

# SIMULATION STUDIES OF DRIVE-BEAM INSTABILITY IN A DIELECTRIC WAKEFIELD ACCELERATOR

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## Abstract

Beam-driven collinear wakefield acceleration using structure wakefield accelerators promises a high gradient acceleration within a smaller physical footprint. Sustainable extraction of energy from the drive beam relies on precise understanding of its long term dynamics and the possible onset or mitigation of the beam instability. The advance of computational power and tools makes it possible to model the full physics of beam-driven wakefield acceleration. Here we report on the long-term beam dynamics studies of a drive beam considering the example of a dielectric waveguide using high fidelity particle-in-cell simulations performed with WARPX.

## INTRODUCTION

In a beam-driven collinear wakefield accelerator (CWA), a high-charge drive beam propagates through a slow-wave medium to generate wakefields, which then accelerate a trailing witness beam. The slow wave medium can be plasma, dielectric waveguides or metallic waveguides with corrugation. In the case of using electromagnetic waveguides. Large amplitude wakefields can be generated using waveguides with a small aperture excited by a high-charge bunch. For example, the loss factor  $\kappa$  of a cylindrical waveguide is inversely proportional to the square of its aperture size  $a$ ,  $\kappa \propto a^{-2}$  [1]. In addition to using high charge, drive beams with asymmetric current profile are being considered in CWAs to enhance the transformer ratio,  $\mathcal{R} \equiv |E_+/E_-|$ , where  $E_+$  is the maximum accelerating field behind the drive bunch, and  $E_-$  is the maximum decelerating field within the drive bunch. However, beam-breakup (BBU) instability caused by associated strong transverse wakefields (with transverse loss factor  $\kappa_{\perp} \propto a^{-3}$ ) is one of the main challenges toward the practical realization of a CWA. Hence, efficient modeling of the beam dynamics inside a CWA is important for designing and building a practical CWA.

Advances in computer simulation software in accelerator modeling along with the availability of large computing resources are enabling first-principle electromagnetics simulations of beam dynamics in a CWA. For such simulations, particle-in-cell (PIC) simulations is a popular technique being used to model beam dynamics. In this paper, we report preliminary simulation studies of the drive-beam dynamics inside a CWA consisting of dielectric waveguides with superimposed external focusing. The model is implemented

using a finite-different time-domain (FDTD) PIC algorithm in the WARPX open-source electromagnetics framework being developed for accelerator modeling [2].

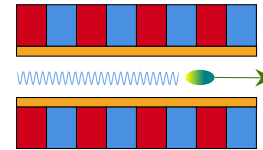


Figure 1: A dielectric wakefield accelerator consists of a dielectric waveguide embedded in a series of interleaving focusing (red) and defocusing (blue) quadrupole magnets. A drive beam (green ellipse) generates wakefields (blue sinusoidal waves) by passing through the waveguide.

## DIELECTRIC-WAKEFIELD ACCELERATOR

We considered a CWA with dielectric waveguides, which we shall refer as a dielectric wakefield accelerator (DWA). This accelerator consists of a dielectric waveguide embedded in a series of interleaving focusing and defocusing (FD) quadrupole magnets for beam transport. Wakefields are generated by propagating a drive beam inside the accelerator. A schematic diagram of a DWA appears in Fig. 1. Earlier studies of a DWA were performed in Ref. [3] using a two-particle model tracking code to address the limit of accelerating gradient. In addition, simulation performed with the particle-tracking program ELEGANT [4] indicates that a FODO lattice with a tapered quadrupole-magnet strength and asymmetric shaped drive beams with a large energy chirp can suppress the BBU instability [5]. Further investigation performed using a two-particle model provided further guidance on possible external-focusing configurations [6]. The latter paper specifically demonstrated that a drive beam with a large energy chirp and a tapered focusing-defocusing (FD) channel could suppress the BBU instability. Correspondingly, we considered a drive beam with an asymmetric current profile and a DWA with FD channels to conduct our simulation studies.

The dielectric waveguide considered throughout this paper is a multi-mode structure with fundamental-mode frequency  $f \simeq 148$  GHz; see the structure parameters listed in Table 1. We considered using quadrupole without tapering focusing strength to test the limit of the drive beam stability. Table 1 shows properties of a DWA that we used in

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our simulation studies. A betatron phase advance of  $0.1\pi$  in both planes was selected in order to minimize the transverse beam-size variation.

For the drive beam, we considered using a doorstep distribution proposed in Ref. [7]. The beam distribution was generated using Eq. 6 of ref. [8] and consistent with the one achieved from start-to-end simulations in Ref. [8]. Since current profile with sharp edges will cause numerical noises in PIC simulations, we patched both ends of the beam with a Gaussian function of user-defined variable  $\sigma_1$  to have smooth head and tail edges. Parameters of the drive beam are shown in Tab. 1.

Table 1: Properties of the DWA Structure and Drive-bunch Distribution used in the Simulations. The parameters  $\xi$  and  $\sigma_1$  are defined in Eq. 6 of Ref. [8].

Property of DWA	Value	Unit
Inner radius	1	mm
Outer radius	1.2	mm
Dielectric constant	3.8	-
Wavelength of the first mode	2.03	mm
Frequency of the first mode	147.97	GHz
Quadrupole gradient	1000	T/m
Phase advance	$0.1\pi$	-
Property of the drive beam	Value	Unit
Total charge	10	nC
$\xi$	$0.2525\lambda$	-
Total length	$\lambda$	-
Normalized emittance	10	$\mu\text{m}$
Energy of the reference particle	1	GeV
Chirp	-49	$\text{m}^{-1}$
Intrinsic relative energy spread	0.001	-
$\sigma_1$	$0.015\lambda$	-

## WARPX MODEL

The model was implemented in WARPX and uses the macroscopic electromagnetic solver. Beam distribution was generated in openPMD data format [9, 10] and used as an input in WARPX. In order to increase precision and reduce running time, a moving-window approach is implemented where the field are only computed over a limited computational domain surrounding and moving with the drive bunch. Despite the cylindrical symmetry of the problem, the model was implemented in a Cartesian domain to properly account for 3D beam-dynamics effect arising from the interaction with high-order multipole components of the wakefields. The key parameters used in WARPX appear in Table 2. In order to gain confidence in our implementation we first considered a long moving window (including the bunch and few periods of the radiation field behind) with a total length of  $3.75\lambda$  ( $\lambda$  being the fundamental-mode wavelength); the resulting longitudinal wakefield computed over

Table 2: Parameters of WARPX used for Simulation Studies.

Parameter	Value	Unit
Version	22.04	-
Number of cells in $x, y$	128	-
Number of cells in $z$	1920	-
Billinear filter	True	-
Number of passes in $x, y$	3	-
Number of passes in $z$	4	-
CFL number	1	-
Boundary condition in $x, y, z$	PEC	-
Simulation domain in $x, y$	$2 \times \text{outer radius}$	-
Cell size in $z$	$\lambda/512$	-
Rigid injection	True	-
Initial position in $z$	1.2	mm
Rigid injection plane	5	cm
Time step	$\lambda/512c$	-

a plane  $E_z(x, y = 0, s)$  is displayed in Fig. 2. Figure 3 compares the on-axis longitudinal wakefield  $E_z(s, x = y = 0)$  obtained from WARPX with the field computed by convolving the numerically-computed Green's functions with the drive-beam charge distribution [11]. The fields agree up to the first peak of the wakefield while some minor deviations are observed further away from the bunch.

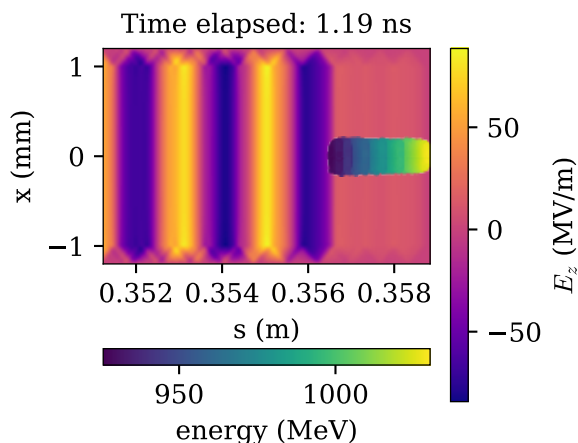


Figure 2: Snapshot of the drive beam traveling in the DWA with corresponding longitudinal field  $E_z(x, y = 0, s)$  field.

## SIMULATION OF A LONG DWA

Long term simulation of a drive beam in a DWA were performed over a length of 7.45 m. The moving-window length and the number of cells in  $z$  are set to respectively 1.25 $\lambda$  and 640 to reduce the computational cost.

Figure 4 shows a 2D snapshot of the drive beam and the longitudinal wakefield at the end of the simulation. The onset of the BBU instability is observed at the tail edge of the beam. To further investigate the instability, the drive beam

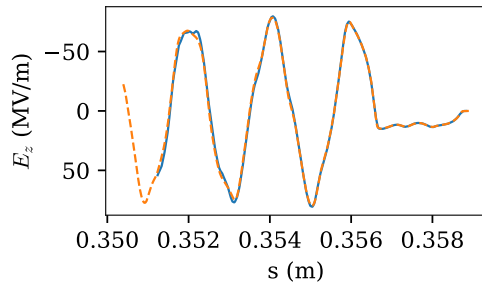


Figure 3: Comparison of  $E_z(x = y = 0, s)$  extracted from WARPX (blue line) and from Green's function approach (orange dashed line).

was divided into 6 longitudinal slices for further analysis as shown in Fig. 5. Figure 6 shows the evolution of slices'

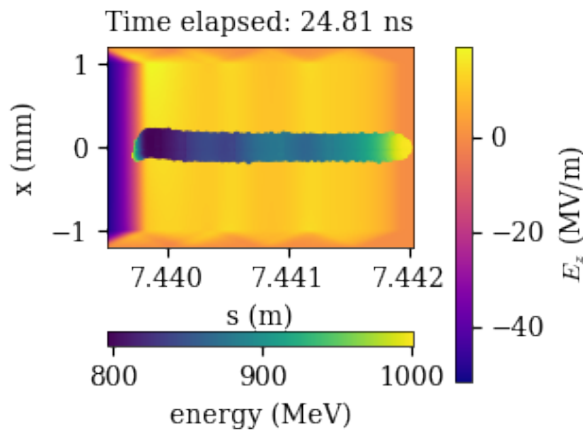


Figure 4: Snapshot of the drive beam traveling in the DWA with corresponding longitudinal field  $E_z(x, y = 0, s)$  field at the end of the simulation. The onset of BBU can be observed at the tail of the beam.

centroids along the accelerator beamline and confirm that the centroid of the slice 5 oscillate with an exponentially-growing amplitude. Similar features are observed on the evolution of the centroid associated with slice 0 albeit in a sub-micron level; see in Fig. 7. These results indicate that beam breakup instability due to misalignment occurred in all slices, but differed in growing amplitude. It confirms that the instability mainly affects the tail of the bunch. In the present case no initial misalignment was introduced and the observation of the onset of the BBU instability is the result of small misalignments (coming from numerical noise) at the sub-micron level. Such a misalignment will most likely be present in drive beam produced from realistic accelerator.

## CONCLUSION

In summary, we have successfully implemented FDTD/PIC simulations of a drive beam passing through a

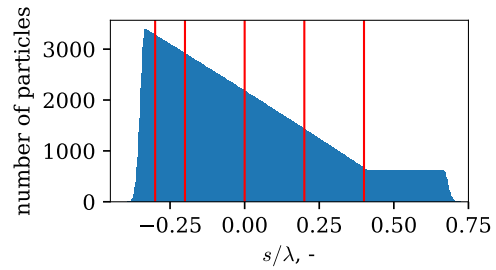


Figure 5: Longitudinal slices for the analysis shown in Fig. 6. Slice #0 corresponds to the head (right) of the bunch.

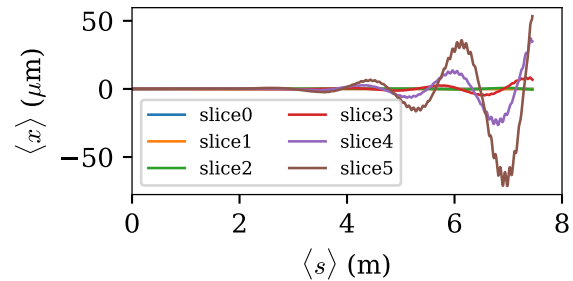


Figure 6: The evolution of beam centroid for all slices.

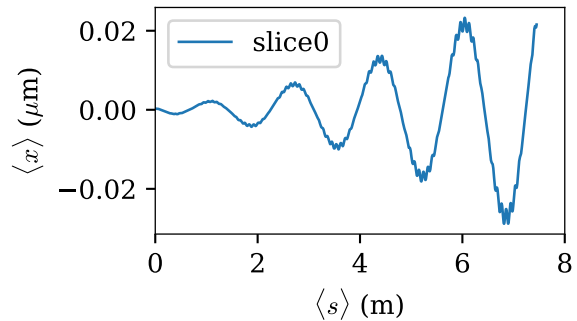


Figure 7: The evolution of beam centroid of slices 0.

DWA using the WARPX framework. Long-term simulations performed over a 7.45 m length suggest that the drive beam exhibits the onset of a beam breakup instability throughout its propagation in the DWA where the centroids of longitudinal slices exhibit periodic transverse oscillation with exponentially-growing offsets. As expected, the centroid oscillation amplitudes are larger toward the tail of the bunch. So far our simulations have focused on a specific betatron phase advance. The tools developed will be further used to explore the impact of different parameters and possible mitigation of this BBU instability.

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