

# IDENTIFICATION OF INTRA-BUNCH TRANSVERSE DYNAMICS FOR MODEL BASED WIDEBAND FEEDBACK CONTROL AT CERN SUPER PROTON SYNCHROTRON\*

O.Turgut<sup>†</sup>, C. Rivetta, J. Dusatko and J. D. Fox, SLAC, Menlo Park, CA 94305, USA  
 W. Hofle, CERN, Geneva, Switzerland

## Abstract

Multi-input multi-output (MIMO) feedback design techniques can be helpful to stabilize intra-bunch transverse instabilities induced by electron-clouds or transverse mode couplings at the CERN Super Proton Synchrotron (SPS). These MIMO techniques require a reduced order model of intra-bunch dynamics. We estimate a linear reduced order MIMO models for transverse intra-bunch dynamics and use these models to design model based MIMO feedback controllers. The effort is motivated by the plans to increase currents in the SPS as part of the HL-LHC upgrade. Parameters of the reduced order models are estimated based on driven beam SPS measurements. We study different types of controllers. We test the model based designs using macro particle simulation codes (CMAD and HEADTAIL) and compare its performance with FIR filters tested during beam measurements of the feedback system in SPS machine development (MD) studies.

## INTRODUCTION

Electron clouds and machine impedance can cause intra-bunch instabilities at the CERN Super Proton Synchrotron (SPS). The high current operation of the SPS for LHC injection requires mitigation of these problems. Modern control techniques can be used to stabilize the bunch. These techniques are powerful tools allowing us to evaluate and understand the performance and the limits of the system beforehand. Yet, they require reduced order models of intra-bunch dynamics to design optimal or robust controllers. System identification techniques can be used to get these required reduced order models.

The feedback system senses the vertical positions at multiple locations within the nanosecond-scale bunch. Control filters use these measurements to calculate correction signals and apply them back onto the bunch using the kicker as an actuator. A 4 Gs/Sec. digital feedback system has been developed to process the motion signals and generate the correction actions [1]. Due to the very fast intrinsic time requirement of the system, a parallel computation control filter architecture has been used developed.

In this paper, we show the use of system identification techniques to estimate parameters of linear models representing single bunch dynamics. We define the form of the reduced order model. We show an example of identification

applied to data from SPS measurements. We use reduced order models to design model based controllers. We compare a model based IIR controller with an existing FIR filter for a specific case using nonlinear macro particle simulation codes.

## REDUCED ORDER MODEL AND IDENTIFICATION

Any linear dynamical system can be represented in state space matrix form. A discrete time system sampled at every revolution period  $k$  with  $p$  inputs and  $q$  outputs is represented by

$$\begin{aligned} X_{k+1} &= AX_k + BU_k \\ Y_k &= CX_k \end{aligned} \quad (1)$$

where  $U \in R^p$  is the control variable (external excitation),  $Y \in R^q$  is the vertical displacement measurement,  $A \in R^{n \times n}$  is the system matrix,  $B \in R^{n \times p}$  is the input matrix, and  $C \in R^{q \times n}$  is the output matrix. For a MIMO system, the model order  $n$  determines the complexity.

$$Y(z) = [D^{-1}(z)N(z)] U(z) \quad (2)$$

where  $[]$  represents the transfer function matrix ( $\in R^{q \times p}$ ) for a system with  $p$  inputs and  $q$  outputs in  $z$  domain.  $D(z)$  and  $N(z)$  represent denominator and numerator of discrete time transfer function matrix between input-output couples.

$$\begin{bmatrix} N_r & | & -D_r \end{bmatrix} \begin{bmatrix} U(k) \\ Y(k) \end{bmatrix} = 0 \quad (3)$$

Given the input  $U(k)$  and output  $Y(k)$  signals, the estimation of the transfer function coefficient matrices  $N_r$  and  $D_r$  is obtained by solving the last linear equation using time domain data. Assuming full observability of the system, we can represent our state space in discrete time observable canonical form. This enables us to represent the system with the minimum number of model parameters [2] [3] [4].

## SPS Measurements - Bunch Dynamics Identification

Multiple MDs have been conducted at the CERN SPS ring driving the bunch with different excitations (open loop) and testing feedback controllers to stabilize the bunch dynamics (closed loop). Those measurements were conducted using a single bunch in the machine with intensities of about  $1 - 2 \times 10^{11}$  protons at the injection energy, 26 GeV and Q26 lattice configuration. The driven tests with different excitation signals have been designed such that the kicking signal

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<sup>†</sup> oturgut@stanford.edu

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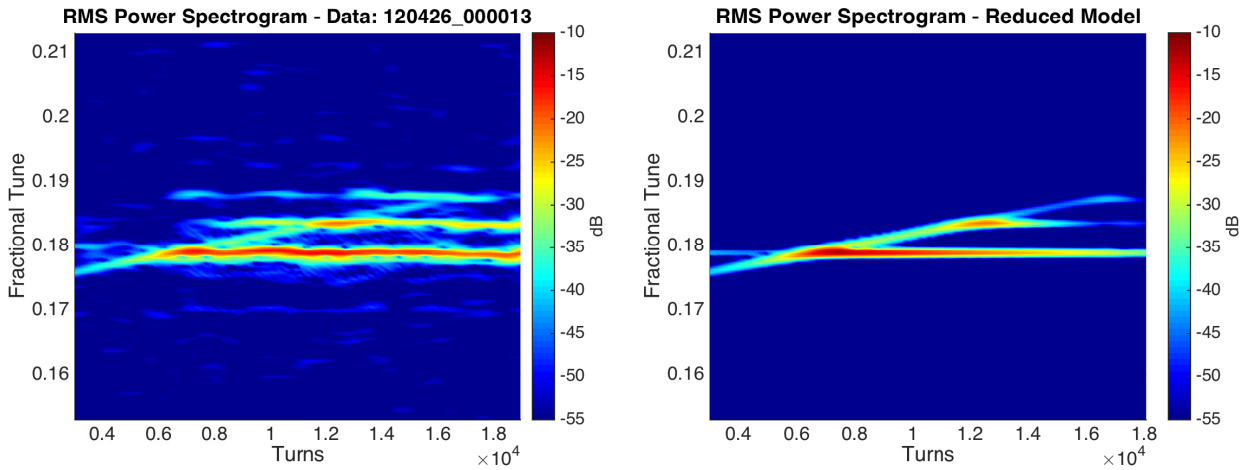


Figure 1: On the left we see the spectrogram of physical measurement showing chirp excitation where we excite mode 0, mode 1, and mode 2 excitation around turns ~ 7000, ~ 11500, and ~ 17000 respectively. On the right, we see the same excitation and analysis applied to the reduced order model capturing linear dynamics.

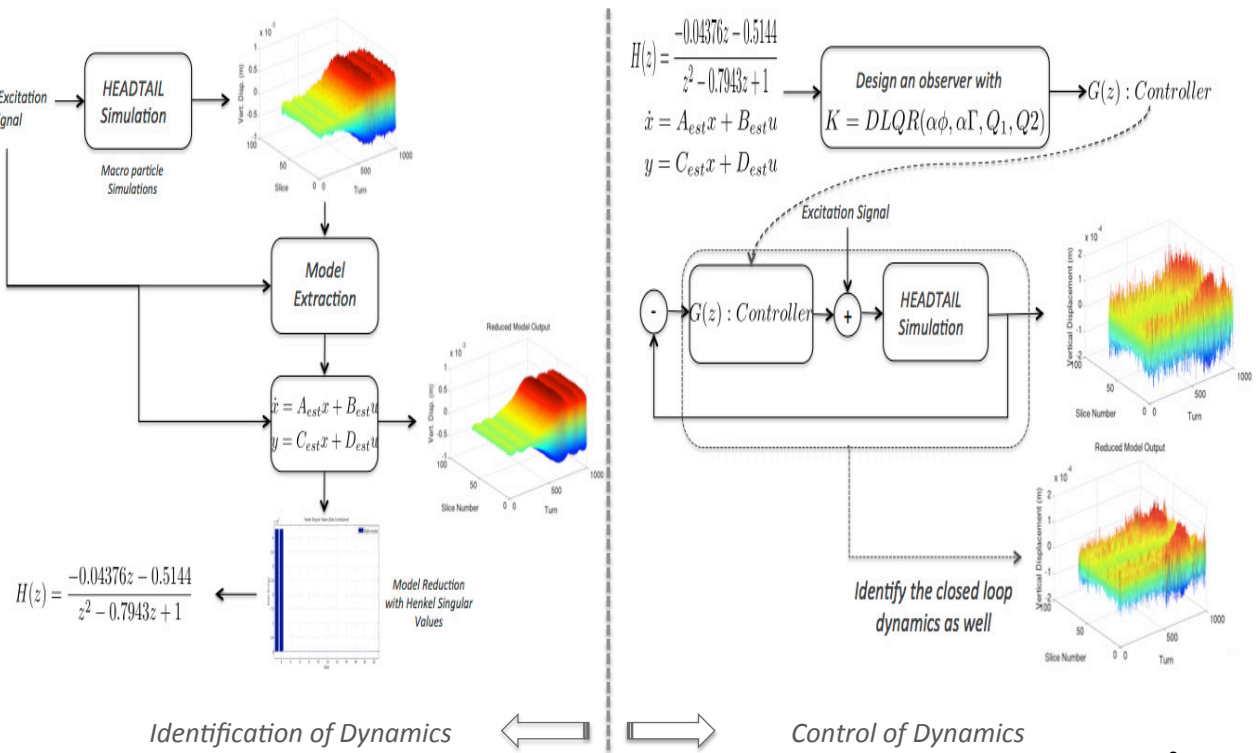


Figure 2: Identification of the intra-bunch dynamics and model based controller based on the identified model. In this example, open loop simulation data from the nonlinear macro particle code HeadTail is used to get the model of the bunch for mode 0 dynamics. A controller is designed based on the model. The controller is tested using HeadTail simulation for mode 0 and mode 1 dynamics.

		<i>Model Based IIR</i>	<i>5 Tap FIR</i>
<i>Open Loop Dynamics</i>	Mode 0	$-0.000 \pm 0.185i$	
<i>Closed Loop Dynamics</i>	Mode 0	$-0.0074 \pm 0.183i$	$-0.0074 \pm 0.185i$
	Mode 1	$-0.0037 \pm 0.199i$	$-0.0026 \pm 0.2i$

Figure 3: Comparison of the closed loop dynamics in between model based IIR is compared with FIR filter. Closed loop eigenvalues are close for both filters however more study is required to understand robustness, required control power and implementation complexity.

is a persistent input for the system [5], and the collected data can be used to study the identification algorithms and quantify a reduced parametric model of the beam dynamics. Similarly, data obtained from macro-particle simulation codes (CMAD-HEADTAIL) has been used to test the identification algorithms and compare the dynamic model results with those obtained from machine measurements [4].

The bandwidth and power limitations during the measurements forced us to set our reduced model to detect low order modes corresponding to frequencies up to the second sideband ( $2f_s$ ) around the betatron frequency ( $f_\beta$ ). We use both mode 0 (constant signal along longitudinal axis) and mode 1 (tailored to have head tail shape across the bunch) excitation signals for which the amplitude is modulated by a sine wave whose frequency changes linearly [1]. The chirp covers the range  $f_\beta \pm 2f_s$  in  $\sim 15000$  turns.

Tailoring the reduced model to the low-order modes, it is possible to use 4 coupled  $2^{nd}$  order differential equations to capture the low-order dynamics of the bunch (mode 0 - barycentric motion and mode  $\pm 1$  - head-tail motion). The input-output relationship (momentum kick to vertical displacement) of the bunch is defined by a  $4 \times 4$  MIMO system with  $p = 4$ ,  $q = 4$  and  $n = 8$ . We set the input and output vector dimension (equation 1) to 4 for each sampling instance  $k$  to incorporate data with the MIMO model. The measurement set-up acquires 16 samples across the bunch at each sampling time  $k$  for the momentum kick and the vertical displacement signals. To do identification with the corresponding MIMO model, each sample in vectors  $U_k$  and  $Y_k$  ( $\in R^{4 \times 1}$ ) is calculated averaging either 4 or 8 consecutive non overlapping samples of the 16 samples long original data (e.g  $U(1, k), Y(1, k)$  is the average of samples 1-4,  $U(2, k), Y(2, k)$ , the average of samples 5-8 ... etc).

Figure 1 shows frequency domain representation of vertical motion with strong excitation of both mode 0 and  $1^{st}$  sideband (mode 1). We also see some motion around  $2^{nd}$  sideband. On the left, we see the spectrogram of the driven measurement with clear mode 0, mode 1, and mode 2 motion around turns  $\sim 7000$ ,  $\sim 11500$ , and  $\sim 17000$ . On the right side, we show the spectrogram of bunch's vertical motion predicted by the reduced model. It is important to notice from the measured data (Fig. 1 - left) the effect of nonlinearities in the bunch. The spectrogram analysis of the measured signal shows that the  $1^{st}$  and  $2^{nd}$  ( $f_\beta + 2f_s = 0.189$ ) sidebands are excited before the chirp excitation drives the bunch motion at that particular frequency. As expected, our linear model is able to capture dominant characteristics and linear dynamics such as motions at mode 0, mode 1 and mode 2 tunes, but not the effect attributed to the non-linearity in the bunch.

## MODEL BASED CONTROLLER DESIGN APPROACH

MIMO model based optimal control architectures are evaluated using identified reduced order models. We analytically estimate the closed loop dynamics and validate the results

using nonlinear macro particle simulation codes. Figure 2 shows model based controller design procedure and validation of the closed loop dynamics using the HEADTAIL simulation [6].

FIR filters show robust characteristics for a certain band of frequencies and we used a 5 tap FIR filter to control mode 0 ( $f_\beta$ ) and mode 1 ( $f_\beta + f_s$ ) dynamics during all SPS MDs [7]. It works very well for these modes. At this early stage in the model based controller design studies, we use a subset of model based controllers for the bunch dynamics. We start with modeling the bunch dynamics for mode 0 and design the model based controller using this model. The model based IIR controller is compared with this FIR filter in terms of closed loop performance for mode 0 and mode 1 dynamics. Model based IIR is designed using discrete linear quadratic regulator methods. We use the model based IIR and FIR controller to control mode 0 and mode 1 dynamics in HEADTAIL simulation. Results in Fig. 3 show that model based IIR performs very similar to FIR for mode 0 dynamics and introduces more damping to mode 1.

## CONCLUSION AND FUTURE WORK

Model-based control design techniques for intra-bunch instabilities require a reduced model of the intra-bunch dynamics. We show initial results of the identification of those models. We identify parameters of a reduced order model that captures mode 0, mode 1 and mode 2 dynamics from the CERN SPS machine measurements. The natural tunes, damping values and the separation of modes associated with the motion seen in measurements are estimated correctly using a linear model. Similar studies have also been conducted using nonlinear macro particle code simulation data. Results show that model based controller design technique can also be a promising approach for wideband feedback application. Compared to the existing diagonal FIR control architecture, this approach has advantages and disadvantages in terms of performance, sensitivity and implementation complexity. These have to be studied and understood in detail. Future work is also aimed at estimating more internal modes with the wideband kicker. Optimal and robust controllers will be extended using identified reduced order models with higher modes. These new model based control architectures will be compared with the existing parallelized control filter architecture in terms of performance, processing power and complexity requirements. We continue to evaluate new controllers using macro particle simulations and plan tests in the SPS with single bunch control throughout 2015.

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