# OPTIMIZED DIELECTRIC LOADED WAVEGUIDE TERAHERTZ LINACS

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## Abstract

Dielectric loaded waveguides (DLW) powered by multi-cycle terahertz (THz) pulses have shown promising performance as a compact Linear Accelerator (LINAC) due to higher breakdown fields at THz frequencies compared to conventional RF components. By changing the dielectric dimensions one can control phase and group velocities of the THz pulse inside the DLW. Since optimum waveguide dimensions are dependent on initial electron energy, THz pulse energy, and etc., it is worthwhile to determine optimum values for different conditions in order to maximize final kinetic energy. In this work, we present a combined analytical/numerical guide to determine the optimum DLW parameters for single on-axis electron acceleration. We also introduce graphic representations to visualize optimum designs for different initial electron and THz pulse energies.

## INTRODUCTION

Due to low breakdown electric fields in the RF regime, conventional Radio Frequency (RF) accelerators need to operate at limited acceleration gradient. By increasing the frequency one can take advantage of operating at higher fields [1]. Therefore, THz driven LINACs offer new capabilities when compared to RF- and optical driven accelerators. So far different techniques have been used to accelerate electrons by THz radiation [2-7]. Among these methods, cylindrical DLWs are very attractive due to easier fabrication and better field uniformity in comparison with rectangular ones [5]. Since the phase velocity of usual metallic waveguides is higher than the speed of light, a dielectric layer is added to reduce the phase velocity. Also, the group velocity is significantly lower than the phase velocity. Therefore, in this case a multi-cycle pulse is ideal to increase the interaction length between electron bunch and electromagnetic fields. In the following, we propose a method to optimize cylindrical DLW LINACs. In the simulations we assume a rectangular THz pulse powering the DLW. If this pulse is long enough and narrow band, we can neglect the dispersion effects in the waveguide but we must still take into account the envelope velocity, i.e. group velocity, of the pulse within the waveguide.

## ELECTROMAGNETIC CALCULATION

We assume low bunch charge, such that beam loading and wakefields can be neglected. In this case, Maxwell's equations decouple from the equation of motion of charges. We use an analytical/numerical method to analyse the performance of a cylindrical DLW LINAC. The $TM_{01}$ mode of

a DLW is the optimum mode for acceleration with the highest longitudinal electric field on axis.

The fields of the $TM_{01}$ mode can be written as follows [8]:

$$E_z(z,t) = \left[A_i J_0\left(k_{\rho i}\rho\right) + B_i Y_0\left(k_{\rho i}\rho\right)\right]e^{j(\omega t - k_z z)} \quad (1)$$

$$E_\rho(z,t) = -\frac{jk_z}{k_{\rho i}}\left[A_i J_1\left(k_{\rho i}\rho\right) + B_i Y_1\left(k_{\rho i}\rho\right)\right]e^{j(\omega t - k_z z)} \quad (2)$$

$$H_\phi(z,t) = -\frac{j\omega\varepsilon_{di}}{k_{\rho i}}\left[A_i J_1\left(k_{\rho 2}\rho\right) + B_i Y_1\left(k_{\rho i}\rho\right)\right]e^{j(\omega t - k_z z)} \quad (3)$$

Where i=(1,2) represents the layer number. $k_{\rho i}$ and $k_z$ are the transverse and longitudinal components of the wave vector $k$. $J_m$ and $Y_m$ are the first and second kind of Bessel functions of order m and $\varepsilon_{di}$ is the permittivity of the i-th layer. This formula can be used for each layer of a cylindrical concentric structure. By applying boundary conditions for continuity of the tangential fields on the interface of the layers, we are able to write the dispersion relation. By solving the dispersion relation for a given frequency, we can calculate the mode distribution and wave vector $k_z$ within the DLW. Subsequently, phase velocity, group velocity, and absorption coefficient are given directly by the following equations.

$$v_{ph} = \frac{\omega}{\beta} \quad (4)$$

$$v_g = \frac{\partial\omega}{\partial\beta} \quad (5)$$

$$\beta = Re\{k_z\}, \alpha = Im\{k_z\} \quad (6)$$

Figure 1 shows phase and group velocity for a DLW loaded by a dielectric with a refractive index of 1.95 (fused silica) surrounded by a copper layer with conductivity 6e7 S/m at the frequency of 300 GHz. The DLW thickness is designed such that the phase velocity is optimized for maximum energy gain. Therefore, we need to know the field amplitude to be able to calculate the energy gain

In order to calculate the electric field on a cross section we must know the total Energy coupled to the DLW. Total power flow from a cross section is calculated by integrating the z-component of the Poynting vector over the total cross section area.

Figure 2 shows the longitudinal component of the electric field on axis of the DLW for a total input power of 1W. So far, we have calculated electromagnetic fields inside the DLW. We are now prepared to solve the equation of motion for a single on axis electron.

$$P_z = \int \frac{1}{2} E_r \times H_\varphi{}^* \quad (7)$$

MC3: Novel Particle Sources and Acceleration Techniques
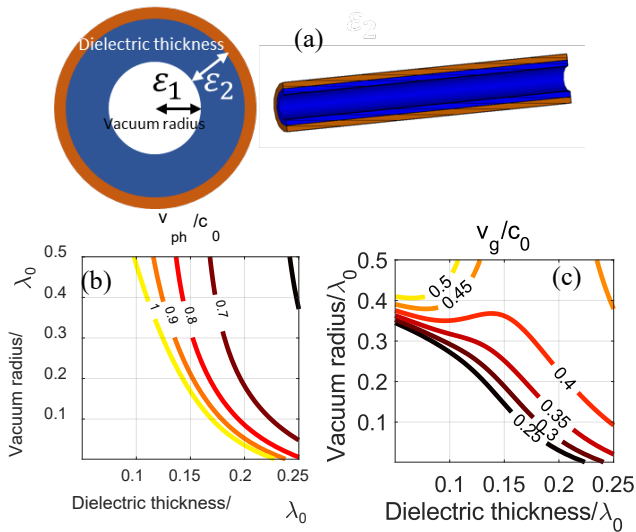
A15: New Acceleration Techniques

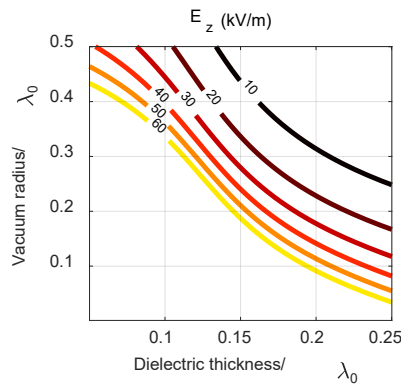Figure 1: (a) DLW LINAC structure (b) phase velocity (c) group velocity for DLW.



Figure 2: Longitudinal component of the electric field on the axis of the DLW for an input power of 1 W.

## LINEAR ACCELERATION

The DLW is powered by a multicycle THz pulse and utilizes the traveling wave for accelerating electrons. The most important challenge here is to keep the electron bunch in a single half-cycle otherwise there will be also deceleration. Since the velocity of electrons increases during the acceleration and the THz pulse has a constant phase velocity, the position of an electron relative to the THz wave drifts during acceleration. It implies that electrons must be injected in the front of the half-cycle to gain maximum possible energy. They will be lagging continuously while the electron velocity is less than the phase velocity of the THz wave. After significant acceleration, the electrons move faster than the phase velocity of the THz wave and start to lead the half-cycle and finally will leave the accelerating half-cycle towards the decelerating one. DLW length and pulse width are the important parameters here to prevent electrons from deceleration. In the next section we will optimize DLW parameters for maximum on axis energy gain.

## DESIGN PARAMETERS

Due to the large number of parameters, one has to fix some of them while optimizing the remaining ones. Here we assume that fused silica and copper are used as dielectric and conducting material of the Linac operating at 300 GHz. Going to higher frequencies allows us to apply stronger electric fields which may lead to an even higher acceleration gradient. However due to smaller dimensions at higher frequencies, space charge effects limit the amount of charge as well as the beam quality. The losses in the metal tube are taken into account in the simulations which leads to a drop in the final electrons energy when compared to the lossless structure. From the acceleration point of view, there is no optimum value for the vacuum radius i.e. the smaller the vacuum radius, the higher the acceleration for a given THz energy. Therefore, vacuum radius is dictated by other criteria such as bunch charge and beam quality. Here we choose 200 μm vacuum radius and find the optimum values for the linac parameters while scanning the THz energy and the injection energy of the electrons.
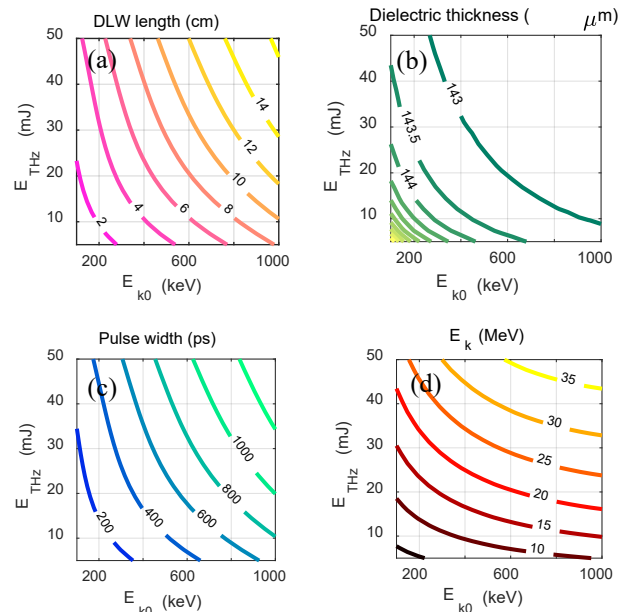


Figure 3: (a-c) Optimum parameters for DLW and THz pulse and (d) maximum achievable final energy as a function of THz energy and initial kinetic energy of the electron.

The important parameters that need to be optimized are dielectric thickness, DLW length and THz pulse duration. Changing the thickness of the dielectric can control the phase velocity of the wave in the tube which plays a key role in the acceleration process. DLW length and THz pulse width are optimized in such a way that electrons leave the negative half-cycle and the whole THz pulse simultaneously. Since all these parameters affect the dynamics of the beam inside the THz wave, we need to optimize the parameters concurrently. Figure 3 (a-c) show the optimum values for the DLW length, dielectric thickness and the THz pulse width as a function of THz energy and initial kinetic energy of the electrons, while the maximum achievable final

energy is shown in Fig. 3 (d) for different THz pulse and initial electron energy. One can employ these graphic representations as an guideline to design a DLW LINAC.

## CONCLUSION

In order to design a DLW LINAC one needs to solve Maxwell's equations as well as the relativistic equation of motion within the DLW. Since there are large number of parameters which determine the final beam properties, we need an efficient method to get an overview on optimum device parameters. In this paper we have used an analytical/numerical method to simulate DLW LINACs efficiently and produced graphic representations for the LINAC parameters given available THz pulse energy and electron injection energy.

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