## Abstract

New series of RFQ vane shapes are under investigation by introducing more terms in addition to the two term potential. Because they can incorporate with the feature of the trapezoidal shape modulation with less multipole © components, higher acceleration efficiency is expected.

## INTRODUCTION

RFQs have basically been designed with so-called two term potential [1]. The two term potential has the simplest form that has minimum terms of acceleration and focusing:
左 $U_{2}(r, \psi, z)=\frac{V}{2}\left\{X\left(\frac{r}{a}\right)^{2} \cos 2 \psi+A I_{0}(k r) \cos (k z)\right\}$,
where $A=\frac{m^{2}-1}{m^{2} I_{0}(k a)+I_{0}(m k a)}, X=1-A I_{0}(k a), k=\pi / L c$ $\frac{1}{0}$ and $a$ is the minimum radius at $\mathrm{z}=0$ (see Fig. 1). The vane 3 surface profile can be defined by the equipotential surface of $U_{2}$. These parameters are defined at each cell, which o may make discontinuities between cells.

The acceleration term $A$ and the focusing term $X$ are the functions of only m and $\mathrm{Lc} / \mathrm{a}$, whose contour plots are shown in Fig. 2. They are not monotonic with $m$ in the short cell length region. This is the main reason why $m$ is limited up to 2 or 3 for practical cases. The effective $\dot{\sim}$ acceleration factor should include the transit time factor.

Typical vane profiles based on this expression are shown in Fig. 3. As can be seen, the vane shape becomes ugly when the modulation factor $m$ becomes large since the large modulation at $\mathrm{Lc} / \mathrm{a}$ is not realistic. Fringe areas have to be truncated for real vanes in any case. This will $\dot{r}$ be discussed later in this paper.


Figure 1: Definitions of vane parameter.


Figure 2: Acceleration term A and focusing term X .


Figure 3: Typical vane profiles based on the two term potentials.

## SIX TERM POTENTIAL

Additional 4 terms were introduced to improve the accelerating efficiency:
$U_{6}(r, \psi, z)=$
$\frac{V}{2}\left\{\cos 2 \psi\left(X_{0}\left(\frac{r}{a}\right)^{2}+X_{1} I_{2}(k r) \cos (k z)+X_{2} I_{2}(2 k r) \cos (2 k z)\right)\right.$
$\left.+A\left(\alpha_{1} I_{0}(k r) \cos (k z)+\alpha_{2} I_{0}(3 k r) \cos (3 k z)+\alpha_{3} I_{0}(5 k r) \cos (5 k z)\right)\right\}$
where
$X_{0}$ : Constant Q term for conventional RFQ A01,
$X_{1}$ : Inter Cell Continuity (new),
$X_{2}$ : IH DTL type Q (finger) \& Trapezoidal Shape A21,
A: Basic Accelerating Term A10,
$a_{1}$ : Trapezoidal Shape A30,
$a_{2}$ : Trapezoidal Shape A30,
$a_{3}$ : Trapezoidal Shape A50 (new) and $a_{1}+a_{2}+a_{3}=1$.
Among these terms, $X_{1}$ and $a_{3}$ are newly introduced to the so-called eight term potential set [2]. $X_{1}$ is set from the vane heights of either side of the cell so that the intercell continuity is recovered. a3 enhances the accelerating factor This expression has no explicit higher multipole term than the quadrupole, while the Bessel functions have inherent nonlinearities on $r$. A03, A12, A23, A32 are omitted since they are multipole components higher than quadrupole. Setting $X_{\mathrm{n}}$ and $\alpha_{\mathrm{n}}$ by following conditions, $U_{6}$ is a function of only $m, L c / a$ and the physical coordinates. They are set by the conditions at the vane ridges so that their curvatures (second order derivatives) along z axis are zero at both the ends and their values are defined at the vane surface points (see Fig. 4). The middle point location is a kind of free parameter, while the vane shape becomes ugly if the point approached to the center. This middle point condition may be substituted by other condition. The number of higher components may be added whose coefficients are set by the similar way.

The acceleration term $A$ and the focusing term $X$ are the functions of only m and $\mathrm{Lc} / \mathrm{a}$, whose contour plots are shown in Fig. 5. They are monotonic with $m$ in the region while right bottom area is invalid where the resulted vane surface shapes become singular and they should not be used. It would be safe to limit $m$ to follow the following condition: $m<0.7 L c / a+0.5$. In the valid area 6 -term construction shows better acceleration factor.
The ratio of the effective acceleration factors for the 2term potential and the 6 -term potential that include the transit time factors is shown in Fig. 6. The newly


Figure 4: Conditions for vane parameters.


Figure 5: Acceleration term A and focusing term X.


Figure 6: The ratio of the effective acceleration term AT.
proposed vane series shows better effective acceleration むifactor.

Typical vane profiles based on this expression are shown in Fig. 7. Higher order terms make the fringes wavy. Since the fringe areas have to be truncated for real vanes in any case. As can be seen, the vane shape 0 becomes ugly when the modulation factor $m$ becomes $\stackrel{\sim}{\circ}$ large since the large modulation at $\mathrm{Lc} / \mathrm{a}$ is not realistic.

Typical vane cross sections along $z$ axis with parameters $L c / a(=k)$ and $m$ are shown in Fig. 8. The quarter circles denote $r=1.4 a$ area, where not big effect can be given onto the axis further from this radius. Hence the vane shape will be truncated at this point and connected to a slope line. It can be seen that the shape of the vane ridges $\underset{\sim}{\circ}$ in the transverse plane does not change much along $z$ axis. Since the most closest areas $(r=a)$ of a vane should make a dominant potential effects on the axis, we may substitute the vane ridge cross section with the constant cross section at the most closest point $(r=a)$. While this procedure will result some discontinuity between the cells, the ridge line should keep the continuity because of ${ }_{3}$ the adjustment of $X_{1}$, and the step may not become a big problem.
Examples of the vane profiles with $k=2, m=1.2$ and 1.5 are shown in Fig. 9. The wavy shapes seen in Fig. 7 are eliminated, while the ridge lines are the same. The case of $\mathrm{m}=1.8$ in Fig. 7 has the broken ridge line, which should not be included in a real design.
This truncation procedure may introduce extra multipoles and other effects such as less accelerating factor. These effects are under investigation.


Figure 9: Constant cross sectional vanes with $k=2, m=1.2$ and 1.5 .

## REFERENCES

[1] R. H. Stokes, K. R. Crandall, J. E. Stovall and D. A. Swenson, RF Quadrupole Beam Dynamics, IEEE Trans. Nucl. Sci. NS-26, 3, 1979, pp.3469-3471
[2] K. R. Crandall, "Effects of Vane-Tip Geometry on the Electric Fields on Radio-Frequency Quadrupole Linacs", Los Alamos National Laboratory Technical Note, LA-9695-MS, 1983


Figure 7: Typical vane profiles based on the six term potentials.


