ISBN: 978-3-95450-227-1 ISSN: 2673-5490 doi:10.18429/JACoW-IPAC2022-MOPOMS050

RIGOROUS APPROACH FOR CALCULATION OF RADIATION OF A CHARGED PARTICLE BUNCH EXITING AN OPEN-ENDED DIELECTRICALLY LOADED WAVEGUIDE*

S. N. Galyamin†, St. Petersburg State University, St. Petersburg, Russia
S. S. Baturin, ITMO University, St. Petersburg, Russia

Abstract

First, recent results on radiation of a Cherenkov mode at the open end of a dielectric-lined circular waveguide (including a three-layer case) are presented. Second, rigorous solution is presented for the case of a charged particle bunch exiting the open end of a waveguide with unifiorm dielectric filling.

INTRODUCTION

Among prospective applications of dielectric-filled waveguides and Cherenkov effect one can mention dielectric wakefield acceleration [1–3], bunch manipulation [4–6] and beam-driven radiation sources [7–9]. Mentioned cases typically involve interaction of both EM waves and charged particle bunches with an open end of certain open-ended waveguide structure loaded with dielectric. Convenient rigorous approach for the circular waveguide geometry has been presented recently [10, 11] (internal excitation in the form of a slow waveguide mode has been used). However, problems with more complicated layered filling [9] and excitation in the form of a charged particle bunch require similar analytical solution. These are main topics of the present paper.

OPEN-ENDED WAVEGUIDE WITH DIELECTRIC LINING

First, we briefly discuss a two-layer open-ended waveguide with PEC walls excited by single waveguide mode (details can be learned from [11]), see Fig. 1. A φ-symmetric TM problem is considered in the harmonic regime with time dependence in the form $H_\varphi(\rho,z,t) = H_{\omega\varphi}(\rho,z)\exp(-i\omega t)$. Single symmetrical TM_{0l} mode is incident on the open end while the reflected field inside the waveguide $H^{(r)}_{\omega\varphi}$ is decomposed into a series of such modes propagating in the opposite direction (z-dependence for the incident mode is $\sim\exp(ik_{zl}z)$) with unknown "reflection coefficients" $\{M_m\}$ that should be determined:

$$H^{(r)}_{\omega\varphi} = \sum\nolimits_{m=1}^{\infty} M_m e^{-ik_{zm}z} \times \begin{cases} J_1(\rho\sigma_m)/\sigma_m & \text{for } \rho<b, \\ [J_1(\rho s_m)Y_0(as_m) - Y_1(\rho s_m)J_0(as_m)] \\ \times J_1(b\sigma_m)/[\sigma_m\psi_0(s_m)] & \text{for } b<\rho<a, \end{cases} \quad (1)$$

where J_ν and Y_ν are Bessel and Neumann functions, transverse wave numbers σ_m and s_m are determined by dispersion equation (Eq. (4) in [11]),

$$\psi_0(s_m) = J_1(bs_m)Y_0(as_m) - J_0(as_m)Y_1(bs_m), \quad (2)$$

longitudinal wave numbers $k_{zm} = \sqrt{k_0^2-\sigma_m^2} = \sqrt{k_0^2\varepsilon - s_m^2}$, $\mathrm{Im}k_{zm} > 0$, $k_0 = \omega/c + i\delta$ ($\delta\to 0$ is responsible for small dissipation), c is the light speed in vacuum.

After a series of calculations involving field matching, deriving Wiener-Hopf equation, factorization (see [11] for details) we arrive at the following infinite linear system:

$$\sum\nolimits_{m=1}^{\infty} W_{pm}M_m = M^{(i)}w_p, \quad p = 1,2,\dots, \quad (3)$$

$$W_{pm} = \left(k_{zm}\varepsilon^{-1} + \alpha_p\right)\eta_m(\alpha_p) - \frac{\zeta_m(\alpha_p)}{k_{zm}-\alpha_p} + u_p \times \sum_{q=1}^{\infty}\left[\left(\frac{k_{zm}}{\varepsilon} - \alpha_q\right)\eta_m(\alpha_q) - \frac{\zeta_m(\alpha_q)}{k_{zm}+\alpha_q}\right]v_{pq}, \quad (4)$$

$$w_p = \left(k_{zl}\varepsilon^{-1} - \alpha_p\right)\eta_l(\alpha_p) - \frac{\zeta_l(\alpha_p)}{k_{zl}+\alpha_p} + u_p \times \sum_{q=1}^{\infty}\left[\left(\frac{k_{zl}}{\varepsilon} + \alpha_q\right)\eta_l(\alpha_q) - \frac{\zeta_l(\alpha_q)}{k_{zl}-\alpha_q}\right]v_{pq}, \quad (5)$$

$$u_p = \kappa_+(\alpha_p)G_+(\alpha_p)J_1(j_{0p})a/(2ij_{0p}),$$

$$v_{pq} = \kappa_+(\alpha_q)G_+(\alpha_q)j_{0q}\left[a^2\alpha_q J_1(j_{0q})(\alpha_p+\alpha_q)\right]^{-1},$$

$M^{(i)}$ is amplitude constant for the incident mode, $G(\alpha) = \pi a\kappa J_0(a\kappa)H_0^{(1)}(a\kappa) = G_+(\alpha)G_-(\alpha)$ (subscripts $\pm$ mean that function is holomorphic and free of poles and zeros in areas $\mathrm{Im}\,\alpha > -\delta$ and $\mathrm{Im}\,\alpha < \delta$, correspondingly), $\kappa = \sqrt{k_0^2-\alpha^2}$, $\kappa_\pm = \sqrt{k_0\pm\alpha}$, $\alpha_q = \sqrt{k_0^2 - j_{0q}^2/a^2}$, $J_0(j_{0m}) = 0$, functions $\Pi(\alpha)$, $\eta_m(\alpha)$, $\zeta_m(\alpha)$ are defined in [11]. For finite p and $m\to+\infty$ we have $W_{pm}M_m = o(m^{-3/2})$, the series (3) converges and can be solved numerically.

For $z>0$ the following representation holds:

$$H_{\omega\varphi} = \sum_{q=1}^{\infty}\Pi(-\alpha_q)\frac{\kappa_+(\alpha_q)G_+(\alpha_q)j_{0q}}{a^2b^{-1}\alpha_q J_1(j_{0q})}\frac{L_q^+(\rho,z)}{2}, \quad (6)$$

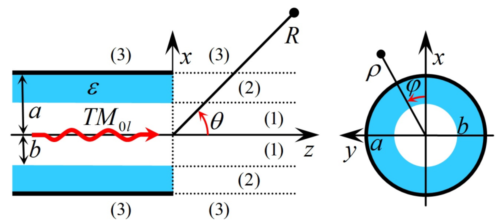

Figure 1: Two-layer problem and main notations.

* Work supported by Russian Science Foundation, Grant No. 18-72-10137
† s.galyamin@spbu.ru

ISBN: 978-3-95450-227-1 ISSN: 2673-5490 doi:10.18429/JACoW-IPAC2022-MOPOMS050

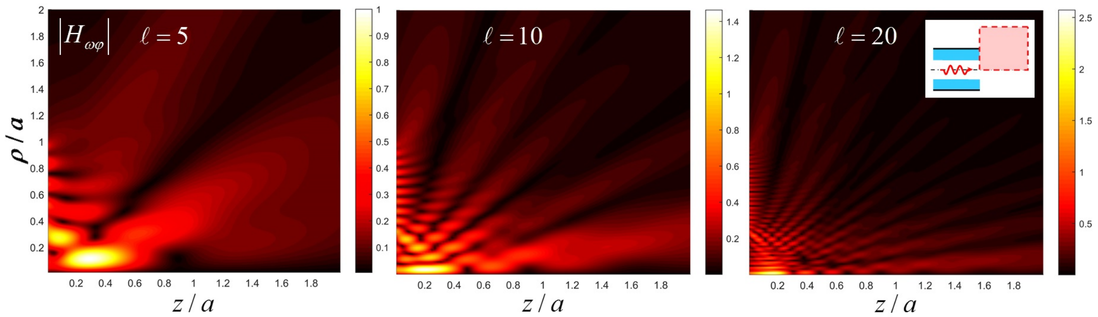

Figure 2: Near-field distribution of $|H_{\omega\varphi}|$ for the cases of incident Cherenkov mode with number $l = 5, 10, 20$, calculation parameters are: $a = 0.24$ cm, $b = a/3$, $\varepsilon = 2$, $f_5^{CR} = 397$ GHz, $f_{10}^{CR} = 864$ GHz, $f_{20}^{CR} = 1.81$ THz. Constant $M^{(i)}$ is chosen so that incident mode carries unity power, all plots are normalized to the maximum value of $|H_{\omega\varphi}|$ for $l = 5$.

where L_q^+ is defined by Eq. (47) in [10].

Figure 2 shows near-field distribution over the region $0 < z < 2a$, $2 < \rho < 2a$. The mode frequency f was chosen to be equal to the frequency of 5-th, 10-th and 20-th Cherenkov mode produced by a moving charge having Lorentz factor $\gamma = 7$. One can clearly see penetration of waveguide modes to the vacuum area and formation of main and lateral lobes of the radiation patterns.

For a three-layer case, see Fig. 3, formulation of the problem and its solution are in general similar to those for a two-layer case, see [12] for details. In particular, an infinite system for reflection coefficients similar to (3) can be obtained and solved numerically, field representation (6) is also valid for this case (with substitution $a \to d$ and more complicated form of $\Pi(\alpha)$).

Figure 4 shows how radiation of the 1-st Cherenkov mode changes with an increase in thickness of the third layer (parameters are chosen in accordance with paper [9] where possibilities to enhance directivity and reduce reflection of the capillary-based beam-driven source of THz radiation by adding the third layer with permittivity ϵ just slightly larger than unity are investigated). As one can see, the position of radiation maximum ($39°, 35°, 21°$) and its width ($2\Delta\theta_{0.7} = 52°, 35°, 25°$) decrease twice while field magnitude increases 2.5 times with an increase in the thickness of the third layer from 0.1mm to 0.8mm, reflection (S_{11}) also decreases essentially.

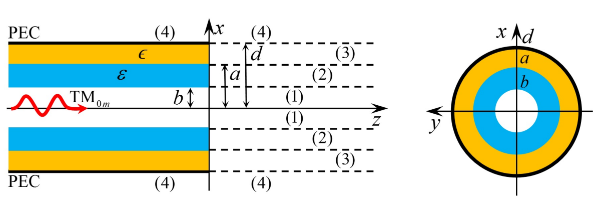

Figure 3: Geometry of a three-layer problem.

UNIFORM FILLING AND EXCITATION BY A MOVING CHARGE

Here we discuss a problem with simpler filling (see Fig. 5) but excitation in the form of a point charged particle q moving along the waveguide axis with velocity $c\beta$, $\varepsilon\beta^2 > 1$ (generalization to the case of a thin prolonged bunch can be made straightforwardly). Incident field inside the waveguide ($\rho < a$, $z < 0$) is

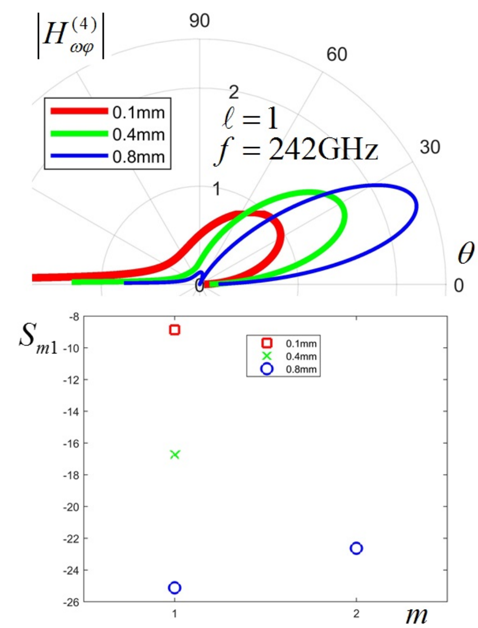

Figure 4: Far-field pattern (top) and S-parameters (bottom) for the 1-st Cherenkov mode ($f_1^{CR} \approx 240$GHz, the mode carries inity power in each case) exiting the three-layer structure, Fig. 3. Parameters: $b = 0.4$mm, $a = 0.55$mm, $\varepsilon = 3.8$ (fused silica), $\epsilon = 1.01$, $\gamma = 10$, $d - a$ is indicated in the legend.

$$H_{\varphi\omega}^{(i)} = \frac{iqs}{2c} e^{ik_0 z/\beta} \left[H_1^{(1)}(\rho s) - \frac{H_0^{(1)}(as)}{J_0(as)} J_1(\rho s) \right], \quad (7)$$

where $s = \sqrt{k_0^2/\beta^2(\varepsilon\beta^2 - 1)}$, $\mathrm{Im}s > 0$. In vacuum, we define an incident field in the area $z > 0$ only:

$$H_{\varphi\omega}^{(i0)} = \frac{iqs_0}{2c} e^{ik_0 z/\beta} H_1^{(1)}(\rho s_0), \quad (8)$$

ISBN: 978-3-95450-227-1 ISSN: 2673-5490 doi:10.18429/JACoW-IPAC2022-MOPOMS050

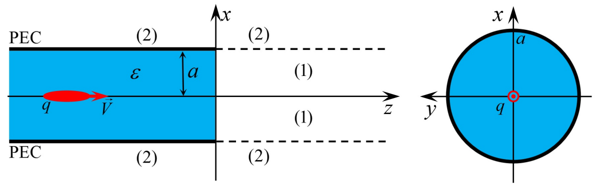

Figure 5: Geometry of the problem with uniform filling and excitation by moving charge.

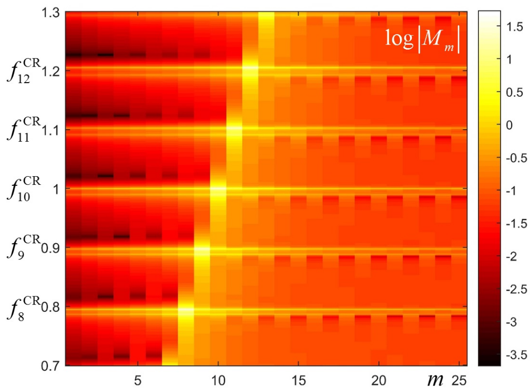

Figure 6: Absolute value of the coefficients of reflected modes excited by a point charge exiting the open-end waveguide, see Fig. 5, $a = 0.24$ cm, $\varepsilon = 2$, $f_{10}^{CR} = 615$ GHz.

where $s_0 = i\sigma_0$, $\sigma_0 = k_0\sqrt{\beta^{-2} - 1}$, $\mathrm{Re}\sigma_0 > 0$. Reflected field is decomposed as usual

$$H_{\varphi\omega}^{(r)} = \sum_{m=1}^{\infty} M_m J_1\left(\frac{\rho j_{0m}}{a}\right) e^{-ik_{zm}z}, \quad (9)$$

where $k_{zm} = \sqrt{k_0^2\varepsilon - j_{0m}^2 a^{-2}}$, $\mathrm{Im}k_{zm} > 0$, coefficients $\{M_m\}$ should be determined.

After a series of derivations we obtain the following system for $\{M_m\}$:

$$\sum_{m=1}^{\infty} W_{pm}^q M_m = w_p^q, \quad p = 1, 2, \dots, \quad (10)$$

$$W_{pm}^q = J_1(j_{0m})\left[\zeta_{m+}(\alpha_p) + \frac{\delta_{mp} i a\left(\frac{k_{zm}}{\varepsilon} + \alpha_m\right)}{\kappa_+(\alpha_m)G_+(\alpha_m)}\right], \quad (11)$$

$$w_p^q = \frac{q}{c\pi a}\frac{\zeta_{0+}(\alpha_p)}{J_0(as)} + J_1(j_{0p})\phi_p i a\frac{\frac{k_0}{\varepsilon\beta} - \alpha_p}{\kappa_+(\alpha_p)G_+(\alpha_p)}, \quad (12)$$

$$\zeta_{0+}(\alpha) = \frac{G_+(\alpha_0)\kappa_+(\alpha_0)\left(\frac{k_0}{\varepsilon\beta} + \alpha_0\right)}{2\alpha_0(\alpha_0 + \alpha)}, \quad (13)$$

$$\zeta_{m+}(\alpha) = \frac{G_+(\alpha_m)\kappa_+(\alpha_m)\left(\frac{k_{zm}}{\varepsilon} - \alpha_m\right)}{2\alpha_m(\alpha_m + \alpha)}, \quad (14)$$

$$\phi_p = \frac{iqs}{2c}\frac{4ij_{0p}}{\pi a s J_1^2(j_{0p})}\frac{1}{(as)^2 - j_{0p}^2}, \quad (15)$$

$\alpha_0 = \sqrt{k_0^2 - s^2}$, $\mathrm{Im}\alpha_0 > 0$.

The obtained solution describes all radiation processes occuring at the open end including radiation of Cherenkov modes, transition radiation at the dielectric-vacuum interface and diffraction radiation from the PEC edge of the waveguide. For example, Fig. 6 shows frequency spectrum of M_m (in the range $\pm 30\%$ with respect to the 10-th Cherenkov frequency f_{10}^{CR}) for a waveguide with parameters from paper [10]. One can see that a coefficient with given m possesses a strong maximum for $f = f_m^{CR}$ which is natural since the incident field inside the waveguide (7) possesses the same maximum.

REFERENCES

[1] B.D. O'Shea *et al.*, "Observation of acceleration and deceleration in gigaelectron-volt-per-metre gradient dielectric wakefield accelerators", *Nature Communications*, vol. 7, p. 12763, 2016. doi:10.1038/ncomms12763

[2] C. Jing *et al.*, "Electron acceleration through two successive electron beam driven wakefield acceleration stages", *Nucl. Instrum. Methods Phys. Res., Sect. A*, vol. 898, pp. 72-76, 2018. doi:10.1016/j.nima.2018.05.007

[3] D. Wang *et al.*, "Phase control with two-beam interferometry method in a terahertz dielectric wakefield accelerator", *Appl. Phys. Lett.*, vol. 111, p. 174102, 2017. doi:10.1063/1.4999959

[4] L. Zhao *et al.*, "Femtosecond Relativistic Electron Beam with Reduced Timing Jitter from THz Driven Beam Compression", *Phys. Rev. Lett.*, vol. 124, p. 054802, 2020. doi:10.1103/PhysRevLett.124.054802

[5] E.C. Snively *et al.*, "Femtosecond Compression Dynamics and Timing Jitter Suppression in a THz-driven Electron Bunch Compressor", *Phys. Rev. Lett.*, vol. 124, p. 054801, 2020. doi:10.1103/PhysRevLett.124.054801

[6] F. Mayet *et al.*, "Longitudinal phase space synthesis with tailored 3D-printable dielectric-lined waveguides", *Phys. Rev. Accel. Beams*, vol. 23, p. 121302, 2020. doi:10.1103/PhysRevAccelBeams.23.121302

[7] S.N. Galyamin, A.V. Tyukhtin, S. Antipov, and S.S. Baturin, "Terahertz radiation from an ultra-relativistic charge exiting the open end of a waveguide with a dielectric layer", *Opt. Express* vol. 22, p. 8902, 2014. doi:10.1364/OE.22.008902

[8] D. Wang *et al.*, "Non-perturbing THz generation at the Tsinghua University Accelerator Laboratory 31 MeV electron beamline", *Rev. Sci. Instrum.*, vol. 89, p. 093301, 2018. doi:10.1063/1.5042006

[9] S. Jiang, W. Li, Z. He, Q. Jia, and Lin Wang, "Intrinsically reducing divergence angle of Cherenkov radiation from dielectric capillary", *Opt. Lett.*, vol. 45, pp. 5416–5419, 2020. doi:10.1364/OL.401947

[10] S.N. Galyamin, V.V. Vorobev, and A.V. Tyukhtin, "Diffraction at the Open-Ended Dielectric-Loaded Circular Waveguide: Rigorous Approach", *IEEE Trans. Microwave Theory Techn.*, vol. 69, pp. 2429–2438, 2021. doi:10.1109/TMTT.2021.3064343

[11] S.N. Galyamin, V.V. Vorobev, "Diffraction at the Open End of Dielectric-Lined Circular Waveguide", *IEEE Trans. Microwave Theory Techn.*, vol. 70, pp. 3087–3095, 2022. doi:10.1109/TMTT.2022.3167639

[12] S.N. Galyamin, "Cherenkov Wakefield Radiation from an Open End of a Three-Layer Dielectric Capillary", doi:10.48550/arXiv.2205.03986