ISBN: 978-3-95450-168-7 doi:10.18429/JACoW-IPAC2015-TUPWA039

TRANSVERSE GRADIENT UNDULATOR-BASED HIGH-GAIN-FELS — A PARAMETER STUDY

A. Bernhard*, E. Burkard, V. Afonso Rodríguez, C. Widmann, A.-S. Müller, KIT, Karlsruhe, Germany

Abstract

Transverse gradient undulators (TGU) have recently been discussed as sources for High Gain Free Electron Lasers (FEL) driven by electron beams with an elevated energy spread as for example generated in storage rings or wakefield accelerators. In this contribution we present the results of a parameter study based on the one-dimensional TGU-FEL theory making realistic assumptions on the key parameters achievable for the transverse gradient undulator. We show for which parameter areas LWFA-driven TGU-FELs are virtually technically feasible today and which technical improvements would be required to employ the concept for a laboratory-scale X-Ray FEL.

INTRODUCTION

Is a "table-top" high-gain X-ray free-electron laser driven by a laser wakefield accelerator employing a transverse gradient undulator scheme realistic?

Almost immediately after the experimental demonstration of laser wakefield acceleration (LWFA) of plasma electrons in the highly non-linear regime [1–3] the discussion started how this acceleration scheme could be used to realize compact free-electron lasers fitting into a normal laboratory and delivering laser-radiation in the X-ray regime [4–6] usable e.g. for advanced medical imaging techniques.

The main obstacle on the way to such an admittedly appealing scenario is the comparably large energy spread of the LWF-accelerated electrons (1 % to 10 %). Grüner et al. [6] pointed out that the unfavorable effect of the energy spread could be compensated by the high peak current of the short-bunched LWFA electrons. However, the assumptions the authors made on achievable LWFA beam- as well as undulator parameters have not proved realistic so far.

More recently more elaborated high-gain FEL schemes for electron sources with increased energy spread have been proposed that employ a special preparation of the electron beam phase space distribution by means of magnetic chicanes. Such schemes rely either on bunch decompression [7, 8] or on a transverse spectral dispersion matched to a transversely varying undulator field amplitude. These transverse-gradient undulator high-gain FEL schemes have been discussed both for self amplification of spontaneous emission (SASE) [9, 10] and high-gain harmonic generation (HGHG) [11] scenarios.

In our study we investigated the TGU-SASE case based on the 1-D theory described in [9, 10], searching for optimized and technically feasible TGU parameters for several sets of LWFA beam parameters, moving from beam properties that can be routinely achieved today to beam properties that have been achieved in individual cases to properties that would actually be required.

* axel.bernhard@kit.edu

ACCESSIBLE PARAMETER SPACE

The TGU-FEL scheme implicates modifications of the 1D-FEL equation [10] through the introduction of a modified Pierce parameter

$$\rho_{\mathrm{TGU}} = \rho_{\mathrm{FEL}} \left(1 + \frac{\eta^2 \sigma_\delta^2}{\sigma_x^2}\right)^{-\frac{1}{6}} \tag{1}$$

and an effective energy spread

$$\sigma_{\delta,\mathrm{eff}} = \sigma_\delta \left(1 + \frac{\eta^2 \sigma_\delta^2}{\sigma_x^2}\right)^{-\frac{1}{2}}. \tag{2}$$

Here, ρ_{FEL} is the unmodified Pierce parameter, σ_δ the energy spread, σ_x the transverse beam size (approximated as constant in 1D-theory) and η the dispersion function which is presupposed to be matched to the TGU parameters through the relation

$$\eta = \frac{2 + K_0^2}{\alpha K_0^2}, \tag{3}$$

where K_0 is the undulator parameter at the transverse position for the central beam energy and $\alpha = \frac{1}{K_0} \frac{\partial K}{\partial x}\Big|_{x=0} =: \frac{1}{K_0}\alpha_K$ the relative transverse K-gradient, i.e. the linear approximation $K(x) = K_0(1+\alpha x)$ is assumed to be admissible.

The crucial parameters entering into the TGU-FEL equation are therefore the undulator period length λ_u, influencing both the unmodified Pierce parameter and the undulator parameter, the undulator flux density amplitude $\tilde{B}_0 = \tilde{B}(x = 0)$ and the transverse gradient α_K on the one hand, the transverse beam size σ_x on the other hand. In the following, we discuss our considerations on the accessible ranges for these parameters.

TGU Parameters

Technically, transverse gradient undulators are realized by a transverse variation of the undulator gap. Several possible TGU geometries have been discussed in [12, 13] particularly for superconducting TGUs. Among those the TGU geometry consisting of two cylindrically shaped halves provides the highest achievable transverse field gradients. Indeed we consider the statement adequate that the cylindrical superconducting TGU defines the upper limits of the technically achievable crucial TGU parameters for a given gap and period length [14].

To estimate these limits we use the analytic expressions for the field of a cylindrical TGU given in [12], combined with a 2D finite element calculation used to determine the optimum

ISBN: 978-3-95450-168-7 doi:10.18429/JACoW-IPAC2015-TUPWA039

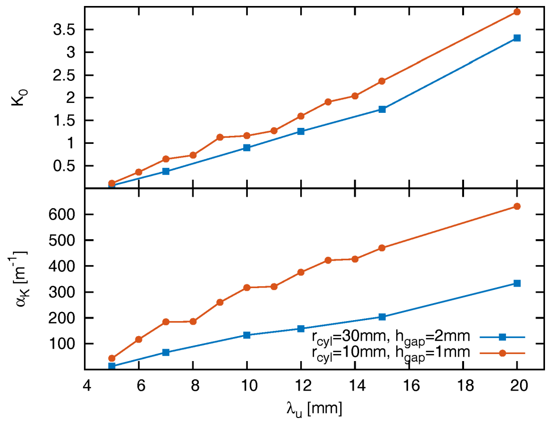

Figure 1: Undulator parameter K_0 and transverse gradient α_K as a function of period length for two superconducting cylindrical TGU design parameter sets.

critical current density and in turn the optimum pole tip field for a given period length, assuming a superconducting TGU (SCTGU) with iron poles, wound with a Nb-Ti multifilament wire as described in [13]. We assume the SCTGU to be operated at 80 % of the critical current on the load line. Considering the flux density amplitude as a function of x we choose the inflection point of $\tilde{B}(x)$ as origin of the x-axis and as the transverse position of the electrons with the nominal beam energy E_0.

Figure 1 shows the resulting undulator parameter and transverse K-gradient at $x = 0$ as a function of period length. Two designs in terms of gap width (closest distance of the two cylinders) and pole/winding radius were considered: one that has been realized in our laboratory [15] and one that represents the technical limit particularly regarding the winding radius of the superconducting wire. The results for the latter we refer to as the technical limit of the state of the art.

Since our discussion is about LWFA-driven X-ray FELs, we aim at radiation wavelengths below 10 nm. LWFA beam energies range from 0.1 GeV to 1 GeV. In turn, the shortest possible undulator period lengths are favoured and we restrict our parameter study to period lengths below 15 mm and conclude already at this point that hard X-rays ($\lambda \leq 1$ Å) are realistically out of reach of the technologies discussed here.

Transverse Beam Size

Since the transverse field gradient in a TGU exhibits a periodically alternating polarity, a TGU can be thought of as a FODO structure in the horizontal direction [16]. Accordingly, matching conditions for the horizontal beta function exist that result in a beta function inside the TGU oscillating with constant amplitude and period λ_u. Such a solution would always be favorable for very long undulators since no additional external focusing would be required. However, if we consider table-top FELs with gain lengths in the order of a few decimeters and undulator lengths L_u of a few meters, the matching condition for a drift space with a beam waist in the center of the undulator and $\beta_{x,\mathrm{waist}} = L_u/2$ turns out

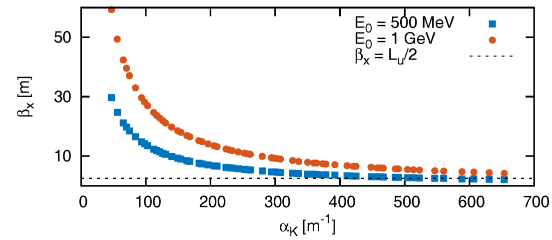

Figure 2: Maximum values of the horizontal beta function satisfying the condition $\beta_x(z_0) = \beta_x(z_0 + \lambda_u)$, $\beta'_x(z_0) = \beta'_x(z_0 + \lambda_u) = 0$ inside the TGU calculated for different sets $\{\frac{\partial B_y}{\partial x}, \lambda_u\}$ and plotted as a function of α_K. The dashed line represents the condition $\beta_{x,\mathrm{waist}} = L_u/2$ for the undulator length $L_u = 5$ m

Table 1: Variation Ranges of Beam and Undulator Parameters

E_0	[GeV]	0.5, 1.0	K_0		0.5…2.5
σ_δ		0.1, 0.01	λ_u	[mm]	5…15
I_P	[kA]	2, 10, 50			
σ_x	[µm]	100, 50, 10			

to yield smaller beam sizes in virtually all cases of interest. This conclusion can be drawn from Fig. 2 where the constant beta function amplitude is plotted versus the K-gradient for two beam energies and compared to the drift-space matching condition for $L_u = 5$ m. In the following we will therefore assume the beam size to be determined by the latter condition and set $\beta_x = 2.5$ m in all calculations presented below.

METHOD OF THE PARAMETER STUDY

The parameter study was performed in two steps [17]. In the first step a fixed dispersion of $\eta = 0.02$ m is assumed. For the beam parameters E_0, σ_δ, the peak current I_P and σ_x all combinations of the values summarized in Table 1 were applied (we omit our calculations for lower beam energies here, because of our focus on X-ray FELs). The most conservative of these values (i.e. the lowest beam energies and peak currents and the largest beam sizes and energy spreads) we consider routinely achievable at LWFAs today. For each combination of beam parameters the undulator period length and the undulator parameter were randomly varied in the ranges $\lambda_u = 5$ mm to 15 mm and $K_0 = 0$ to 2.5. The resulting gain length is calculated using the 1D-equations of [9, 10]. The figure of merit is to achieve a gain length $L_g < 0.5$ m. Additionally we investigated in how far the modified FEL resonance condition

$$\sigma_{\delta,\mathrm{eff}} \ll \rho_{\mathrm{TGU}} \Leftrightarrow \frac{\rho_{\mathrm{TGU}}}{\sigma_{\delta\mathrm{eff}}} = \frac{\rho_{\mathrm{FEL}}}{\sigma_\delta}\left(1 + \frac{\eta^2\sigma_\delta^2}{\sigma_x^2}\right)^{\frac{1}{3}} \gg 1 \quad (4)$$

is fulfilled.

In the second step, for a distinct batch of parameter sets also the dispersion was varied. For this step parameter sets were chosen that yield, additional to a gain length below

6th International Particle Accelerator Conference IPAC2015, Richmond, VA, USA JACoW Publishing
ISBN: 978-3-95450-168-7 doi:10.18429/JACoW-IPAC2015-TUPWA039

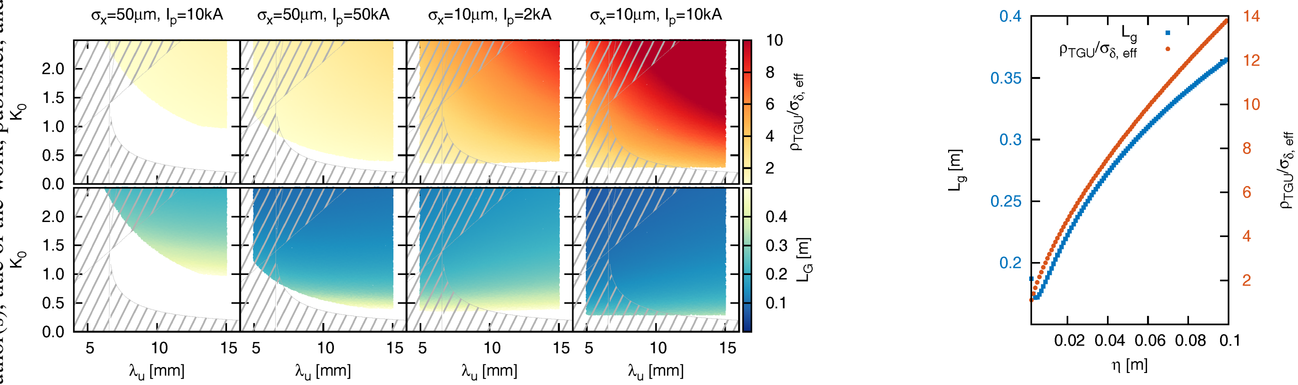

Figure 3: Gain length and resonance condition for $\eta = 0.02$ m, $E_0 = 500$ MeV and $\sigma_\delta = 0.01$ (left), the hatched area is technically inaccessible; gain length and resonance condition as a function of dispersion for the case $\sigma_x = 10\,\mu$m, $I_P = 2$ kA, and $\lambda_u = 11$ mm, $K_0 = 1.19$ (right).

0.5 m, also a radiation wavelength in the soft X-ray regime, i.e. $\lambda \lessapprox 10$ nm.

RESULTS

Figure 3 (left part) shows the results for two beam parameter sets with two peak current values each, representing the most conservative and the "optimum" case, respectively, where the term "optimum" refers to the highest ratio $\rho_{TGU}/\sigma_{\delta,\mathrm{eff}}$ and the shortest gain lengths achieved in the parameter space under investigation. The hatched area indicates the technically inaccessible parameter space which is determined by $K_{0,\max}(\lambda_u)$ (Fig. 1 top), and $\alpha_{K\max}(\lambda_u)$ (Fig. 1 bottom) together with Equation 3. We emphasize that already the most conservative LWFA parameters yielding at all solutions with acceptable gain lengths are quite challenging. Neither of these solutions fulfills the resonance condition Eq. 4.

In general (and not surprisingly), low initial energy spreads, beam sizes, beam energies and high peak currents are favored, both by the requirement of short gain lengths and by the FEL resonance condition. For the undulator, the gain length requirement calls for high K-values, whereas the resonance condition additionally favors longer period lengths. These tendencies as well as the preference for low electron energies obviously go against the aim of generating radiation in the X-ray regime.

To investigate in the second step of our study the influence of the dispersion, we filtered results with $L_g < 0.5$ m for all combinations of beam parameters shown in Table 1 for the shortest wavelengths of the radiation generated. With the fixed dispersion $\eta = 0.02$ m chosen for the first step of our study, neither of these results strictly fulfills the resonance condition Eq. 4. As an example, Figure 3 (right) shows the gain length and the ratio $\rho_{TGU}/\sigma_{\delta,\mathrm{eff}}$ under variation of η for the parameter set $E_0 = 0.5$ GeV, $\sigma_\delta = 0.01$, $\sigma_x = 10\,\mu$m, $I_P = 2$ kA, and $\lambda_u = 11$ mm, $K_0 = 1.19$. For the different parameter sets investigated, the position of the minimum gain length varies between 0.005 m and 0.5 m. In all cases, at minimal gain length the resonance condition is only poorly fulfilled. By increasing the dispersion the ratio $\rho_{TGU}/\sigma_{\delta,\mathrm{eff}}$ can be increased at the cost of a likewise increased gain length which might however be admissible.

Table 2: Parameter Sets Satisfying $\rho_{TGU}/\sigma_{\delta,\mathrm{eff}} \geq 5$ and $\lambda \lessapprox 10$ nm. For all of these sets $\sigma_x = 10\,\mu$m and $\sigma_\delta = 1$ % applies. Gain lengths range between 0.3 and 0.5 m

E_0 [GeV]	I_P [kA]	η [mm]	λ_u [mm]	K_0	α_K [m^{-1}]	λ [nm]
0.5	2	62	11	1.2	47	9.9
0.5	2	83	6.6	0.5	54	3.9
0.5	10	61	5.8	0.3	114	3.2
1.0	50	72	5.9	0.3	97	0.8

The parameter set forming the basis of Figure 3 turns out to be the only among the investigated sets for which $\rho_{TGU}/\sigma_{\delta,\mathrm{eff}} \geq 10$, a gain length below 0.5 m and a radiation wavelength below 10 nm are achievable at the same time. Table 2 summarizes the parameter sets for which the same conditions with a relaxed resonance requirement $\rho_{TGU}/\sigma_{\delta,\mathrm{eff}} \geq 5$ are met. Among these, only the first two are in the technically feasible range on the part of the undulator, and the assumptions made for the LWFA beam size and energy spread are very optimistic.

CONCLUSIONS

Transverse gradient undulators enable Free Electron Laser schemes involving electron sources with relatively large energy spreads, particularly laser wakefield accelerators, providing gain lengths in the order of a few decimeter. Still a table top X-ray free electron laser employing a TGU is a challenging enterprise. Our study shows, based on the 1D TGU-FEL theory, that if above the gain length and radiation wavelength minimization also the fulfillment of the modified resonance condition (Eq. 4) is ensured, even a soft X-ray TGU-FEL requires a substantial improvement on the side of the typical LWFA beam parameters. How strictly the TGU-FEL resonance condition applies, however, needs to be validated by 3D simulations in future.

ISBN: 978-3-95450-168-7 doi:10.18429/JACoW-IPAC2015-TUPWA039

REFERENCES

[1] S. P. D. Mangles, C. D. Murphy, Z. Najmudin, A. G. R. Thomas, J. L. Collier, A. E. Dangor, E. J. Divall, P. S. Foster, J. G. Gallacher, C. J. Hooker, D. A. Jaroszynski, A. J. Langley, W. B. Mori, P. A. Norreys, F. S. Tsung, R. Viskup, B. R. Walton, and K. Krushelnick. Monoenergetic beams of relativistic electrons from intense laser-plasma interactions. *Nature*, 431:535, 2004.

[2] C. G. R. Geddes, Cs. Toth, J. van Tilborg, E. Esarey, C. B. Schroeder, D. Bruhwiler, C. Nieter, J. Cary, and W. P. Leemans. High-quality electron beams from a laser wakefield accelerator using plasma-channel guiding. *Nature*, 431:538, 2004.

[3] J. Faure, Y. Glinec, A. Pukhov, S. Kiselev, S. Gordienko, E. Lefebvre, J.-P. Rousseau, F. Burgy, and V. Malka. A laser-plasma accelerator producing monoenergetic electron beams. *Nature*, 431:541, 2004.

[4] D. A. Jaroszynski, R. Bingham, E. Brunetti, B. Ersfeld, J. Gallacher, B. van der Geer, R. Issac, S.P. Jamison, D. Jones, M. de Loos, A. Lyachev, V. Pavlov, A. J. W. Reitsma, Y. Saveliev, G. Vieux, and S. M. Wiggins. Radiation sources based on laser-plasma interactions. *Philosophical Transactions of the Royal Society A*, 364:689–710, 2006.

[5] C. B. Schroeder, W. M. Fawley, E. Esarey, and W. P. Leemans. Design of an XUV FEL Driven by the Laser-Plasma-Accelerator at the LBNL LOASIS Facility. http://escholarship.org/uc/item/1sp9h9w2, 2006.

[6] F. Grüner, S. Becker, U. Schramm, E. Eichner, M. Fuchs, R. Weingartner, D. Habs, J. Meyer-Ter-Vehn, M. Geissler, M. Ferrario, L. Serafini, B. van der Geer, H. Backe, and W. Lauth. Design considerations for table-top, laser-based VUV and X-ray free electron lasers. *Applied Physics B*, 86:431–435, 2007.

[7] A. R. Maier, A. Meseck, S. Reiche, C. B. Schroeder, T. Seggebrock, and F. Grüner. Demonstration Scheme for a Laser-Plasma-Driven Free-Electron Laser. *Phys. Rev. X*, 2:031019, Sep 2012.

[8] T Seggebrock, AR Maier, I Dornmair, and F Grüner. Bunch decompression for laser-plasma driven free-electron laser demonstration schemes. *Phys. Rev. ST Accel. Beams*, 16(7):070703, 2013.

[9] Zhirong Huang, Yuantao Ding, and Carl B. Schroeder. Compact X-ray Free-Electron-Laser from a Laser-Plasma Accelerator Using a Transverse-Gradient Undulator. *Phys. Rev. Lett.*, 109:204801, 2012.

[10] Panagiotis Baxevanis, Yuantao Ding, Zhirong Huang, and Ronald Ruth. 3D theroy of a high-gain free-electron laser based on a transverse gradient undulator. *Phys. Rev. ST Accel. Beams*, 17:020701, 2014.

[11] Haixiao Deng and Chao Feng. Using Off-Resonance Laser Modulation for Beam Energy Spread Cooling in Generation of Short-Wavelength Radiation. *Phys. Rev. Lett.*, 111:084801, May 2013.

[12] Golo Fuchert, Axel Bernhard, Sandra Ehlers, Peter Peiffer, Robert Rossmanith, and Tilo Baumbach. A novel undulator concept for electron beams with a very large energy spread. *Nuclear Instruments and Methods in Physics Research A*, 672:33–37, 2012.

[13] V. Afonso Rodriguez, T. Baumbach, A. Bernhard, A. Keilmann, P. Peiffer, R. Rossmanith, C. Widmann, M. Nicolai, M. Kaluza, and G. Fuchert. Design optimization for a non-planar undulator for the JETI-laser wakefield accelerator in Jena . In *Proceedings of the IPAC 2011*, 2011.

[14] A. Bernhard, N. Braun, V. Afonso Rodriguez, P. Peiffer, R. Rossmanith, C. Widmann, and M. Scheer. Radiation emitted by transverse gradient undulators. not published, 2015.

[15] V. Afonso Rodriguez, A. Bernhard, A. Grau, P. Peiffer, R. Rossmanith, M. Weber, C. Widmann, A. Will, M.C. Kaluza, M. Nicolai, A. Sävert, and M Reuter. Construction and first magnetic field test of a superconducting transversal gradient undulator for the laser wakefield accelerator in Jena:w. In *Proceedings of the International Particle Accelerator Conference IPAC2014*, 2014.

[16] C. Widmann, V. Afonso Rodriguez, A. Bernhard, N. Braun, B. Härer, O. Jäckel, M. Nicolai, P. Peiffer, M. Reuter, T. Rinck, R. Rossmanith, A. Sävert, M. Scheer, W. Walter, T. Baumbach, and M. Kaluza. Non-linear beam transport optics for a laser wakefield accelerator. In *Proceedings of the International Particle Accelerator Conference IPAC2015*, page TUPWO013, 2013.

[17] Eva Burkard. Freie-Elektronen-Laser auf Basis supraleitender Transversalgradient-Undulatoren — Eine Parameterstudie. Bachelor's Thesis, Karlsruhe Institute of Technology, 2015.