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EIC CRAB CAVITY MULTIPOLE ANALYSIS AND THEIR EFFECTS ON DYNAMIC APERTURE*

Q. Wu†, Y. Luo, B. Xiao, Brookhaven National Laboratory, Upton, NY, USA
S. U. De Silva, Old Dominion University, Norfolk, VA, USA
Z. Li, SLAC, Menlo Park, CA, USA

Abstract

Crab cavity is essential for retrieving the loss in luminosity due to the large crossing angle in the two colliding beam lines of the Electron Ion Collider (EIC). Due to the asymmetric design of the proton beam crab cavity, the fundamental mode consists of contributions from higher order multipoles. These multipole modes may change during fabrication and installation of the cavities, and therefore affect the local dynamic aperture. Thresholds for each order of the multipoles are applied to ensure dynamic aperture requirements at these crab cavities. In this paper, we analysed the strength of the multipoles due to fabrication and installation accuracies and set limitations to each procedure to maintain the dynamic aperture requirement.

INTRODUCTION

The concept of using an RF deflector to compensate for the geometrical loss in the colliding beams due to fast sep-aration from the designed crossing angle has been intro-duced by Robert Palmer in 1988 [1]. As concept only relates to colliders that pursue high luminosity, these RF deflectors are not developed as popular as other RF resonators in traditional accelerators. Figure 1 shows the various crab cavities that have been built and operated in the past [2-5].

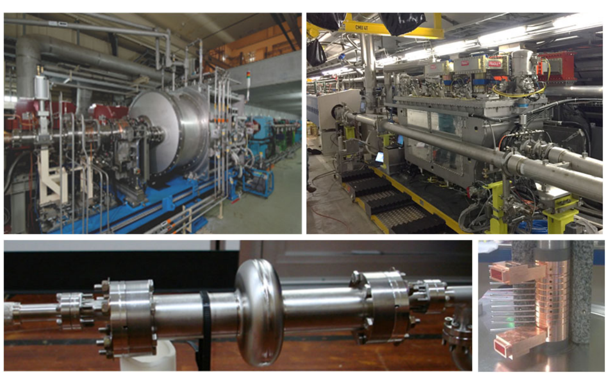

Figure 1: Difference crab cavities fabricated and operated. Top left: Squashed crab cavity at 509 MHz operated in the HER ring @ KEK. Top right: Double Quarter Wave crab cavity at 400 MHz operated in the SPS ring @ CERN. Bottom left: Single-cell prototype of the 3.9 GHz 9-cell crab cavity for ILC. Bottom right: 12-cell copper travelling-wave structure for crabbing in CLIC.

The future Electron Ion Collider [6] has a design goal of reaching a luminosity of 10^{34} cm^{-2} sec^{-1}. The EIC interaction region has a crossing angle of 25 mrad, Fig. 2 which is more than 40 times larger than the LHC. With such a large crossing angle, the EIC benefits from the LHC experience as well as encounters many challenges while demonstrating the crabbing scheme. The crab cavity of the EIC will have a full SRF system with 197 MHz and 394 MHz resonators combined. To reach the peak luminosity, both frequencies must be considered for beam-cavity interaction studies. This paper will focus on the main crab cavity of 197 MHz for hadron storage ring, as shown in Fig. 3. The 394 MHz crab cavity requires future work.

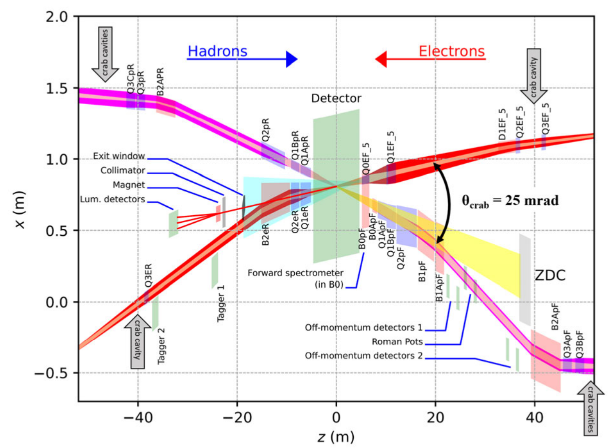

Figure 2: EIC interaction region layout with 25 mrad full crossing angle.

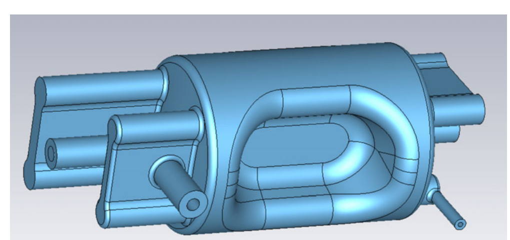

Figure 3: 197 MHz crab cavity model with horizontal coupling for EIC.

CRAB CAVITY MULTIPOLES

The transverse crabbing effect in the RF structure can be decomposed into a series of EM fields that imitate the magnets in the machine, with the time-dependent dipole mode (crabbing mode) dominating. The coefficient of each mode can be evaluated through the Lorentz force of the transverse EM field or analysing the longitudinal electric field using the Panofsky-Wenzel Theorem [7].

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† qiowu@bnl.gov

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Using the PW Theorem, for particles with speed c, the transverse momentum change inside the crab cavity can be expressed as

$$\Delta p_{\perp} = e^{\frac{i\omega z}{c}+\Psi}\left(\frac{iq}{\omega}\nabla_{\perp}\int_{0}^{L} E_s(r,\theta,s)\, e^{\frac{i\omega s}{c}} ds\right)$$

where $\omega = 2\pi f$, f is the frequency of the cavity, z is the coordinate along longitudinal direction, s is the coordinate along the trajectory, r is the distance in radial direction, q is the charge of the particle, and Ψ is the phase advance of the crab cavity.

If we expand the integral inside the parentheses, the equation above can be expressed as

$$\Delta p_{\perp} = e^{\frac{i\omega z}{c}+\Psi}\left(\sum_{n=1}^{\infty}\frac{r^n}{n}(b_n \cos n\theta + a_n \sin n\theta)\right)$$

where θ is the angle in transverse plane, with horizontal plane as 0 degree. b_n and a_n are the normal and skew coefficients of each order of the modes, e.g. n=1 is dipole, n=2 is quadrupole, n=3 is sextupole, etc..

The higher order coefficients (n>2) provide unwanted effects during the beam-cavity interaction, but cannot be avoided due the asymmetrical structure of the cavity and the RF auxiliaries coupling into the cavity. These multipolar components of the crabbing mode are always in phase with the crabbing dipole.

These modes can cause instabilities inside the bunches as they travel downstream, and will limit the dynamic aperture (DA) of the IR region. Therefore, the multipolar modes should be carefully evaluated with each version of the cavity design, and should be limited below a threshold to obtain an acceptable DA.

Multipole Coefficient Calculation

The multipolar field coefficients b_n and a_n can be calculated from the following equations:

$$b_n = \frac{1}{\pi}\int_{0}^{2\pi}\frac{n}{r^n}A(r,\theta)\cos n\theta\, d\theta$$

$$a_n = \frac{1}{\pi}\int_{0}^{2\pi}\frac{n}{r^n}A(r,\theta)\sin n\theta\, d\theta$$

where

$$A(r,\theta) = \frac{iq}{\omega}\nabla_{\perp}\int_{0}^{L} E_s(r,\theta,s)\, e^{\frac{i\omega s}{c}} ds$$

The fundamental mode field distribution can be obtained from the CST Studio [8] with consideration of the convergence from previous studies [9]. The EIC crab cavities are required to provide a horizontal crabbing kick for the collision, which leads to the skew coefficients a_n are negligible. Therefore, the studies only focus on the dominating normal multipolar components coefficients b_n.

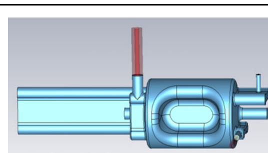
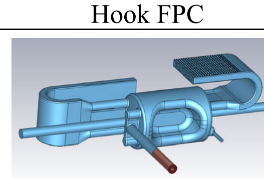
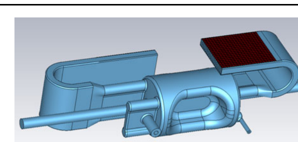
By comparing different design versions of the EIC 197 MHz crab cavity, Table 1, the change in multipole strength can be linked to a specific change in the cavity shape or coupler type as shown in their 3D models. All three designs share the same cavity main body, but their Fundamental Power Couplers are coupling differently into the cavity field.

All multipole coefficients are scaled to 25.8 MV crabbing voltage. This is the total voltage required on one side of the interaction point to completely compensate the crossing angle of 25 mrad in EIC.

With b_1 representing the main dipole crab kick, the table shows E probe FPC design has a quadrupole b_2 that is one order of magnitude less than the other two designs, as well as a 6-fold decrease in this version in octupole b_4. Two designs with horizontal FPC have much smaller b_6 compared to the vertical FPC design. The differences in the three FPC coupling schemes contributed to the change in these even number higher order multipole strength. This can be verified by the multipole coefficient plot along the beam axis as shown in Fig. 4. From the b_3 distribution plot, the main contribution to the sextupole component is from the two gaps between the cavity end plates and the center deflection plates.

Verifying the source of each higher order multipole in the cavity design is essential for decreasing their strength when necessary. Some multipoles are originated from the same design feature, and changing the design would have to ensure all of the multipoles are under the threshold.

Table 1: Normal Multipole Coefficients for Various Cavity Design. All Normalized to 25.8 MV Crabbing Voltage

Type	Vertical FPC	Hook FPC	E Probe FPC
3D Model			
b_1 $[mT\cdot m]$	85.48	85.47	85.70
b_2 $[mT]$	2.27	1.91	0.19
b_3 $[mT/m]$	1356.18	1333.64	1342.72
b_4 $[mT/m^2]$	19.55	19.03	-3.35
b_5 $[mT/m^3]$	-71717.47	-72285.03	-71760.90
b_6 $[mT/m^4]$	34457.83	5721.66	2186.21

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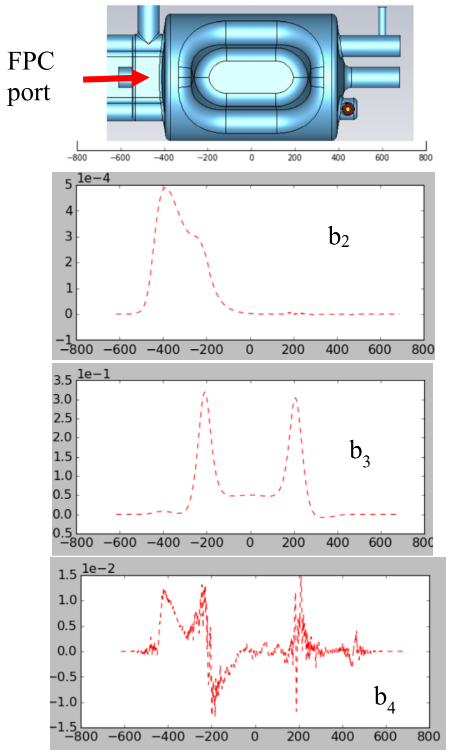

Figure 4: b_2, b_3 and b_4 strength along beam axis in the 197 MHz crab cavity for EIC. Vertical axis is in arbitrary units. Horizontal axis is distance in mm, with zero at the center of the cavity.

DYNAMIC APERTURE TRACKING

We use SimTrack [10] with a parameter scan of the multi-polar components of the crabbing mode for DA tracking. These studies have assumed the crabbing kick happens at the center of the installation region in the lattice, and all multipoles are treated as a time dependent thin length mag-net. SimTrack implements exact 6D Hamiltonian and in-cluded nonlinear field errors and optics correction before DA tracking.

Based on the limitation of the achievable voltage from the SRF cavities, four 197 MHz crab cavities of the same design are planned to be installed on each side of the interaction point providing a total crabbing voltage of 25.8 MV. The summation of all cavity multipoles confined the DA of the IR section along with other components, and the dynamic aperture is affected by the nonlinear effect introduced by the higher order magnet fields. However, comparing with traditional magnets in the ring, the multipoles generated by the crabbing kick is time dependent during the interaction with each bunch. The traditional magnets are not capable of cancelling the effect of the crab cavity multipoles. Therefore, it is important to set a threshold to the crab cavity multipoles to allow sufficient DA in the IR.

To achieve high luminosity for physics experiments, the EIC hadron beam has unequal transverse emittance with a ratio of 11:1 for horizontal to vertical bunch size. The DA is described by the mean amplitude in terms of root mean square width of the bunch (σ) at different bunch cross-section radial angles. When the flat beam passing through the round beam pipe with a diameter of 10 cm, the DA is constraining the horizontal direction by far more than the vertical direction. Such beam distribution indicates that the limitation of the DA comes from horizontal direction. In the DA studies, the tracking covers from 0°to 60° with respect to the horizontal plane.

The threshold of the multipoles is then defined by the minimum acceptable DA for the IR region, which currently is set at 6σ. In the tracking process, each multipole interacts with hadron beam the same location on each turn. DA is calculated at the end of the defined tracking period. With 10^6 turns, the DA for crab cavity multipoles b_2 and b_3 are plotted in Fig. 5. To learn the DA sensitivity to different multipolar fields, each tracking of multipole strength scan is completed with only one type of the multipoles.

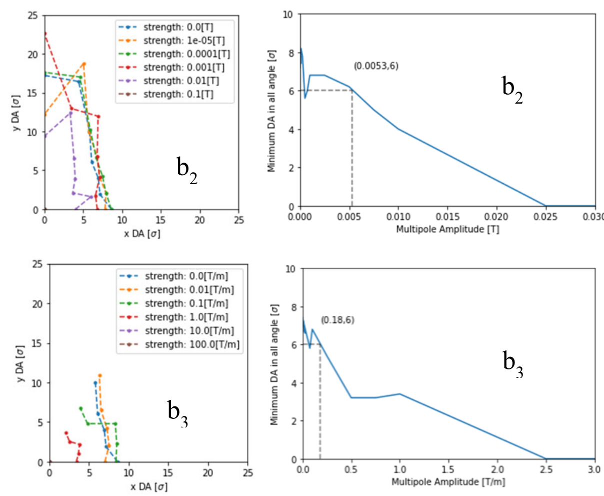

Figure 5: DA vs multipole coefficients b2 and b3.

On the right-hand side of Fig. 5, the thresholds of quadrupole and sextupole coefficients at 6σ are labeled for comparison with Table 1.

The minimum quadrupole coefficient b_2 among the three designs in Table 1 is from the E Probe FPC with a value that is 27 times less than the threshold, which leaves a large safety margin during operation. However, b_3 of all three designs are very similar due to the same cavity main body design, and the value is 7 times greater than the threshold. The relatively small threshold might be caused by the two sets of crab cavities on both sides of the interaction point do not have an exact phase difference of 180 degrees. This still need future study as well as the higher order multipoles with n>3.

CONCLUSION

Different design versions of the 197 MHz crab cavity for EIC have been compared as guidance to the design selection. The multipolar coefficients also compared with the threshold set by DA tracking, and the sextupole strength exceeds the current threshold simulations. Further study to resolve the issue is needed, as well as verification of the higher order multipoles.

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REFERENCES

[1] R. B. Palmer, "Energy Scaling Crab Crossing and the Pair Problem", SLAC-PUB-4707, 1988.

[2] K. Hosoyama, "Experiences with KEK-B Crab Cavity", Beam Dynamics Mini-Workshop on Deflecting/Crabbing Cavity Research and Applications in Accelerators, Lanzhou, China, July 2012.

[3] T. Capelli, "WP4-RFD cryomodule design for SPS tests", 11th HL-LHC Collaboration Meeting, CERN - 19-22 October 2021.

[4] P. McIntosh, "ILC Crab Cavity (ILC-CC) System Overview", presented at *LCWS'08*, Chicago, Nov. 2008. https://agenda.linearcollider.org/event/3346/contributions/12277/attachments/9739/16145/ILC_CC_Overview_and_further_work.pdf

[5] G. Burt *et al.*, "Prototype Development of the CLIC Crab Cavities", in *Proc. LINAC'14*, Geneva, Switzerland, Aug.-Sep. 2014, paper THPP013, pp. 856-859.

[6] C. Montag *et al.*, "Electron Ion Collider Design Status", presented at IPAC'22, Bangkok, Thailand, May 2022, paper WEPOPT044, this conference.

[7] W. K. H. Panofsky and W. A. Wenzel, "Some Considerations Concerning the Transverse Deflection of Charged Particles in Radio-Frequency Fields", *Rev. Sci. Instrum.*, vol. 27, p. 967, 1956. doi:10.1063/1.1715427

[8] CST Microwave Studio Suite, 2020. https://www.3ds.com/products-services/simulia/prod-ucts/cst-studio-suite/.

[9] Q. Wu, S. U. De Silva, Y. Luo, J. A. Mitchell, and B. P. Xiao, "EIC Crab Cavity Multipole Analysis", in *Proc. IPAC'21*, Campinas, Brazil, May 2021, pp. 2589-2591. doi:10.18429/JACoW-IPAC2021-WEPAB006

[10] Yun Luo, "SimTrack: A compact c++ code for particle orbit and spin tracking in accelerators", *Nucl. Instrum. Methods Phys. Res. Sec. A*, vol. 801, pp. 95-103, Nov. 2015. doi:10.1016/j.nima.2015.08.014