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INFLUENCES OF THE TRANSVERSE MOTIONS OF THE PARTICLES TO THE RECOMBINATION RATE OF A CO-PROPAGATING ELECTRON-ION SYSTEM*

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Abstract

For a system with the ion beam co-propagating with the electron beam, such as a traditional electron cooler or a Coherent electron Cooler (CeC), the recombination rate is an important observable for matching the energy of the electrons with the ions [1, 2]. In this work, we have developed the analytical expressions to investigate how the recombination rate depends on the energy difference of the two beams, with the influences from the transverse motions of the particles being considered. The analytical results are then applied to analyse the measured recombination data collected during the CeC experiment in RHIC run 21 and RHIC run 22.

INTRODUCTION

The CeC experiment at RHIC targets on cooling of Au^{+79} ions with electrons at the energy of $\gamma = 28.5$ [3]. One of the major challenges in the experiment is to ensure that the electron beam has the same energy as that of the ion beam. A common technique employed for the energy matching is by measuring the recombination rate. Since the recombination rate is maximized when the velocities of the two beams coincide, the optimal energy of the electrons can be identified by scanning the energy of the electrons while monitoring the recombination rate. How fast the recombination rate drops with the energy deviation from the optimal energy depends on the 3D velocity spreads of the electrons and the ions. During RHIC run 21, it was found in the CeC experiment that the measured recombination rate drops much slower than what one would expect solely from the measured energy spread of the electrons and the ions. One of the major candidates responsible for the discrepancy is the transverse motion of the particles which had been neglected from the analytical estimates.

In this work, we derived analytical formula to calculate the recombination rate with transverse motion of the particles taken into account. The analytical results are then used to fit the data collected during the CeC experiment in run 21 and run 22.

DERIVATION OF RECOMBINATION RATE

The general form of the recombination rate is given by the following integral over the velocity distribution of the electrons and the ions [2].

$$\alpha_r = \frac{\int_{-\infty}^{\infty} d^3v_i d^3v_e f_e(v_e) f_I(v_i) |\vec{v}_e - \vec{v}_i| \sigma(|\vec{v}_e - \vec{v}_i|)}{\int_{-\infty}^{\infty} d^3v_i d^3v_e f_e(v_e) f_I(v_i)}, \tag{1}$$

where $\sigma(|\vec{v}_e - \vec{v}_i|)$ is the recombination cross section which depends on the relative velocity of an ion with respect to an electron, and $f_e(v_e)$ and $f_I(v_i)$ are the velocity distributions of the electrons and the ions.

Gaussian Transverse Velocity Distribution

We assume that the velocity distribution of electrons is

$$f_e(v_e) = \frac{1}{2\pi\beta_{e,\perp}^2} \exp\left(-\frac{v_{e,x}^2 + v_{e,y}^2}{2\beta_{e,\perp}^2}\right) f_{e,z}(v_{e,z}) \tag{2}$$

and that of ions is

$$f_I(v_i) = \frac{1}{2\pi\beta_{i,\perp}^2} \exp\left(-\frac{v_{i,x}^2 + v_{i,y}^2}{2\beta_{i,\perp}^2}\right) f_{i,z}(v_{i,z}), \tag{3}$$

with $\beta_{e,\perp}$ and $\beta_{i,\perp}$ being the transverse velocity spread of the electrons and the ions. Inserting Eq. (2) and Eq. (3) into Eq. (1) yields

$$\alpha_r = \frac{\int_{-\infty}^{\infty} d^3v v\sigma(v) e^{-m_e(v_x^2+v_y^2)/2kT_{ei}} \int_{-\infty}^{\infty} f_{e,z}(v_z + v_{i,z}) f_{i,z}(v_{i,z}) dv_{i,z}}{(2\pi kT_{ei}/m_e) \int_{-\infty}^{\infty} f_{e,z}(v_z + v_{i,z}) f_{i,z}(v_{i,z}) dv_z dv_{i,z}} \tag{4}$$

where we defined the effective temperature parameter

$$T_{ei} = \frac{m_e}{2k}\left(\beta_{e,x}^2 + \beta_{e,y}^2 + \beta_{i,x}^2 + \beta_{i,y}^2\right). \tag{5}$$

Longitudinally Cold Electrons and Ions

Typically, the longitudinal velocity spread in the beam frame is much smaller than the transverse velocity spread and hence we can take the delta function for the longitudinal velocity distribution, i.e. $f_{e,z}(v_{e,z}) = \delta(v_{e,z} - v_{z0})$ and $f_{i,z}(v_{i,z}) = \delta(v_{i,z})$. In this case, Eq. (4) becomes

$$\alpha_r = \frac{1}{2\pi kT_{ei}/m_e} \int_{-\infty}^{\infty} d^3v v\sigma(v) e^{-m_e(v_x^2+v_y^2)/2kT_{ei}} \delta(v_z - v_{z0}). \tag{6}$$

The recombination cross section for an electron moving with velocity $\vec{v}$ with respect to the ion is [2]

$$\sigma(v) = A\frac{2h\nu_0}{m_e v^2}\left[\ln\left(\sqrt{\frac{2h\nu_0}{m_e v^2}}\right) + \gamma_1 + \gamma_2\left(\frac{m_e v^2}{2h\nu_0}\right)^{1/3}\right], \tag{7}$$

where $A = 2.11\times10^{-22} cm^2$, $h\nu_0 = Z^2 \times 13.6eV$, Z is the

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charge number of the ion, $\gamma_1 = 0.1402$ and $\gamma_2 = 0.525$. Inserting Eq. (7) into Eq. (6) yields

$$\alpha_r = Ac\frac{h\nu_0}{kT_{ei}}\sqrt{\frac{2h\nu_0}{m_e c^2}}\,\mathrm{e}^{m_e v_{z0}^2/(2kT_{ei})} \times \int_{\frac{m_e v_{z0}^2}{2h\nu_0}}^{\infty}\frac{\mathrm{e}^{-h\nu_0 y/(kT_{ei})}}{\sqrt{y}}\left(\gamma_1+\gamma_2 y^{1/3}-\frac{1}{2}\ln y\right)dy \quad . \tag{8}$$

Influences of Orbital Angle

If the electrons merge into ions with an angle, the average transverse velocity of the electrons in the beam frame is nonzero. In this case, Eq. (4) becomes

$$\alpha_r = \frac{Ach\nu_0}{kT_{ei}}\sqrt{\frac{2h\nu_0}{m_e c^2}}\,\mathrm{e}^{m_e\left(v_{z0}^2-v_{x0}^2\right)/(2kT_{ei})}\int_{\frac{m_e v_{z0}^2}{2h\nu_0}}^{\infty}\frac{\mathrm{e}^{-h\nu_0 y/(kT_{ei})}}{\sqrt{y}}\times \left(\gamma_1+\gamma_2 y^{1/3}-\frac{1}{2}\ln y\right)I_0\left(\frac{v_{x0}\sqrt{2m_e h\nu_0}}{kT_{ei}}\sqrt{y-\frac{m_e v_{z0}^2}{2h\nu_0}}\right)dy \quad , \tag{9}$$

where v_{x0} is the average horizontal velocity of the electrons with respect to the ions and $I_0(x)$ is the modified Bessel function.

Gaussian Longitudinal Velocity Distribution

In the previous sections, we assume that, in the co-moving frame of the ions, the longitudinal velocity spreads of the electrons and the ions are negligible compared to their transverse velocity spreads. To investigate how the longitudinal velocity spread may affect the recombination rate, we take the Gaussian longitudinal velocity distribution, i.e.,

$$\alpha_r = \frac{Ach\nu_0}{kT_{ei}\eta}\sqrt{\frac{2h\nu_0}{m_e c^2}}\,\mathrm{e}^{\frac{m_e v_{z0}{}^2\left(1-\eta^2\right)}{4kT_z\eta^2}}\int_0^{\infty}\left(\gamma_1+\gamma_2 y^{2/3}-\ln y\right) \times \mathrm{e}^{-\frac{h\nu_0 y^2}{kT_{ei}}}\left\{erf\left[\eta\sqrt{\frac{h\nu_0}{2kT_z}}\left(y+\sqrt{\frac{m_e}{2h\nu_0}}\frac{v_{z0}}{\eta^2}\right)\right] +erf\left[\eta\sqrt{\frac{h\nu_0}{2kT_z}}\left(y-\frac{v_{z0}}{\eta^2}\sqrt{\frac{m_e}{2h\nu_0}}\right)\right]\right\}dy \quad , \tag{10}$$

where $kT_z = m_e\left(\beta_{e,z}^2+\beta_{i,z}^2\right)/2$ and $\eta=\sqrt{1-2T_z/T_{ei}}$. It is worth noting that for $T_{ei} < 2T_z$, η is imaginary but according to the relation, $erf(i\cdot x) = i\cdot erfi(x)$, the error function in Eq. (10) also returns an imaginary value and hence the expression is real and still valid for calculating the recombination rate. Eq. (10) has a singularity at $T_{ei} = 2T_z$ and in this case, the following expression can be used

$$\alpha_r = \frac{Ah\nu_0}{m_e v_{z0}}\sqrt{\frac{2h\nu_0}{\pi kT_z}}\,\mathrm{e}^{\frac{-m_e v_{z0}{}^2}{4kT_z}} \times\int_0^{\infty}\frac{\gamma_1+\gamma_2 y^{1/3}-\frac{1}{2}\ln y}{\sqrt{y}}\mathrm{e}^{-\frac{h\nu_0 y}{2kT_z}}\sin h\left(\frac{v_{z0}\sqrt{m_e h\nu_0 y}}{\sqrt{2kT_z}}\right)dy \quad . \tag{11}$$

EXPERIMENT RESULTS

The CeC experiment at RHIC targets on cooling the Au+79 ions in the yellow ring with electrons at $\gamma = 28.5$. During RHIC run 21 and 22, the electrons were brought to co-propagate with the ions in the 14 meters long cooling section and the recombination rate were measured while scanning the energy of the electrons. As shown in Table 1,

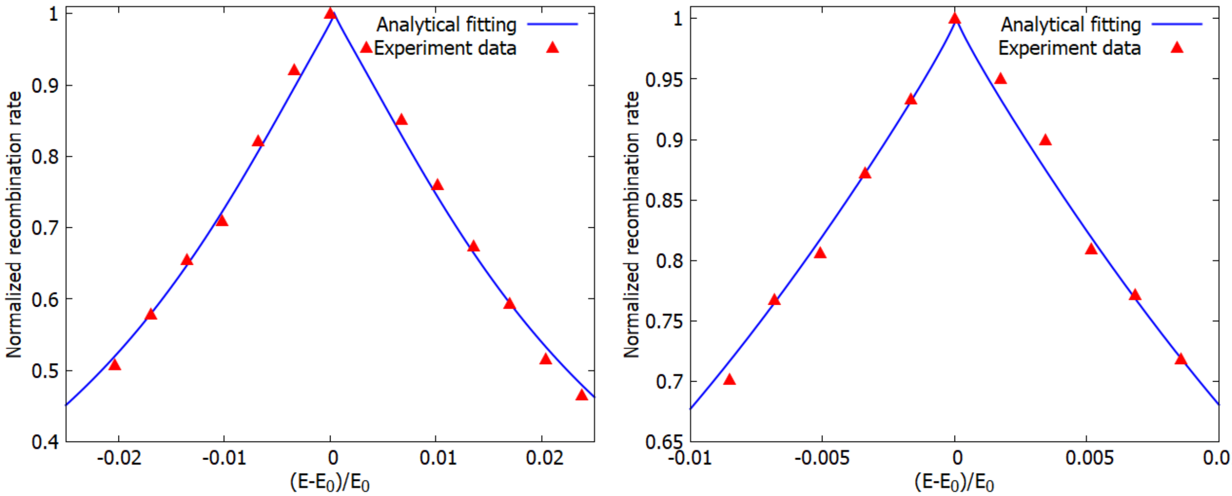

Figure 1: Normalized recombination rate as a function of the relative energy deviation of the electron bunch. (Left) The data were taken during RHIC run 21. The red triangles represent the measured recombination rate normalized to the value at $E_0 = 14.719MeV$ and the blue curve represents the fitting from Eq. (9) with $v_{x0} = 6.6\times10^6 m/s$, which corresponds to 0.77 mrad of orbital angle between the electron beam and the ion beam. (Right) The data were taken during RHIC run 22. The measured recombination rate maximizes at $E_0 = 14.646MeV$ and the fitting curve were calculated with $v_{x0} = 6\times10^6 m/s$, i.e. 0.7 mrad of orbital angle between the two beams.

the designed angular spread at cooling section is about 0.3 mrad for the electrons and 0.14 mrad for the ions, which corresponds to the transverse temperature of the particles in the comoving frame of $kT_{ei} = 46.7eV$. Figure. 1 (red triangles) shows the normalized recombination rate as a function of the electrons' energy as measured in run 21 (Left) and run 22 (Right). The energy of the electrons was adjusted by changing the voltage of the accelerating cavities. As shown in fig. 1 (solid blue curve), Eq. (9) provides reasonable fit for the measured data if we assume that there was an orbital angle between the electrons and the ions with the value of 0.77 mrad for the run 21 measurement and 0.7 mrad for the run 22 measurement. The alignment of the electron beam and the ion beam in the cooling section are achieved with beam position monitors (BPMs), earth field compensation coils and dipole correctors. During the measurements, the beam position readings at the BPMs were within $\pm 1\ mm$ and the distances between two successive BPMs are 1~2 meters. Consequently, we consider it reasonable to assume an angular misalignment in the level of 0.7~0.8 mrad which was responsible for the slow declining in the recombination rate with the energy deviation of the electron beam.

Table 1: Parameters of the CeC Experiment at RHIC

	Electrons	**Ions**
Energy, γ	28.5	
Energy spread R.M.S.	5E-4	1.3E-3
Angular spread, mrad	0.3	0.14

SUMMARY

The recombination rate of the electrons and the ions in a cooling section depends on the 3D velocities of the particles. When the eletron beam and the ion beam are ideally aligned in the transverse planes and their transverse velocity spreads are much smaller than their longitudinal velocity spread in the beam frame, the recombination curve, i.e. the dependence of the recombination rate on the energy deviation of the electrons, has Gaussian shape and the width of the recombination curve is determined by the energy spreads of the two beam [1, 4]. However, when the transverse misalignment is significant or the transverse velocity spread is larger than the longitudinal velocity spread in the beam frame, the recombination curve is more of triangular shape and its width is determined by the orbital angle between the two beams or their angular spreads. For the CeC experiment at RHIC, our analysis shows that the transverse alignment of the two beams determines the behaviour of how recombination rate depends on the electrons' energy.

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