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ALGORITHM OF RECONSTRUCTING PARTICLE DISTRIBUTION IN N-DIMENSIONAL PHASE SPACE FROM PROFILE IN BEAM TRANSPORT

T. Aoki#, Research & Development Group, Hitachi, Ltd., Hitachi, Japan

Abstract

A new method of reconstructing particle distribution from measured profile in beam transport is proposed. In this method, particle distribution in arbitrary dimensional phase space can be reconstructed from profiles in transport system. The particle distribution is obtained by solving a following equation: $I=D\rho$, where I is a counted number of particles at a single channel of the profile monitor, D is a matrix representing relation between I and ρ, and ρ is number of particles included in a single n-dimensional voxel of phase space at start point of the beam transport. We succeeded formulation of matrix D from transportation matrix of the beam transport R, and discovered that D is formulated as piecewise-polynomials of elements of R. We show details of the formulation of D and results of simulations of reconstruction of particle distribution in phase space by this method.

INTRODUCTION

Purpose of this study is to propose a new method of reconstructing particle distribution in phase space. In particle beam therapy, it is demanded to deliver a narrow beam to isocenter in order to realize conformal irradiation. To realize the narrow beam size at isocenter, a tuning of high energy beam transport (HEBT) system and measurement of beam is important. Conventionally, Quadrupole scan method [1] is applied for measurement in HEBT system. Quadrupole scan method is effective when particle distribution in phase space is Gaussian, and we can measure emittance ε, twiss parameters α and β. However, the particle distribution is not Gaussian, it is required that not only measurement of these parameters but also particle distribution in phase space is required. Because the beam energy is larger than 70MeV/u in HEBT, the slit scan method is not applicable for the measurement. Therefore we adopt the new method that improved phase space tomography method [2]. By this phase space tomography method, transverse phase space (x, x') and (y, y') distribution. However in beam tuning in particle therapy, the information about dispersion is need. In this study, we propose the new method which realizes a measurement of particle distribution in arbitrary dimensional phase space.

THEORY

Let $\rho(\boldsymbol{x}; z)$ be the distribution of the particle in phase space at the beam line position z. The phase space coordinates $\boldsymbol{x}$ are $(x, x', y, y', s, \delta)$. x and y are spatial transvers displacements from design orbit. δ is momentum displacement $\Delta p/p$. s is longitudinal displacement from beam centroid. x' and y' are dx/dz and dy/dz respectively. To obtain the distribution at specified point z_0 in HEBT, we measure the number of particles in small voxel of phase space from measured profile in HEBT. We define ρ_i as number of particles in ith voxel in phase space, I_{x_j} as number of particles counted in the jth channel of profile monitor at $x=x_j$. Assuming the distribution in the single voxel is flat, a relation of ρ_i and I_{x_j} become

$$I_{x_j} = \sum_{k=1}^{n} \int_{x_j-\frac{w}{2}}^{x_j+\frac{w}{2}} \phi_k(x) \cdot \rho_k \mathrm{d}x \tag{1}$$

where w is width of profile monitor channel, and $\phi_k(x)$ is projected volume of kth phase space voxel on x-axis. Assembling this relation, we get a equation

$$\boldsymbol{I} \stackrel{\text{def}}{=} \begin{pmatrix} I_1 \\ \vdots \\ I_m \end{pmatrix} = \boldsymbol{D} \begin{pmatrix} \rho_1 \\ \vdots \\ \rho_k \end{pmatrix} \stackrel{\text{def}}{=} \boldsymbol{D\rho} \tag{2}$$

where $\boldsymbol{D}$ is defined as

$$\boldsymbol{D} = \left(d_{ij}\right)_{\substack{1\le i\le k \\ 1\le j\le m}} \tag{3}$$

and

$$d_{ij} = \int_{x_j-\frac{w}{2}}^{x_j+\frac{w}{2}} \phi_i(x) \mathrm{d}x. \tag{4}$$

Therefore, we can obtain the particle distribution ρ by solving the eq.(2). To solve the eq.(2), it is required that the formula for projected volume $\phi_i(x)$ and enough number of measurement of profiles.

Derivation formula for Projected Volume

We derive the formula for projected volume of a N-dimensional parallelepiped constructed by N vectors $\boldsymbol{a}_1$, $\boldsymbol{a}_2$... $\boldsymbol{a}_N$. in this section. At first, In the case of N=1, it is clear that

$$\phi^{(1)}(x) = \frac{1}{c_1} \mathrm{rect}\left(\frac{x-x_c}{c_1 a_1}\right) \tag{5}$$

where c_1 is a cosine of the angle between $\boldsymbol{a}_1$ and x-coordinate, a_1 a length of $\boldsymbol{a}_1$, x_c is a x-coordinate of centroid of the voxel, and rect(x) is the rectangular function defined as

$$\mathrm{rect}(x) \stackrel{\text{def}}{=} \begin{cases} 0, \text{if } |x| < \dfrac{1}{2} \\ 1, \text{if } |x| \ge \dfrac{1}{2} \end{cases} \tag{6}$$

Figure1 shows this situation. The 1 dimensional voxel is a segment of line in phase space. So, the projected volume is a rectangular function.

#takamichi.aoki.my@hitachi.com

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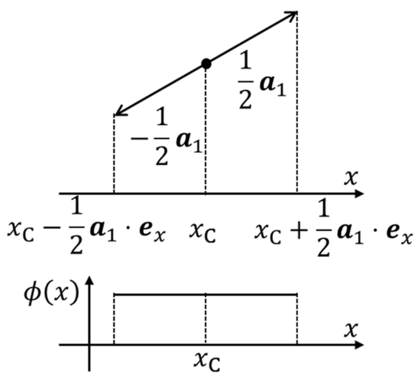

Figure 1: Projected volume of 1 dimensional voxel.

Next, we derive a formula of a higher dimensional voxel by mathematical induction. Assuming the N-dimensional projected volume $\phi^{(N)}(x)$, the projected volume of N+1 dimensional voxel is given by convolution of $\phi^{(N)}(x)$ and $\phi^{(1)}(x)$. So, we get the formula between $\phi^{(N)}(x)$ and $\phi^{(1)}(x)$;

$$\phi^{(N+1)}(x) = \int_{-\infty}^{\infty} \phi^{(N)}(t) \cdot \frac{h_{N+1}}{|c_{N+1}a_{N+1}|}\mathrm{rect}\left(\frac{x-x_c-t}{|c_{N+1}a_{N+1}|}\right)\mathrm{d}t. \quad (7)$$

solving eq.(7) under the condition of Eq.(5). We get $\phi^{(N)}(x)$ as

$$\phi^{(N)}(x) = \frac{V}{\prod_{k=1}^{m} c_{s_k}a_{s_k}} \overbrace{\int_{-\infty}^{x}\cdots\int_{-\infty}^{x}}^{m\text{ integrals}} \sum_{\sigma=\pm 1}\left[\left\{\prod_{l=1}^{m}\sigma_{s_l}\right\} \cdot \delta\left(x - x_C + \frac{1}{2}\sum_{l=1}^{m}\sigma_{s_l}c_{s_l}a_{s_l}\right)\right]\mathrm{d}^m x \quad (8)$$

where V is N-dimensional volume of the voxel, m is a number of vectors which is not orthogonal to x-axis in { $\boldsymbol{a}_1, \boldsymbol{a}_2 \dots \boldsymbol{a}_N$ }, s_l is index of these vectors, summation is get under all cases of σ_{s_l} is 1 or -1. This equation means that the projected volume is polynomial of degree less than N. Fig(2)-(4) shows the projected volume in the case of N=2, 3, and 4. Finally we succeeded the formulation of projected N-dimensional volume.

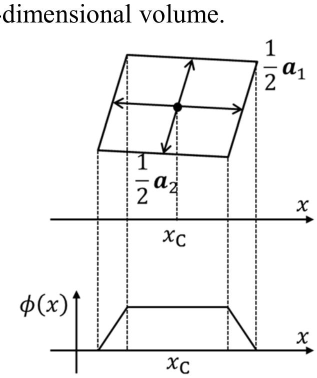

Figure 2: Projected volume of 2 dimensional voxel.

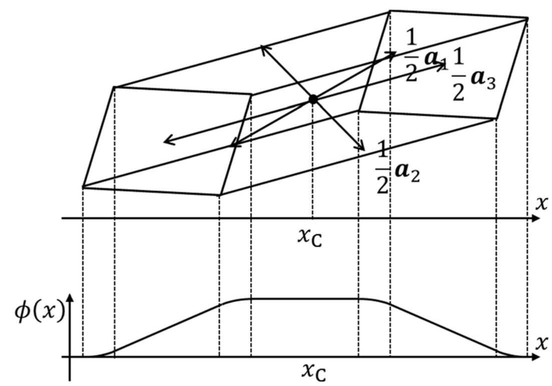

Figure 3: Projected volume of 3 dimensional voxel.

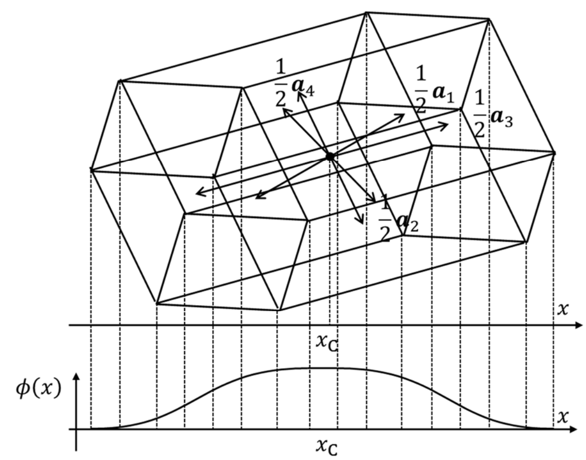

Figure 4: Projected volume of 1 dimensional voxel.

Reconstruction Process

We show the process of reconstruction of the particle distribution in phase space. The distribution can be obtained by solving of eq.(2). Generally, the equation derived from the only one case of profile measurement is indeterminate system. Therefore it is required that the equation is contracted from plural measurements. The measurements must be done under different condition each other. The changing condition of measurements is realized by changing transfer matrix of HEBT upstream of profile monitor, such as changing quadrupole magnet current. Required number of measurements depends on number of channel of profile monitor m and number of voxels k which divide phase space to be reconstructed. So, obviously the more measurements is required to reconstruct the distribution at higher resolution.

These instructions are a typical implementation of the requirements. Manuscripts should have:

SIMULATION

We simulate of reconstructing particle distribution by the new method. Assuming that HEBT system showed in Fig.5, we reconstructed the particle distribution at start point (S) from measured profile at profile monitor (M). This HEBT has one bending magnet (BM) and two

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straight sections. There are three quadrupole magnets (Q1-3 and Q4-6) in each straight section.

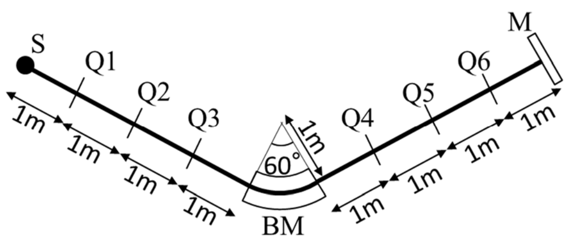

Figure 5: Assumed HEBT for simulation.

We assumed particle distribution which Fig. 6 indicates as initial particle distribution at the start point (S). The initial distribution is rectangular distribution including 1000 particles. On the momentum beam, maximum of x and x' are 1 mm and 1 mrad respectively. Dispersion function η and η′ are 1 m and 0.1 respectively. The momentum displacement δ distributes between $\pm 10^{-3}$, and the dispersion function η and η′ are 0.98m and 0.07 respectively.

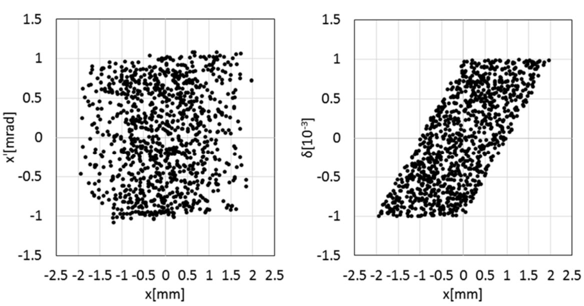

Figure 6: Initial particle distribution.

We calculated profile at monitor (M) in the in 32 cases which varies focal length of quadrupole. The focal length is 2m or 16m, Q1, Q3, Q5, and Q7 are focusing magnets, the others are defocusing magnet. Then we reconstruct the initial particle distribution from calculated profiles. The property of phase space voxel is shown in Table 1. Phase space of $|x| < 3.5$ mm, $|x'| < 1.25$mm and $|\delta| < 1.25\times10^{-3}$ at start point is divided into 125 (5 × 5 × 5) voxels. As the result of simulation, the distribution indicated in Fig. 7-8 is obtained. The results of dispersion function η and η′ are 0.71m and 0.15 respectively.

Table 1: Phase Space Voxel

Direction	Width	Num. of Voxels
x	1.4 mm	5
x'	0.5 mrad	5
δ	0.5×10^{-3}	5

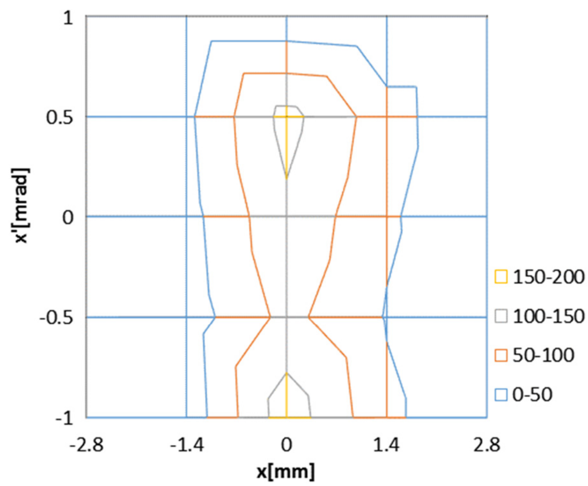

Figure 7: Reconstructed particle distribution on (x, x').

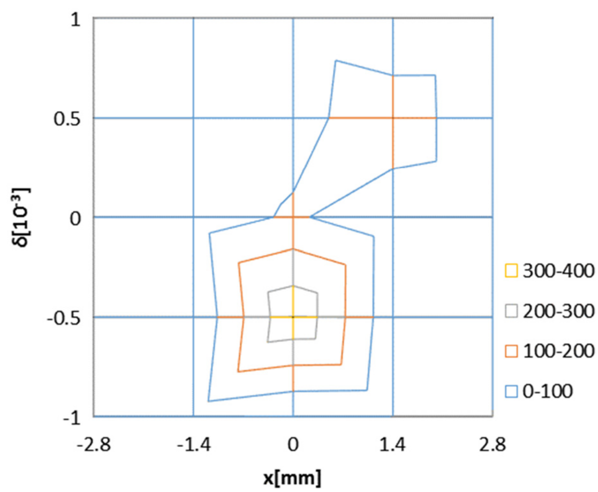

Figure 8: Reconstructed particle distribution on (x, δ).

CONCLUSION

We propose new method of reconstructing particle distribution from measured profiles. The new method is basically verified by simulation of measurement in HEBT. As result of simulation, reconstruction of particle distribution in phase space divided into 125 voxels was succeeded. In higher resolution reconstruction is need for practical use.

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[1] J. T. Seeman, *Handbook of Accelerator Physics and Engineering*, (Singapore: World Scientific, 1998), p 559- 560.
[2] C. B. McKee, et al., “Phase space Tomography of relativistic electron beams”, NIM, A358, p264-267 (1995).