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ON THE CHARACTERIZATION OF A CCR SOURCE*

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Abstract

Peak and spectral brightness of a resonant long-range wakefield extractor are evaluated. It is shown that the brightness is dominated by beam density within the slow wave structure and antenna gain of the outcoupling. Far field radiation patterns and brightness of circular and high-aspect-ratio planar radiators are compared. A possibility to approach the diffraction limited brightness is demonstrated. Role of group velocity in designing of the Cherenkov source is emphasized. The approach can be applied for design and characterization of various structure-dominated sources (e.g., wakefield extractors with gratings or dielectrics, or FEL-Cherenkov combined sources) radiating into a free space using an antenna (from microwave to far infra-red regions). The high group velocity structures can be also effective as energy dechirpers and for diagnostics of microbunched relativistic electron beams.

INTRODUCTION

For generation of a relatively narrow bandwidth mm-sub-mm wave radiation resonant Cherenkov radiation can be an attractive alternative to undulator radiation due to the exceptional compactness of the radiator and capability to operate at lower energies. Such a coherent Cherenkov radiation (CCR) source is usually supplied by an antenna and employs a circular [1] or planar configuration [2,3]. Most of FIR FEL sources do not employ radiating structure and operate in a diffraction limited mode due to attainability of low emittance electron beams (namely, with geometric emittance lower than the emittance of diffraction limited photon beam $\lambda/2\pi$).

An attempt is made to pave an engineering path for characterization of structure-dominated sources employing radiators (or extractors). The sources comprising an antenna are not limited by CCR mechanism but may also include undulator with fast or slow-wave structure (e.g., mm-wave Bragg resonator and waveguide), or Cherenkov-FEM combined sources [4]. .

ANTENNA GAIN AND BRIGHTNESS

Unlike unbounded FEL and undulator the radiation pattern from a structure-dominated source is formed by an antenna, whereas the beam serves only as a launcher for the structure. It is convenient to express the power density S and brightness B via the modal antenna directivity D_n or gain G_n [5] assuming nearly normal orientation of the detector with respect to the incident rays:

$$S=\frac{dP_{ff}}{dA}=\frac{1}{4\pi R_d^{\,2}}\sum_n G_n P_n\,,\quad B=\frac{d^2P_{ff}}{dAd\Omega}\approx\frac{1}{4\pi A_A}\sum_n G_n P_n\,, \qquad (1)$$

where R_d is the distance between the source and the detector (object), $R_d >> W_A$, W_A is the maximum antenna dimension, $R_d >> W_A^{\,2}/\lambda$, A_A is the antenna aperture area, $G_n=(1-|\Gamma_n|^2)\eta_n D_n$, $D_n=4\pi P_{ff}^{-1}/dP_{ff}/d\Omega$, P_{ff} is the modal power of far-field radiation, $d\Omega$ and dA are the infinitesimal small solid angle and area, and P_n is the modal power on the transition from the slow-wave structure to the antenna having return loss Γ_n and modal efficiency η_n.

The modal energy and power radiated at the structure exit can be calculated analytically [6,7]. For a single microbunch and single mode operation the peak brightness can be calculated as follows:

$$B_{1b}=\frac{G_n}{4\pi A_A}\frac{\omega}{4}\frac{r}{Q}|v_{gr}|\cdot\left(\frac{q\Phi}{1-\beta_{gr}/\beta}\frac{1-\exp(-\alpha L)}{\alpha L}\right)^2, \qquad (2)$$

where $\omega=2\pi f$ is the circular frequency of the resonant Cherenkov radiation $\omega=h(\omega)v$, $h(\omega)$ is the wavenumber defined by the structure dispersion, q is the bunch charge, L is the structure length, $r=E_z^{\,2}/(dP/dz)$ is the shunt impedance, $\beta=v/c$, $v_{gr}=\beta_{gr}c$, is the group velocity, Q is the Q-factor, $Q|\beta-\beta_{gr}|>>1$, $\alpha=\pi f/Qv_{gr}$ is the attenuation, and $\Phi=\exp(-(\omega\sigma_t)^2/2)$ is the formfactor for a Gaussian bunch having r.m.s. duration σ_t.

For a long ($t>t_f=L/v_{gr}$) train of coherent microbunches with interval T_b less than the drain time $T_d=L(v_{gr}^{\,-1}-v^{-1})$ the brightness can be calculated as follows:

$$B_{train}=\frac{G_n}{4\pi A_A}\frac{\omega}{4}\frac{r}{Q}\frac{1}{|v_{gr}|}\left|I\Phi L\frac{1-e^{-\alpha L(1+ia_s)}}{\alpha L(1+ia_s)}\right|^2, \qquad (3)$$

where I the beam current within the train, $a_s=2Q(f/f_b-1)/(1-v_{gr}/v)$ is the generalized detuning, and $f_b=1/T_b$ is the frequency of the microbunched train (or its resonant sub-harmonic).

The spectrum FWHM of the radiation induced by a single microbunch is determined by the inversed radiation pulse length which is equal to the drain time. Therefore the peak spectral brightness for that case is evaluated as:

$$\frac{\Delta B_{1b\_Peak}}{\Delta f}\approx\frac{G_n}{4\pi A_A}\frac{\omega}{4}\frac{r}{Q}\frac{L}{1-\beta_{gr}/\beta}\cdot\left(q\Phi\frac{1-\exp(-\alpha L)}{\alpha L}\right)^2. \qquad (4)$$

For a long (~steady state) coherent train of microbunches the spectral brightness is proportional to the train duration $>t_f$ at $T_b^{\,2}<<T_d^{\,2}$ and neglecting jitter within the train. In transient mode (i.e. for pulse lengths comparable to the t_f) the spectral brightness under these conditions can be estimated as product of (3) and the filling time t_f.

For some applications an averaged brightness can be more important rather than the peak brightness. For the "single bunch" mode of radiation the averaged brightness

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and spectral brightness are calculated using energy radiated by a single bunch:

$$\Delta B_{1b\_Aver} = \frac{G_n}{4\pi A_A} \frac{\omega}{4} \frac{r}{Q} \frac{L}{1-\beta_{gr}/\beta} f_{rate} \left(q\Phi \frac{1-\exp(-\alpha L)}{\alpha L} \right)^2, \quad (5)$$

$$\frac{\Delta B_{1b\_Aver}}{\Delta f} = \frac{G_n}{4\pi A_A} \frac{\omega}{4} \frac{r}{Q} \frac{L^2}{|v_{gr}|} f_{rate} \left(q\Phi \frac{1-\exp(-\alpha L)}{\alpha L} \right)^2, \quad (6)$$

where f_{rate} is the equivalent repetition rate (number of pulses radiated per observation time).

Note the "single bunch" mode also includes train of non-overlapping radiation pulses, $T_b \geq T_d$ (e.g., a train of bunches generated from a thermionic gun and microbunched in a magnetic compressor). At $T_b \geq T_d$ the jitter between the microbunches does not have any effect on the peak brightness and the peak spectral brightness. The averaged spectral brightness can be affected only indirectly by the corresponding jitter spectral component.

One can also see from Eqs. (2), (4), and (5) that for this mode of operation a high group velocity might be preferable to maximize the averaged and peak spectral brightness and especially the peak brightness. On the other hand, low group velocity appears to be impractical for shorter wavelength (e.g., IR) CCR source because of much tighter tolerances, aperture limitation, power flow effect (at $\beta_{gr} \to 0$), and difficulty in matching.

CIRCULAR AND PLANAR RADIATOR

In hypothetical "FEL approximation", when $A_A \to A_{beam}$, and for the same group velocity, gain and antenna aperture there is no obvious advantage of the planar configuration over the circular one in terms of brightness. Note, for given beam current density the FEL brightness is proportional to product of beam beta functions $\beta_x \beta_y$, whereas for given e-beam brightness it is proportional to product of beam transverse emittances $\varepsilon_x \varepsilon_y$. Thus in the last case, which is more relevant to an FEL having relatively long interaction distances (in beta function scale), using of high aspect ratio flat beams is not useful in terms of radiation brightness.

However, the situation is different in a short, structure-dominated radiating system, such as a CCR source. Let us compare circular and planar radiators attached to high gain antenna (see Fig. 1). They have the same length L=1.75", operating frequency f=0.5 THz, phase velocity β=0.98, iris thickness ~51 µm, structure period ~140 µm, and the same (minimum) aperture gap 0.8 mm.

We simulated far-fields (see Fig. 1) of the antennae launched with a monopole mode in the port (simulated numerically with GdfidL code [8]). Unlike planar configuration the antenna gain for the circular configuration is noticeably lower (by factor of ~4 for our example). It is limited by the field distribution over the donate-shaped pattern, which cannot be as sharp as the two knife-edge radiation beams produced by the planar configuration (see Fig. 2). Since the antenna opening angle is limited by a narrow range of angles where reflections are low, we plotted in Fig. 3 the antenna gain and the G/A_A ratio vs. the antenna length L_A for fixed angle antenna openings. One can see from Fig. 3 that optimum length for maximum gain (or radiator flux density) is much longer than that for maximum G/A_A ratio (that, in turn, determines radiator brightness).

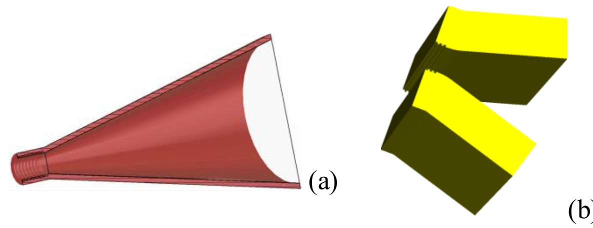

Figure 1: Cut-view of circular (a) and planar (b) radiators with only a few periods of a slow-wave structure and a high gain antenna. The planar radiator width is 1 cm.

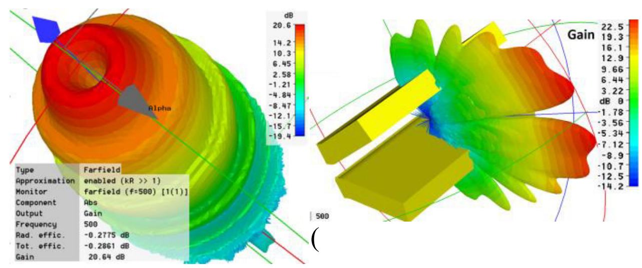

Figure 2: Far-field patterns simulated for the radiators of Fig. 1. Both structures are launched by the lowest monopole mode synchronous to the beam.

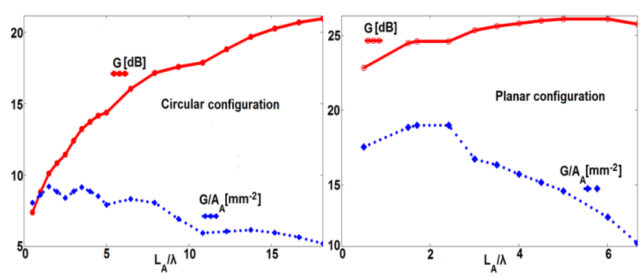

Figure 3: Gain ([dB], top solid curves) and G/A_A [1/mm^2] ratio (dotted curves) for the circular (left) and planar (right) configurations of the radiators shown in Fig. 2 as a function of normalized antenna length L_A/λ. Antenna opening angles are 24.4° for the circular and 32°-34.6° for the planar configuration.

In Table 1 we have compared the planar and circular radiators at moderate charge density 6.25 pC/mm^2 within the radiator apertures. The radiators are partially optimized for maximum flux density and maximum brightness using the plots given in Fig. 3. Note that the shunt impedance is calculated for a 2.6 MeV beam using an eigenmode model of a single period and averaged over beam aperture (which is about uniform for circular configuration and not uniform across the beam 10 mm × 0.8 mm area for the planar configuration). The radiators are characterized for two different variants: maximum gain (and flux density) and maximum gain per antenna aperture (and hence brightness).

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Table 1: An example of comparative characterization of circular and 1 cm wide planar radiators for the structure length L=1.75", minimum gap 0.8 mm, charge density 6.25 pC/mm^2, Φ=0.85, f=0.5 THz, Q=2000, and 7.5·10^{-5} duty factor.

Configuration	β_{gr}	r/Q, kΩ/m	P_{Peak}, kW	B_{limit}, MW/sr·cm^2	B_{Peak}/B_{limit}	dB_{Peak}/df μJ/sr·cm^2	B_{Aver}, W/sr·cm^2
Circular (S=max)	0.61	34.4	0.1	0.028	4.4%	0.11	0.025
Circular (B=max)					26%	0.66	0.15
Planar(S=max)	0.82	0.8	8.5	2.4	31%	22	4.7
Planar (B=max)					54%	40	8.4
UCSB FIR FEL	1	N/A	3	0.83	100%	250	67

For comparison in Table 1 we also give the parameters of UCSB FIR FEL operating at approximately the same frequency and beam energy. In Fig. 4 we plotted power as a function of beam width for fixed charge or charge density for the 1 cm wide planar structure. One can see that wider beams can provide substantial power at reduced space charge effect.

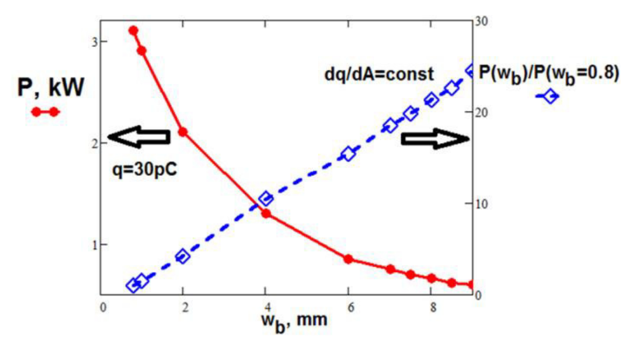

Figure 4: Peak power at fixed charge (solid) and at fixed charge density (normalized to the power at w_b=0.8 mm, dashed) induced in the 1 cm wide planar structure of Fig. 1 by a 130 μm long (r.m.s.) bunch length simulated with GdfidL as a function of beam width w_b.

DISCUSSION

A planar configuration can give more than two orders higher brightness in the "single microbunch" radiation mode at the same current density in spite of about twice less product of shunt impedance and interaction cross-section. The main contributors into the brightness enhancement are higher ratio of antenna gain to antenna aperture (see Fig. 3) as well as almost one order higher product of shunt impedance and interaction cross-section squared that determines the brightness due to coherence. Besides, the group velocity factor $1/(1-\beta_{gr}/\beta)$ is twice higher for the planar design in this example.

In-depth optimization of a CCR radiator system in terms of brightness shall include not only optimal widths of both beam and the structure (for planar configuration) but also beam dynamics. A realistic transverse distribution of a longitudinally-microbunched beam is not flat, but may have a substantial aspect ratio [9]. That can make the dependence of power vs. width given in Fig. 4 non-linear. Nevertheless one can see that the brightness of a planar variant of a CCR source may achieve a substantial fraction of the diffraction limit at least at sub-THz frequencies.

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