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TWO GENERAL ORBIT THEOREMS FOR EFFICIENT MEASUREMENTS OF BEAM OPTICS

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Abstract

Closed-orbit perturbations and oscillating beam solutions in storage rings are closely related. While techniques exist to fit accelerator models to closed-orbit perturbations or to oscillation data, the exploitation of their relation has been limited. In this work, two orbit theorems that allow an efficient computation of optical parameters in storage rings with older hardware are derived for coupled linear beam motion. The monitor theorem is based on an uncoupled case study described by the author in an earlier work [1] and has been generalized as well as simplified in mathematical abstraction to provide a reliable and computationally stable framework for beam optics measurements. It is based on a closed-orbit measurement utilizing 4 dipole correctors (2 for each plane). The corrector theorem allows to obtain parameters of these dipole correctors using two turn-by-turn monitors at almost arbitrary positions in the ring (which do not need to be located in a drift space), so that it possible to uniquely resolve closed orbits into optical parameters without sophisticated lattice models.

INTRODUCTION

To express the orbit theorems in a reasonable and straightforward way, we need to describe coupled optics in the linear phasor model, instead of the polar-like Courant-Snyder parameters. Then, the two orbit theorems are formulated and verified by experimental data obtained using the mapping method, a diagnostic method containing both theorems.

LINEAR PHASOR MODEL

In a storage ring, a closed orbit exists. For any working setup, the particle motion around this orbit is bound and undamped in good approximation. From these assumptions, we can linearize the bound motion around the orbit as a $M = 3$-dimensional harmonic oscillator. In the following, we will not consider synchrotron motion ($M = 2$).

Beam Oscillation (Turn-by-Turn)

The deviations $\vec{r}$ from the closed orbit at each turn n at a given longitudinal position s_j can be modeled [2] as ($\Re$: real part)

$$\vec{r}_n(s_j) = \sum_m^M \Re\left\{\vec{R}_{jm} e^{in\mu_m}\right\}, \tag{1}$$

where the phase advances μ_m correspond to betatron tunes, while $\vec{R}_{jm}$ are vectors of complex oscillation amplitudes (phasors) that we will call *monitor vectors* in the following. The monitor vectors are related to the Mais-Ripken parameterization of beam optics [3] [4] and are a fully coupled representation of spatial linear beam motion.

Closed-Orbit Perturbation (Orbit Response)

The closed orbit can be defined by a fixpoint of the linear transfer map at a given position. The map has a zero-order term $\vec{t}_k$, which corresponds to a dipole kick, and a first-order term which is the transfer matrix $\mathbf{T}_k$. If we assume that the kick is located directly before the corrector, we obtain the closed orbit vector $\langle\vec{r}\rangle$ at position $\tilde{s}_k$ by the equation system

$$\vec{t}_k = (\mathbf{1} - \mathbf{T}_k)\langle\vec{r}\rangle_{jk}$$

A computation [2] including the knowledge that the spatial parts of the eigenvectors of $\mathbf{T}$ are indeed the forementioned monitor vectors $\vec{R}_{jm}$, while the eigenvalues are $e^{i\mu_m}$, leads to a phasor expression for the closed-orbit perturbation

$$\langle\vec{r}\rangle_{jk} = \Re\left\{\sum_m \vec{R}_{jm} E^*_{jkm} D^*_{km}\right\}, \tag{2}$$

The phase jump coefficients $E_{jkm} = \exp\{i\mu/2 \operatorname{sign}(s_j - \tilde{s}_k)\}$ hold numbers on the complex unit circle. These coefficients occur as closed orbits are ring-periodic, and may be interpreted as correcting the "fractional tune" of the betatron oscillations. The corrector parameter D_{km} is a complex quantity that represents the coupling of a given dipole error or corrector k to each oscillation mode m.

ORBIT THEOREMS

Both orbit theorems are based on solving systems of equations for all monitors $1 \leq j \leq J$, or the subset of all turn-by-turn capable monitors $1 \leq f \leq F$, and correctors k. In brief, the first theorem obtains the corrector parameters D_{km} from closed-orbit perturbations turn-by-turn data at monitors f. Then, the second theorem is used to obtain $\vec{R}_{jm}$ at all monitors j.

Corrector Theorem

In a first step towards the knowledge of spatial optical parameters at all BPMs, we state that

- it is possible to compute D_{km} from a set of closed orbit data and turn-by-turn data at $F \geq 2$ turn-by-turn capable monitors

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by solving a set of equation systems[1] based on (2)

$$\begin{pmatrix} \langle\vec{r}\rangle_{1k} \\ \vdots \\ \langle\vec{r}\rangle_{Fk} \end{pmatrix} = \Re\left\{\begin{pmatrix} \vec{R}_{11}E^*_{1k1} & \dots & \vec{R}_{1M}E^*_{1kM} \\ \vdots & \ddots & \vdots \\ \vec{R}_{F1}E^*_{Fk1} & \dots & \vec{R}_{FM}E^*_{FkM} \end{pmatrix}\begin{pmatrix} D^*_{k0} \\ \vdots \\ D^*_{kM} \end{pmatrix}\right\}$$

separately for different correctors k. The systems utilize monitor vectors $\vec{R}_{fm}$ and modal phase advances μ_m (for E coefficients) that can be computed from turn-by-turn data by (1) and Fourier transform-related methods (e.g. FFT followed by fitting).

For $F > 2$, the system is overconstrained and can only be solved in a least-square sense (e.g. by SVD). The remaining error creates an opportunity to check the linearity of the optical system under consideration of monitor errors.

Monitor Theorem

In a second step, we utilize (2) again and conclude that

- with $K \geq 4$ known corrector parameters D_{km}, it is possible to compute R_{jm} at *all* monitors.

To ease notation, one may define the spatial components d of $\vec{R}_{jm}$ resp $\langle\vec{r}\rangle_{jk}$ as R_{jmd} resp. $\langle r\rangle_{jkd}$. This leads to a set of $DJ = 2J$ different equation systems

$$\begin{pmatrix} \langle r\rangle_{j1d} \\ \vdots \\ \langle r\rangle_{jKd} \end{pmatrix} = \Re\left\{\mathbf{G}_j\begin{pmatrix} R^*_{j1d} \\ \vdots \\ R^*_{jMd} \end{pmatrix}\right\},$$

where we have defined J matrices $\mathbf{G}_j$ with the components

$$(\mathbf{G}_j)_{km} = D_{km}E_{jkm}.$$

These systems are only solvable for $\vec{R}_{jm}$ if $K \geq 2M$. For each oscillation mode to consider, two correctors are needed.

A decoupled precursor of the monitor theorem in which the corrector parameters were obtained by turn-by-turn data in a drift space, has been validated experimentally in [1]. As the authors discovered recently, another decoupled precursor of this theorem [5] used lattice model transfer matrices to estimate corrector parameters.

POSTPROCESSING

Invariants of Motion

To normalize the oscillation amplitudes for obtaining e.g. Twiss parameters, one would like to express invariants of motion in form of measurable quantities so that they can be computed. To compute the invariants, the eigenvectors $\vec{Q}_m$ at at least one position in the ring must be known. From the phasor model [2], the eigenvectors can be computed at monitor drift spaces between monitors $j, j+1$ of length L by

$$\vec{Q}_m = \begin{pmatrix} \vec{R}_{j,m} \\ (\vec{R}_{j+1,m} - \vec{R}_{j,m})/L \end{pmatrix}.$$

[1] which can be done by elementary means although $\Re$ is involved

Then, the (real-valued) invariants are given[2] by [2] (cmp. [4])

$$I_m = \frac{\mathrm{i}}{2}\vec{Q}_m^\dagger \begin{pmatrix} & \mathbf{1} \\ -\mathbf{1} & \end{pmatrix}\vec{Q}_m,$$

where $\dagger$ denotes the conjugate transpose of $\vec{Q}_m$. Note that, as $\vec{R}_{jm}$ can be computed at all monitors, one can also utilize non-turn-by-turn capable monitors at drift spaces to obtain the invariants. If more than one drift space between any monitors occurs, the results can be cross-checked.

Connection to Common Parameterizations

Knowing the invariants, one can connect the eigenvectors to the Mais-Ripken parameters [3] [4] via

$$\vec{Q}_1 = \sqrt{I_1}(\vec{z}_1 + \mathrm{i}\vec{z}_2), \quad \vec{Q}_2 = \sqrt{I_2}(\vec{z}_3 + \mathrm{i}\vec{z}_4)$$

Additionally, in decoupled approximation, the monitor vectors can be directly related to Courant-Snyder parameters via $R_{j11} = \sqrt{I_1}\sqrt{\beta_x}\mathrm{e}^{\mathrm{i}\phi_x}$ so that we can obtain two estimates that coincide for strict decoupled motion

$$\beta_x^{\mathrm{A}} = \frac{R_{j11}\cdot R^*_{j11}}{I_1}, \quad \beta_x^{\mathrm{B}} = \frac{\vec{R}_{j1}\cdot\vec{R}^*_{j1}}{I_1}.$$

In the decoupled case, one can also arrive at an equation for closed orbit perturbation in standard notation by defining

$$D_{km} = C_k\sqrt{\beta_k}\mathrm{e}^{\mathrm{i}\phi_k},$$

where C_k is a corrector-and-tune specific constant. Note that ϕ_k can be determined as a complex angle, and that, if the corrector is located in a monitor drift space, β_k and thus C_k can be computed to obtain corrector strengths.

[2] here, the phase space notation (x, y, x', y') is used. For I_m to be invariant, we assume that the transfer matrices are symplectic.

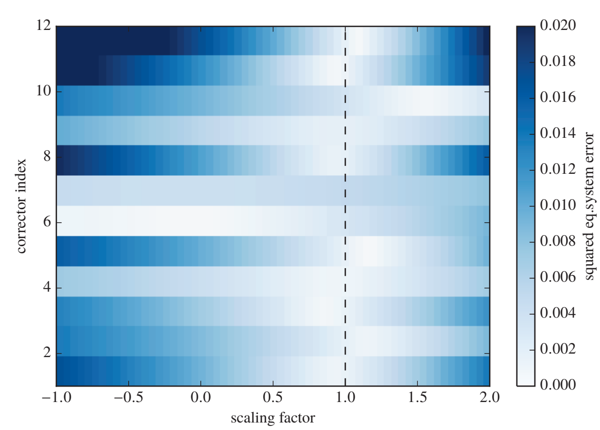

Figure 1: Residual error of corrector equations vs. scaling of time-of-flight compensation. Almost all minima occur in vicinity of scaling factor 1, which is equivalent to activated compensation.

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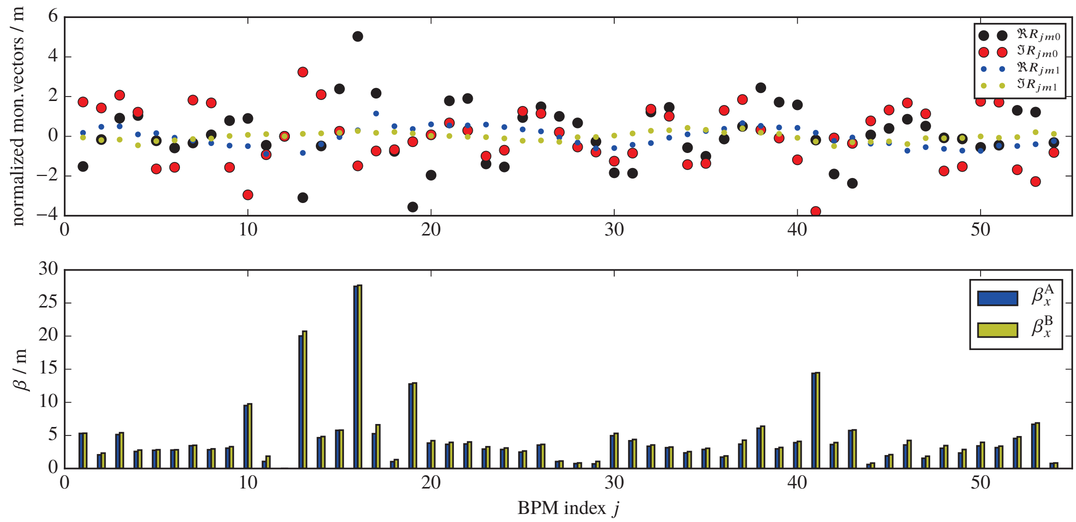

Figure 2: Preliminary results for invariant-normalized optical parameters in mode $m = 0$. The upper plot shows normalized monitor vector components, the lower plot shows two different estimates for the decoupled β_{x} function. ($j = 12$ defunct.)

PRELIMINARY RESULTS

The corrector and monitor theorems are currently being validated as parts of a measurement scheme that we call the mapping method, as it maps known monitor vectors to unknown monitor vectors via corrector parameters. The experiments were performed on the storage ring of the DELTA synchrotron radiation facility [7].

For the experiment, a special set of corrector magnets designed for fast orbit feedback in [6], is used. These magnets have neglible hysteresis effects for the time scale of ≈ 1 Hz, in which the correctors are operated in the following. The only necessary a priori information is the basic ring topology, that is, $\mathrm{sign}(s_j - \tilde{s}_k)$ for all used monitors j and correctors k. For the invariant normalization, it must be known which monitors (if any) are part of a monitor drift space, together with the drift space's length.

Time-Of-Flight Compensation

In our storage ring, as supposedly in many others, the turn-by-turn capable monitor data acquisition for each turn is synchronized by the same trigger, with cable lengths designed to be exactly the same, so that, in theory, all turns at all monitors are acquired in the same moment. This is problematic, as the time of flight for the particles travelling successively through the monitors is not considered.

To compensate this effect, a timing shift $\xi_f T$ (circulation frequency T) can be emulated for the acquisition trigger in the multiturn data by introducing a phase shift $\mu_m \xi_f$ for the measured monitor vectors $\vec{R}_{fm}$.

At DELTA, a set of four turn-by-turn monitor triggers is connected to a single optical fibre cable. To circumvent additional timing errors created by different fibre cable lengths, we limit the application of the corrector theorem to the set around the U250 undulator, which also includes two monitors with the highest signal-to-noise ratio in the ring.

To check the applicability of the corrector theorem, we scale the time-of-flight correction phases for each monitor by the same factor and plot the resulting root of the squared error of the equation system for different correctors in Fig. 1. As can be seen for most correctors, their minimum error occurs in the range of the scaling factor 1, which shows that the time-of-flight compensation is necessary.

Optical Functions

The preliminary results for β_{x} are shown in Fig. 2.[3] The measurements were performed in DELTA's standard operation mode with activated DC injection bump for which no working lattice simulation exists.

We can yet conclude that the results for β_{x} are reproducible throughout different measurement weeks, thus large random errors can be ruled out. Also, tune-scan measurements [8] with active DC bump show comparable results for the U250 region (Fig. 3).

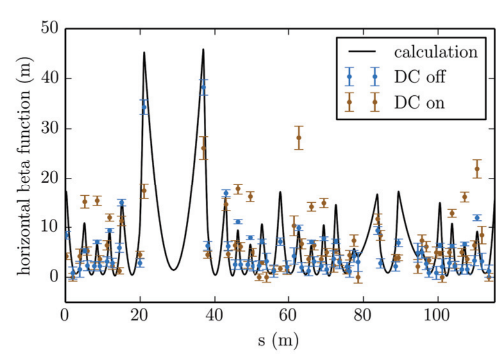

Figure 3: Averaged β_{x} values from tune-scan method without (incl. model data) and with DC bump [8]. The U250 region $s \in [20, 40]$ m corresponds to BPM 13-16 in Fig. 2.

[3] Results for the y plane or phases are not shown due to space limitations.

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