ISBN: 978-3-95450-168-7 doi:10.18429/JACoW-IPAC2015-TUPWI043

CHROMATIC EFFECTS IN LONG PERIODIC TRANSPORT CHANNELS

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Abstract

Long periodic transport channels are frequently used in accelerator complexes and suggested for using in high-energy ERLs for electron-hadron colliders. Without proper chromaticity compensation, such transport channels exhibit high sensitivity to the random orbit errors causing significant emittance growth. Such emittance growth can come from both the correlated and the uncorrelated energy spread. In this paper we present results of our theoretical and numerical studies of such effects and develop a criteria for acceptable chromaticity in such channels.

INTRODUCTION

The subject of emittance growth resulting from non-compensated chromaticity in single pass linac focusing systems was exhaustively studied in the era of linear colliders decades ago. Some of the classical treatment can be found in [1-4] and references therein. Effects described in these papers are fully applicable to energy recovery linacs with non-compensated chromaticity, e.g. a case presented in my paper [5] where it was suggested to use a strongly chromatic lattices to improve the beam's stability in energy-recovery linacs (ERLs), or as proposed for an ERL with FFAG arcs [6]. Recently we realized, that, naturally, such lattices would exhibit a very high sensitivity to orbital errors – the effect described in details in ref. [1-4]. Such errors could, in turn, significantly increase the beam's emittance. These emittance increase can be either stationary or time-dependent – the later is determined if beam orbit is stable or is time dependent.

In this paper we briefly review a simple case when the orbit errors are random. We also use orbit correction approach described in [1-3], and applied to a long chromatic arcs, to define sensitivity to beam-position measurement (BPM) errors.

Finally, we would like to note that there is a continuing effort of finding a robust solution for chromatic FFAG arcs (which operates beams with multiple energies) using various orbit corrections scheme to avoid beam emittance growth [7].

ORBIT DISTORTION IN CROMATIC LATTICE

Let's consider a simple uncoupled motion on a arc (or straight beam-line, then $K_o=0$) comprised of periodic cells with $K_{x,y,o}(s+P)=K_{x,y,o}(s)$:

$$x''+\frac{K_x(s)}{1+\delta}x=K_o(s)-\frac{eB_y(s)}{p_oc(1+\delta)}-\frac{e\delta B_y(s)}{p_oc(1+\delta)};$$

$$y''+\frac{K_y(s)}{1+\delta}y=+\frac{e\delta B_x(s)}{p_oc(1+\delta)};\ K_o(s)\equiv\frac{1}{\rho(s)}.$$

where $\delta B_{x,y}(s)$ are field errors and $\delta=\dfrac{E-E_o}{E_o}$ is the relative energy deviation from the reference (design) energy. It is well known that within the stability range of the periodic cell, one can find periodic reference orbit

$$K_o(s+C)=K_o(s);\ B_y(s+C)=B_y(s)$$

$$\eta^\delta+\frac{K_x(s)}{1+\delta}\eta^\delta=K_o(s)-\frac{eB_y(s)}{p_oc(1+\delta)};\ \eta^\delta(s+C)=\eta^\delta(s)$$

and Courant-Snyder function fully describing transverse oscillation about the reference orbit (here we use y-direction as example):

$$\mathrm{w}_y^\delta(s+C)=\mathrm{w}_y^\delta(s+C);\ \ \beta_y^\delta=\left(\mathrm{w}_y^\delta\right)^2;$$

$$y=a_y\cdot\mathrm{w}_y^\delta(s)\cdot\cos\left(\psi_y^\delta(s)+\varphi_y\right);\ \frac{d\psi_y^\delta}{ds}=\frac{1}{\beta_y^\delta(s)};$$

$$y'=\begin{Bmatrix}a_y\cdot\mathrm{w}_y'^\delta(s)\cdot\cos\left(\psi_y^\delta(s)+\varphi_y\right)\\-\dfrac{a_y}{\mathrm{w}_y^\delta(s)}\cdot\sin\left(\psi_y^\delta(s)+\varphi_y\right)\end{Bmatrix}.$$

There is well known expression for the orbit distortions caused by an arbitrary field errors. Here we rewrite it with emphasis on the energy dependence:

$$\delta y_\delta(s)=\mathrm{w}_y^\delta(s)\int_0^s\sin\left(\psi_y^\delta(s)-\psi_y^\delta(s_1)\right)\mathrm{w}_y^\delta(s_1)\frac{e\delta B_x(s_1)}{p_oc(1+\delta)}ds_1;$$

$$\delta y'_\delta(s)=\int_0^s\begin{Bmatrix}\mathrm{w}_y'^\delta(s)\sin\left(\psi_y^\delta(s)-\psi_y^\delta(s_1)\right)+\\\dfrac{1}{\mathrm{w}_y^\delta(s)}\cos\left(\psi_y^\delta(s)-\psi_y^\delta(s_1)\right)\end{Bmatrix}\mathrm{w}_y^\delta(s_1)\frac{e\delta B_x(s_1)}{p_oc(1+\delta)}ds_1$$

which indicative of the resulting orbit dependence on particle's energy and lattice chromaticity. Naturally, similar treatment is applicable to horizontal motion:

$$\mathrm{w}_x^\delta(s+C)=\mathrm{w}_x^\delta(s+C)$$

$$x=\eta^\delta(s)+a_x\cdot\mathrm{w}_x^\delta(s)\cdot\cos\left(\psi_x^\delta(s)+\varphi_x\right)+\delta x_\delta(s)$$

$$\beta_x^\delta=\left(\mathrm{w}_x^\delta\right)^2;\frac{d\psi_x^\delta}{ds}=\frac{1}{\beta_x^\delta(s)};$$

$$\delta x_\delta(s)=-\mathrm{w}_x^\delta(s)\int_0^s\sin\left(\psi_x^\delta(s)-\psi_x^\delta(s_1)\right)\mathrm{w}_x^\delta(s_1)\frac{e\delta B_y(s_1)}{p_oc(1+\delta)}ds_1.$$

Further in the paper we will use index z both to x and y. To estimate effect of chromaticity and orbit distortions on the beam emittance

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$$\varepsilon^2 = \left\langle (z-\bar{z})^2 \right\rangle \left\langle (z'-\bar{z}')^2 \right\rangle - \left\langle (z-\bar{z})(z'-\bar{z}') \right\rangle^2$$

we separate periodic orbit and the betatron motion from orbit distortion dependence on energy, e.g.

$$z = z_\beta + \Delta z(\delta);\ \langle z_\beta \rangle = 0; \langle z'_\beta \rangle = 0;$$

$$\langle z_\beta^2 \rangle = \beta \cdot \varepsilon_o; \langle z'^2_\beta \rangle = \frac{1+\alpha^2}{\beta} \cdot \varepsilon_o;\ \langle z_\beta z'_\beta \rangle^2 = \frac{\alpha^2}{\beta} \cdot \varepsilon_o$$

$$\Delta\tilde{z} = \Delta z(\delta) - \overline{\Delta z(\delta)};\ \int_{-\infty}^{\infty} f(\delta)\, d\delta = 1$$

$$\overline{\Delta z(\delta)} = \int_{-\infty}^{\infty} f(\delta) \Delta z(\delta)\, d\delta;$$

Simple calculations show that

$$\left\langle (z-\bar{z})^2 \right\rangle == \beta \cdot \varepsilon + \langle \Delta\tilde{z}^2 \rangle$$

$$\left\langle (z'-\bar{z}')^2 \right\rangle = \frac{1+\alpha^2}{\beta} \cdot \varepsilon + \langle \Delta\tilde{z}'^2 \rangle$$

$$\left\langle (z-\bar{z})(z'-\bar{z}') \right\rangle^2 = \begin{Bmatrix} \frac{\alpha^2}{\beta} \cdot \varepsilon_o + 2\sqrt{\frac{\alpha^2}{\beta} \cdot \varepsilon_o}\, \langle \Delta\tilde{z}\Delta\tilde{z}' \rangle \\ + \langle \Delta\tilde{z}\Delta\tilde{z}' \rangle^2 \end{Bmatrix}$$

While the above expression can be used, general expressions become very heavy (see below) and can not fit into this short paper.

$$\overline{\Delta z^2} = \int_0^s ds_2 \frac{e\delta B(s_2)}{p_o c} \int_0^s \frac{e\delta B(s_1)}{p_o c} ds_1 \int_{-\infty}^{\infty} d\delta f(\delta) \frac{\mathrm{w}^\delta(s)\mathrm{w}^\delta(s_1)\sin\left(\psi^\delta(s) - \psi^\delta(s_1)\right)}{(1+\delta)} \cdot \frac{\mathrm{w}^\delta(s)\mathrm{w}^\delta(s_2)\sin\left(\psi^\delta(s) - \psi^\delta(s_2)\right)}{(1+\delta)}.$$

Thus, we will use as a test case a set of localized uncorrelated angular kicks, θ_k, $\langle \theta_i \theta_k \rangle = \theta_i^2 \delta_i^k$

$$\delta x(s,\delta) = \mathrm{w}_N(s_N,\delta) \cdot \sum_{k=1}^{N_{cell}} \theta_k \cdot \mathrm{w}(s_k,\delta) \cdot \sin\left(\psi(s_N,\delta) - \psi(s_k,\delta)\right).$$

For small energy deviation, we can used an expantion [5]

$$\psi^\delta(s) \cong \psi_o(s) + \delta \cdot 2\pi C(s);\ w^\delta(s) = w_o(s) + \delta \cdot u(s);\ w'^\delta(s) = w'_o(s) + \delta \cdot v(s).$$

It is easy to show (see [5] for example) that in case of large chromaticity $2\pi C >> 1$, we can neglect u and v terms above (see exact expressions in [5] on page 074401-5). For a lattice with number of periodic cells, N_{cell}, one will arrive to a simple expression for beam emittance (to be exact, this is the projected beam emittance):

$$\varepsilon_N^2 = \left(\varepsilon_{No} + \gamma_o \beta \sum_{n=1}^{N_{cell}} A_n \theta_n^2 \right) \left(\varepsilon_{No} + \gamma_o \beta \sum_{n=1}^{N_{cell}} B_n \theta_n^2 \right) + O\left(\left(\sigma_\delta \sum_{n=1}^{N_{cell}} \theta_n^2 \right)^2 \right);\quad \beta \equiv \mathrm{w}^2; E_n = \exp\left(-(2\pi n \cdot C\sigma_\delta)^2 \right);$$

$$A_n = \frac{1}{2}(1-E_n)(1-E_n\cos(2n\mu_o)); B_n = \frac{1}{2}(1-E_n)(1+E_n\cos(2n\mu_o)); \frac{A_{N_{cell}}}{N_{cell}} = \frac{\sum_{n=1}^{N_{cell}} A_n\theta_n^2}{\sum_{n=1}^{N_{cell}} \theta_n^2}; \frac{B_{N_{cell}}}{N_{cell}} = \frac{\sum_{n=1}^{N_{cell}} B_n\theta_n^2}{\sum_{n=1}^{N_{cell}} \theta_n^2} \tag{1}$$

where ε_{No} is the beam's initial normalized emittance , $\gamma_o = E_o / mc^2$ is the beam's relativistic factor μ_o is the phase-advance per cell and C is the cell's chromaticity, for x or y direction, correspondingly.

It is easy to check that for achromatic lattice with $C=0$, both $A_n = 0$=0 and $B_n = 0$, and the emittance is preserved. Similarly, for mono-energetic beam, σ_δ=0, emittance is preserved. In contrast, for lattice with large value of the chromaticity and energy spread, $N_{cell} C\sigma_\delta >> 1$, we have

$$E_n = \exp\left(-(2\pi n \cdot C\sigma_\delta)^2 \right) -> 0$$

$$\Delta\varepsilon_N = \varepsilon_N - \varepsilon_{No} \sim N_{cell} \frac{\gamma_o \beta}{2} \langle \theta^2 \rangle.$$

In order to estimate severity of possible emittance growth, we considered few typical examples. *First,* simple estimations shows that for a 5 GeV beam, N_{cell}= 500, and, $\beta \sim 2m$, the RMS angular kicks of 10^{-6} rad could engender a 10-mm mrad growth of normalized emittance.

Second: We also estimated what would be emittance growth induced by random errors (eg. without orbit corrections) in one of the FFAG lattices proposed for eRHIC [8,9] (see Table 1). Lowest energies in both arcs have largest chromaticity per cell. With 1390 cells per pass, FAAG I arc has chromaticities exceeding a thousand and as can ne seen from Fig. 1 even relatively small relative energy spread at 10^{-4} level, effect is significant.

It is easy to estimate that that random kicks with RMS strength of 1 microradian, would lead to about 50 mm mrad increase in normalized emittance for a beam with relative energy spread of 0.0005 or above.

This effect can be studies numerically. Fig. 2 shows normalized emittance growth in the first pass in FFAG II in the presence of random, uncorrected errors of 1 micrometer in x and y quadrupoles' positions. We used code *elegant* [10] for these simulations with the beam initial normalized emittance of 10 mm mrad and relative

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energy spread of 3^x10^{-4}. Regular transverse dispersion is removed from the particles distributions, e.g. without quadruple position errors there is no emittance growth.

Table 1: Cell Parameters

Parameter	FFAG I	FFAG II
ERL pass	1	6
Energy, GeV	1.344	7.944
Q_x, per cell	0.4259	0.3701
Q_y, per cell	0.3618	0.2498
C_x, per cell	-1.205	-0.6049
C_y, per cell	-0.7488	-0.4042
β_x max, m	8.93	5.017
β_y max, m	6.297	4.82
u_x max, m	-139.0	-18.97
u_y max, m	2.209	2.19

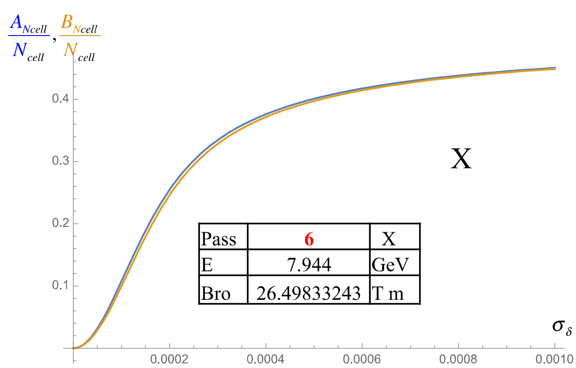


Figure 1: Dependence of coefficient in eq. (1) on the e-beam relative energy spread.

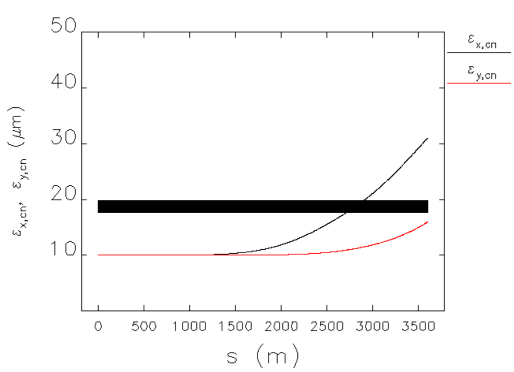


Figure 2: Emittance evolution in the first pass through the FFAG II. RMS quads position error is 1 μm.

The emittance growth depends on the random seed of the errors as can be seen in Fig. 3. The physics behind this apparent emittance growth is a strong and nonlinear dependence of the resulting orbit distortion on the particle's energy. As emphasized in [5], a single kick results in with different energies being spreading in the phase space because of the large variations in the betatron phase advance. Fig.4 illustrate of the "cork-screw" nature of this dependence by projecting the beam distribution on *E-y* and *E-y'* planes of 6D phase space. Similar projections are seen in *E-x*and *E-x'* planes.

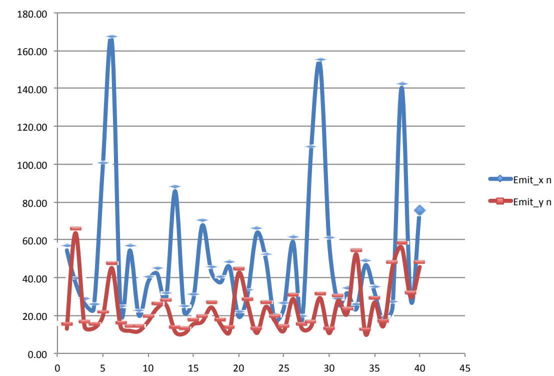
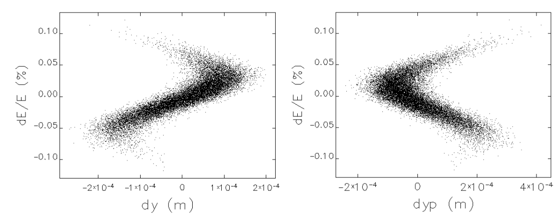


Figure 3: Normalized emittance after the first pass in FFAG for forty random seeds RMS quads position error is 1 μm.



(a) (b)

Figure 4: Correlation between the energy deviation and (a) vertical position, (b) vertical angle.

While the orbit is deterministic, it demonstrates strong sensitivity to the random errors. Hence, any orbit correction mechanism should take into account chromatic nature of the lattice.

As we mentioned in the introduction, this problem was studied in detail for linear colliders. Scientists studying this problem developed a very well-thought through orbit correction algorithms [1-3]. Modifying eq. (34) in ref. [3] for the case of the constant energy, one can arrive to the simple equation of the emittance delusion cased by a corrected orbit in periodic FODO lattice:

$$\Delta\varepsilon_N \approx 32\pi\cdot\gamma_o\cdot N_{cell}\cdot C\frac{\left\langle\ \delta x^2_{BPM}\right\rangle\sigma^2_\delta}{L_{cell}} \tag{2}$$

where $\sqrt{\left\langle\ \delta x^2_{BPM}\right\rangle}$ is an RMS BPM positioning or/and measuring error. Then for the *first case* (see above) with the energy spread of 0.03%, the chromaticity per cell of 0.25, RMS BPM position errors of 0.2 mm and cell length of 2 m, one should expect the emittance growth ~ 0.5 mm mrad. For the *second (FFAG) case*, normalized emittance will grow for about 3 mm mrad per unit of chromaticity per cell. While it is not as serious as random errors, the orbit correction system requires serious consideration. For example, a special achromatic orbit correction algorithm is under development for FFAG lattices [7].

ACKNOWLEDGMENT

We are grateful to Thomas Roser and Dejan Trbojevic (BNL) for their thoughtful comments and suggestions.

ISBN: 978-3-95450-168-7 doi:10.18429/JACoW-IPAC2015-TUPWI043

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