OPTICS CORRECTION FOR THE MULTI-PASS FFAG ERL MACHINE eRHIC[*]

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Abstract

Gradient errors in the multi-pass Fixed Field Alternating Gradient (FFAG) Energy Recovery Linac (ERL) machine, eRHIC, distort the beam orbit and therefore cause emittance increase. The localization and correction of gradient errors are essential for an effective orbit correction and emittance preservation. In this report, the methodology and simulation of optics correction for the multi-pass FFAG ERL machine eRHIC will be presented.

INTRODUCTION

Electron Relativistic Heavy Ion collider (eRHIC) is a new research tool proposed to be built at the existing RHIC facility. The design of the electron machine provides a cost-effective upgrade of RHIC for collision of electron beam with the full array of RHIC hadron beams [1]. Two circular electron accelerators with FFAG lattice are designed to be placed in RHIC tunnel to provide electron beams with top energy at 15.9 and 21.2 GeV [2]. The electron beam will be accelerated from 1.3 GeV to the top energies and energy recovered after collisions. The accelerating passes and decelerating passes in each machine go through the same vacuum chamber thanks to the small dispersion values. The beta functions of each energy differ from each other.

The deviation of the gradient from design will distort beam trajectories and the optical functions. Even though the orbit distortion can be fixed by dipole correctors, the distortion of optical functions will be detrimental for the effectiveness of orbit correction. Therefore, it would be desirable to be able to disentangle dipole errors and quadrupole errors and correct them independently. We adopted the orbit response method (ORM) [3], which has been widely applied for circular rings, and extend it for the multi-pass FFAG case to locate and correct quadrupole errors in eRHIC FFAG machines.

PROPERTIES OF THE LINEAR NON-SCALING FFAG LATTICE

In the eRHIC FFAG design, the magnets are pure focusing and defocusing quadrupoles shifted horizontally relative to a circular orbit. The field experienced by the beam is $G * x$, G is the magnet gradient, x is the horizontal beam position relative to the magnet center. The orbits of beams with different energies are plotted in Fig. 1, with respect to the circular orbit. The energy deviations are calculated with
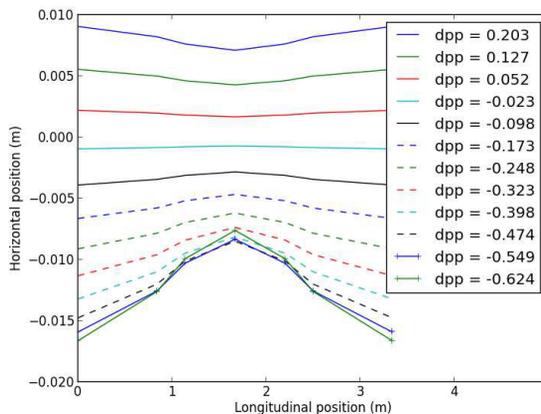
Figure 1: The orbits for beams with different energies in a single FODO cell of the FFAG lattice.

respect to 17.6 GeV. The overall horizontal offsets between beams is ∼25 mm. The beams can therefore fit in the same beam pipe. The close-together beam orbits present a challenge for beam position measurement for individual beams [4].

The beta functions in both planes are plotted in Fig. 2 and 3 for beams with different energies. The responses in terms of beam position changes are expected to be different for beams with different energies.

OPTICS CORRECTION ALGORITHM

In linear FFAG, orbit response deviation depends only on gradient errors linearly. Orbit response deviation from the model can be measured by varying dipole correctors
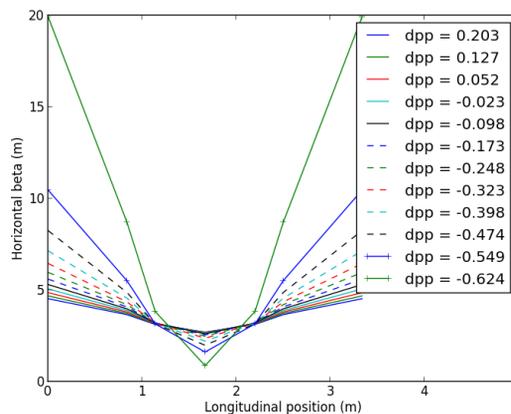

Figure 2: The horizontal beta functions for beams with different energies in a single FODO cell of the FFAG lattice.
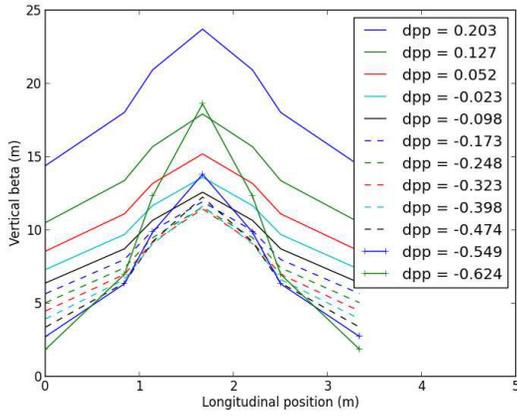
Figure 3: The vertical beta functions for beams with different energies in a single FODO cell of the FFAG lattice.

and recording orbits before and after. The gradient errors can be fitted with knowledge of the model. For a LINAC machine with m BPMs and n correctors, the orbit response matrix is

$$R = \begin{pmatrix} R_{1,1} & R_{1,2} & R_{1,3} & \cdots & R_{1,n} \\ R_{2,1} & R_{2,2} & R_{2,3} & \cdots & R_{2,n} \\ & & \vdots & \vdots & \vdots & \ddots & \vdots \\ R_{m,1} & R_{m,2} & R_{m,3} & \cdots & R_{m,n} \end{pmatrix} \quad (1)$$

where $R_{i,j} = \begin{cases} \sqrt{\beta_i \beta_j} * \sin(\phi_i - \phi_j) & \text{if } \phi_i > \phi_j \\ 0 & \text{if } \phi_i <= \phi_j \end{cases}$

The deviation of the orbit response matrix R, can be put in the form of a vector as $(\triangle R_{1,1}, \triangle R_{1,2} \cdots \triangle R_{2,1}, \triangle R_{2,2} \cdots \triangle R_{m,n-1}, \triangle R_{m,n})^T$ with $m * n$ elements. It is linearly proportional to the gradient errors.

$$\begin{pmatrix} \triangle R_{1,1} \\ \triangle R_{1,2} \\ \vdots \\ \triangle R_{2,1} \\ \triangle R_{2,2} \\ \vdots \\ \triangle R_{m,n-1} \\ \triangle R_{m,n} \end{pmatrix} = \begin{pmatrix} M_{1,1} & M_{1,2} & M_{1,3} & \cdots & M_{1,p} \\ M_{2,1} & M_{2,2} & M_{2,3} & \cdots & M_{2,p} \\ & & \vdots & \vdots & \vdots & \ddots & \vdots \\ M_{mn,1} & M_{mn,2} & \cdots & M_{mn,p} \end{pmatrix} \triangle G$$

$$(2)$$

where $\triangle G$ is a vector of the gradient errors for all quadrupoles. The number of quadrupoles is p. M is a $m * n$ by p matrix.

The model orbit response matrix can be obtained using accelerator simulation codes. The matrix M can be simulated as well. For each quadrupole magnet, one can produce the orbit response matrix with and without some small quadrupole errors. The ratio between the difference of R and the gradient errors corresponds to one column of matrix M, which represents the coefficients to the strength of a given quadrupole. A set of linear equations will be established as Eq. 2, with gradient errors $\triangle G$ as unknowns. The
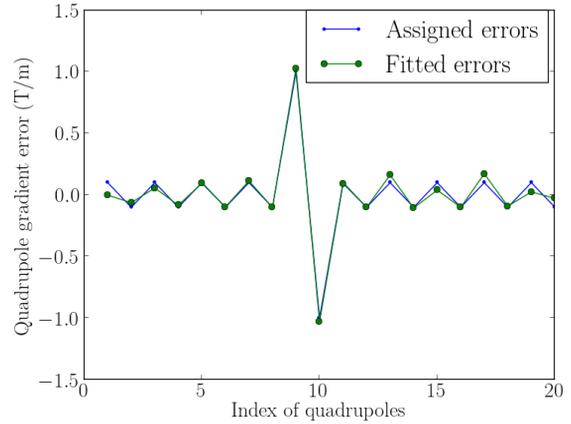

Figure 4: The comparison of the assigned and fitted gradient errors for 20 quadrupole magnets, for the case only the orbit response matrix for the first pass was used in the simulation.

errors can be calculated by linear fitting techniques. In the simulations in next section, the singular value decomposition (SVD) [5] is applied to solve the equations.

In multi-pass machine like eRHIC, the orbit response of all energies will be distorted by gradient errors differently. Therefore, the orbit response deviation for all passes should be measured and be used for more accurate calculation of the gradient errors.

OPTICS CORRECTION SIMULATIONS

The optics correction scheme was demonstrated using MADX-PTC code [6]. For simplicity, there are 10 basic FODO cells, which includes 20 correctors, 20 quadrupole magnets and 5 BPMs, in the lattice for simulation. In principle, dipole errors would not change the orbit responses. Therefore, no dipole errors were assigned in the simulation. There were no BPM calibration or coupling errors. During commissioning, beam may start only be able to pass once in the FFAG lattice. Therefore, we first did the simulation assuming only orbit response matrix for the first pass can be measured. We have less information for quantifying the gradient errors, even though the number of measurements (two times (dual planes) the product of the number of correctors and the number of BPMs) is larger than the number of unknowns (the number of quadrupoles). The simulation results are shown in Fig. 4. The large assigned gradient errors can be found quite precisely (within 5%).

Once the beam can be accelerated and decelerated through all passes, one could measure the orbit response deviation for beams with all energies. Then the number of measurements will increase to be two times of the product of the number of correctors, the number of BPMs and the number of passes. The simulation results for the case of all passes are shown in Fig. 5. All errors can be found with accuracy of +/-5% except for the first two quadrupoles. The reason for less accuracy is that there were correctors inside (not upstream) of the first two quadrupoles, therefore, the
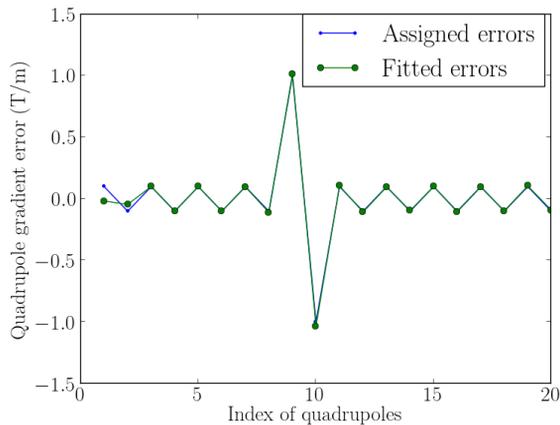
Figure 5: The comparison of the assigned and fitted gradient errors for 20 quadrupole magnets, for the case the orbit response matrix for all the passes were used in the simulation.

orbit response matrix only depends on the errors of the first two quadrupoles weakly.

Further improvement of the simulation can be made to include coupling and BPM calibration errors. The accuracy of find errors is expected to suffer with more unknowns being introduced in the simulation. However, the simulation with all possible errors included will help us better find errors in the real machine. In the simulation, only certain number of eigenvalues was kept so that we match the fitted and assigned errors the best. The method of cutting eigenvalues in the simulation would serve as a guidance when we apply the technique in the experiment.

CHROMATIC EFFECT ON OPTICS CORRECTION

In the previous section, the orbit response refers to the orbit change of the on-momentum particle due to change of a dipole strength. In a machine with chromaticity of single digit, the orbit response of a bunched beam would be very close to that of the on-momentum particle because all particles behaves coherently. This is the reason that there has been no consideration of chromatic effect on optics correction so far. The pure linear FFAG lattice design for eRHIC results in relatively large chromaticities on the order of hundreds. Therefore, the orbit response of the bunched beam, which is measurable by BPMs, is quite different from the orbit response of the on-momentum particle. The comparison of these two is shown in Fig. 6 for the first pass in eRHIC FFAG.

SUMMARY

The orbit response matrix method for finding gradient errors has been chosen for future application in the multipass FFAG eRHIC. The simulation was done for cases that one can measure the orbit response for one energy and all energies. The larger gradient errors can be found accurately
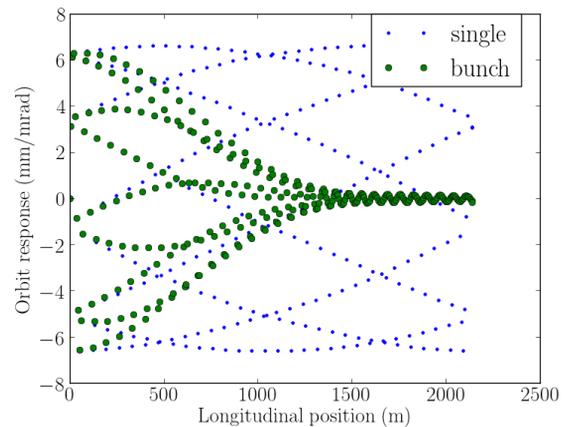

Figure 6: The comparison of the orbit response of the bunched beam and the orbit response of the on-momentum particle.

even for the former case. Better accuracy was achieved for all gradient errors in the latter case. Further improvement of the simulation is underway to include more errors.

So far, the simulation was only done based on single particle orbit response. The measurable orbit response will deviate from the single particle orbit response due to decoherence. Therefore, future simulation with chromaticity effects will be helpful to understand and locate the gradient errors in real case.

ACKNOWLEDGEMENTS

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