = \tau_i - \frac{2\pi\alpha_c}{\omega_0}\delta_{i+1} \quad (12)$$

$$\delta_{i+1} = \delta_i +$$
$$\frac{eV_{\text{RF}}}{E_0}[\sin(h\omega_0\tau_i + \phi_s) + \kappa\sin(hn\omega_0\tau_i + n\phi_h)] - \frac{U_{\text{rad}}}{E_0} \quad (13)$$

where the index i refers to the turn number, V_{RF} is the amplitude of the RF voltage, E_0 is the synchronous particle energy, ϕ_s is the synchronous RF phase, U_{rad} is the total energy lost to radiation in one turn and e is the elementary charge. κ and ϕ_h are parameters of the HC that must be tuned to obtain the flat potential condition [9]. The synchrotron tune with no HC and in the harmonic potential approximation Q_{s0} is determined by the machine parameters but their exact values are not relevant to the following discussion.

A distribution of 10^5 particles, quartic in time and Gaussian in energy, was generated and the synchrotron amplitude and phase of each particle were determined from its position in longitudinal phase space as described above. A coherent head-tail mode was then excited in the beam by assigning each particle the same betatron amplitude and a betatron phase according to Eq. 3.

Figure 1 shows the results of tracking of the $m = 1$ head-tail mode. The destruction of the mode due to the spread in synchrotron frequencies for particles at different time offsets, and therefore, with different synchrotron amplitudes, is clear. Figure 2 shows the resulting decrease in the amplitude of
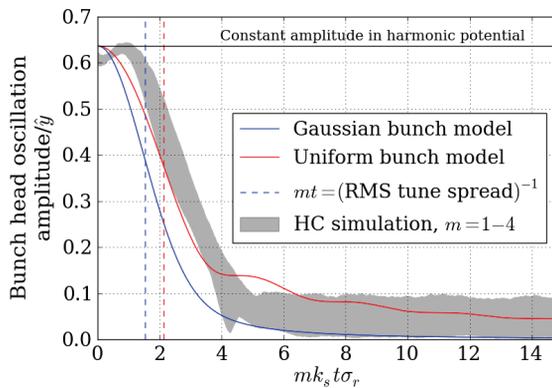

Figure 2: Decoherence of the $m > 0$ head-tail modes in a flat potential with zero chromaticity. The grey band covers all the results of the tracking for modes $m = 1 - 4$.

the bunch head betatron motion, for the case where the chromaticity is zero. Here, the bunch head for a given mode is defined as the sector of phase space between $\theta \pm \Delta\theta = 0 \pm m\pi/2$. Choosing a smaller value for $\Delta\theta$ increases the initial amplitude but does not change the rate of destruction of the coherent head-tail motion. The rate scales with the mode number m, allowing all modes to be shown on the same figure. For $m = 1$, the mean decay time is very close to the inverse of the RMS tune spread ($k_s t \sigma_r = 3/\sqrt{2}$ for the uniform distribution and $k_s t \sigma_r = \sqrt{2/(4-\pi)}$ for the Gaussian).

The HC turn by turn tracking shows a similar decay except there is some difference in the behaviour of the different modes. This is due to the approximation of the single particle motion as an oscillation at a single frequency, which is used to determine the synchrotron amplitude and phase of each particle. In reality, the motion contains harmonics of the synchrotron frequency making it impossible to selectively excite a single head-tail mode using this approximation.

The evolution of the bunch head motion for nonzero chromaticity was obtained by numerical integration of Eq. 8 and is shown in Fig. 3. The behaviour is no longer the same for
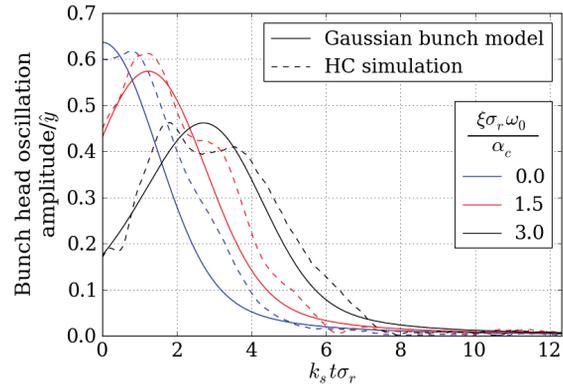

Figure 3: Decoherence of the $m = 1$ head-tail mode in a flat potential with nonzero chromaticity.

all modes and so only the $m = 1$ head-tail mode is shown. The peak at some positive time offset is a consequence of the change in betatron phase along the bunch due to the chromatic modulation. The peak in the bunch tail oscillation has the same time offset but negative. This phase advance smears out the mode duration but also decreases the peak amplitude so the effect this would have on a coupled bunch instability is hard to predict. Again, the Gaussian and Uniform (not shown) bunch models are close to the turn by turn simulation of the double RF system. In all cases, the oscillation amplitude in a harmonic potential remains constant at the same level as at $t = 0$. Modulation at the chromatic frequency reduces the bunch head oscillation amplitude but also introduces a nonzero dipole component.

CONCLUSION

It has been shown that the azimuthal head-tail modes traditionally used to estimate multibunch current thresholds at nonzero chromaticity are destroyed by the synchrotron tune spread introduced by a bunch lengthening HC. In fact, for the case of the MAX IV 3 GeV ring [1] ($k_s = 0.0036$ ms^{-1} ps^{-1}, $\sigma_r = 193$ ps), the lifetime of the $m = 1$ mode is around 3 ms, more than 9 times shorter than the vertical radiation damping time of 28 ms. However, there may be other limiting factors such as radial excitations of the $m = 0$ mode. These have similar profiles to the even numbered azimuthal modes but are usually disregarded in multibunch studies due to their lower interaction with the machine impedance. They are not affected by the synchrotron tune spread since every particle of the same amplitude has the same betatron phase. Future simulation work with multiparticle tracking and possibly, an extended frequency domain code, is required. So far, a linear dependence of the synchrotron tune on the particle amplitude, $\omega_s(r) = k_s r$ as seen in the flat potential condition, has been studied. The above calculations could be extended with higher order terms to see if the form of the tune spread is significant.

REFERENCES

[1] P. F. Tavares et al., "The MAX IV Storage Ring Project", J. Synchrotron Rad. (2014) 21, 862-877.

[2] P. Bosland et al., "Third Harmonic Superconducting Passive Cavities in Elettra and SLS", WSRF XI (2003), TU06.

[3] N. Towne et al., "Bunch Lengthening Harmonic System for NSLS-II", EPAC08 (2008), MOPP152.

[4] M. Georgsson et al., "Landau Cavities at MAX II", Nucl. Instrum. Meth. (1998) A416, 465-474.

[5] P. F. Tavares et al., "Collective Effects in the MAXIV 3 GeV Ring", IPAC2011 (2011), MOPS066.

[6] M. Klein et al., "Investigation of Transverse Beam Instabilities in the MAX IV 3 GeV Ring Using the Multibunch Code mbtrack", TWIICE (2014).

[7] J. L. Laclare, "Bunched Beam Coherent Instabilities", CAS (1987), CERN/87-03 264-327.

[8] A. Hofmann & S. Myers, "Beam Dynamics in a Double RF System", ICHEA XI (1980), CERN ISR-TH-RF/80-26.

[9] P. F. Tavares et al., "Equilibrium Bunch Density Distribution with Passive Harmonic Cavities in a Storage Ring", Phys. Rev. ST Accel. Beams (2014) 17, 064401.