ISBN: 978-3-95450-227-1 ISSN: 2673-5490 doi:10.18429/JACoW-IPAC2022-MOPOST046

ENFORCING THE CONVERGENCE OF LONGITUDINAL BUNCH DENSITY CALCULATION IN THE PRESENCE OF A HARMONIC CAVITY THROUGH ANDERSON ACCELERATION METHOD

I. C. Almeida*, A. P. B. Lima, M. H. Wallner
Brazilian Center for Research in Energy and Materials, Campinas, Brazil

Abstract

Sirius is a 4th generation synchrotron light source at the Brazilian Center for Research in Energy and Materials in Campinas, Brazil. A passive superconducting third harmonic cavity is planned to be installed in the storage ring in order to lengthen the bunches and increase beam lifetime by reducing Touschek scattering while keeping its high brightness. This paper presents the results obtained in applying Anderson acceleration method to enforce the convergence of the self-consistent algorithm used for calculation of the equilibrium longitudinal bunch density in the presence of a harmonic cavity.

INTRODUCTION

New generation synchrotron light sources require low-emittance storage rings in order to increase radiation brightness, which reduces beam lifetime due to Touschek scattering. A common approach to increase beam lifetime without affecting the brightness is to include a higher harmonic cavity in the system in order to lengthen the bunches and reduce the longitudinal bunch density [1]. For Sirius, which has a natural emittance of 0.250 nm−rad and a fundamental RF frequency of 500 MHz, a 1.5 GHz passive superconducting third harmonic cavity is planned to be installed and a beam lifetime increase around 4.5 times the current value is expected.

Since it is known that the maximum lifetime increase is obtained with bunch overstretching , *i.e.*, with a harmonic voltage higher than the one calculated for flat potential [2], a voltage sweep was carried on to find the maximum beam lifetime working point using the self-consistent approach described in [3, 4]. It was observed that above a threshold voltage value the algorithm could not converge and the longitudinal bunch density started to bounce between different fixed states. In this paper, Anderson acceleration method is adopted to enforce the convergence of the self-consistent equilibrium bunch density calculation based on the approach presented in [5].

ANDERSON ACCELERATION

In a passive harmonic cavity the voltage is due only to beam loading, which means that amplitude and phase cannot be controlled independently [3]. Hence, for a given harmonic voltage amplitude, the self-consistent bunch density calculation should converge to an equilibrium state and provide the corresponding cavity detune. However, as will be later shown, above a threshold voltage amplitude, the algorithm does not converge and an equilibrium bunch density cannot be obtained. To overcome this issue, the bunch densities from previous iterations started being used through Anderson acceleration method.

For each iteration k, the electron bunch density of a storage ring is given by:

$$\rho_k(\tau) = Ae^{-\frac{\Phi_k(\tau)}{\alpha^2\sigma_e^2}}, \tag{1}$$

where A is a normalization constant, α is the momentum compaction factor, σ_e is the energy spread, τ is the time deviation with respect to the synchronous particle and $\Phi(\tau)$ is the voltage dependent potential function.

Since the voltage of a harmonic cavity depends on the bunch density, a fixed point problem is established and the following map can be defined:

$$g_k = g(\rho_k) = Ae^{-\frac{\Phi(\tau,\rho_k)}{\alpha^2\sigma_e^2}}. \tag{2}$$

The new bunch density can be calculated as a linear combination of the maps of previous iterations. Anderson acceleration method solves a constrained linear least-squares problem to find the coefficients of the linear combination [5]:

$$f_k = g_k - \rho_k,$$
$$(\alpha_0^k, \alpha_1^k, ..., \alpha_{m_k}^k) = \operatorname{argmin}\left|\sum_{j=0}^{m_k}\alpha_j^k f_{k-m_k+j}\right|^2, \tag{3}$$
$$\sum_{j=0}^{m_k}\alpha_j^k = 1.$$

Therefore,

$$\rho_{k+1} = \sum_{j=0}^{m_k}\alpha_j^k g_{k-m_k+j} \quad \text{for} \quad k \geq 0, \tag{4}$$

where $m_k = \min(k, m)$ with $m \geq 1$ being the number of previous iterations desired to be taken into account in the the current step. For $k = 0$, an initial condition must be given and the constraint relation provides $\alpha_0^0 = 1$.

An extra degree of flexibility can be added by introducing the relaxation parameter $\beta_k \leq 1$:

$$\rho_{k+1} = \beta_k\sum_{j=0}^{m_k}\alpha_j^k g_{k-m_k+j} + (1-\beta_k)\sum_{j=0}^{m_k}\alpha_j^k\rho_{k-m_k+j}. \tag{5}$$

* iago.almeida@cnpem.br

ISBN: 978-3-95450-227-1 ISSN: 2673-5490 doi:10.18429/JACoW-IPAC2022-MOPOST046

RESULTS

The results presented in this section were based on Sirius's nominal parameters shown in Table 1. Considering the range of values from known harmonic cavities [6–8], the R/Q and quality factor were taken to be $R/Q = 90$ and $Q_0 = 2\times10^8$, respectively. For the implementation of Anderson acceleration, the following parameters were considered: $m_k = 5$, $\beta_0 = 0.1$ and $\beta_k = 1$ for $k \geq 1$, which allows a relaxation during the first iteration. The initial condition is the flat potential bunch density and the convergence criteria used when Anderson acceleration was implemented is given by the Euclidean norm of the difference between the bunch density of the current and the previous iterations. The tolerance was set to 1×10^{-6} and the maximum number of iterations allowed was set to 2000.

Table 1: Nominal Sirius Storage Ring Parameters

Parameter	Symbol	Value
Energy	E_0	3 GeV
RF frequency	f_{RF}	499.6638 MHz
RF voltage	V_{cav}	3 MV
Energy loss per turn	U_0	871.01 keV
Revolution period	T_0	1.729 μs
Circumference	C	518.39 m
Moment compaction	α	1.645×10^{-4}
Energy spread	σ	8.400×10^{-4}
Beam current	I_{av}	350 mA

For a harmonic voltage sweep up to 1.2 MV, Fig. 1 compares the harmonic cavity detune angle without and with Anderson acceleration implemented in the self-consistent algorithm [3]. The blue curve shows for each voltage value the resultant detune angle of the last 50 iterations from 10000 iterations. Up to a threshold value $V_{th} = 969$ kV the self-consistent algorithm successfully converges and a single detune value is obtained for the equilibrium longitudinal bunch density. Above this voltage, the equilibrium cannot be found and the bunch density bounces between different states, creating a spread in the detune angle. On the other hand, the red curve shows the detune angle as a function of the harmonic cavity voltage when Anderson acceleration was considered. The resultant smooth function indicates that an equilibrium longitudinal bunch density was reached for each voltage value.

The lifetime increase, *i.e.*, the ratio between the beam lifetime in the presence and in the absence of the harmonic cavity, and the peak current of the longitudinal bunch density are shown in Figs. 2 and 3, respectively, as functions of the harmonic cavity voltage. It can be seen that with Anderson acceleration it is possible to obtain the equilibrium bunch densities that provide the maximum lifetime increase and the minimum peak current. It is worth noting that these two cases occur at different voltage values, requiring a trade-off when choosing the operating point of the harmonic cavity.

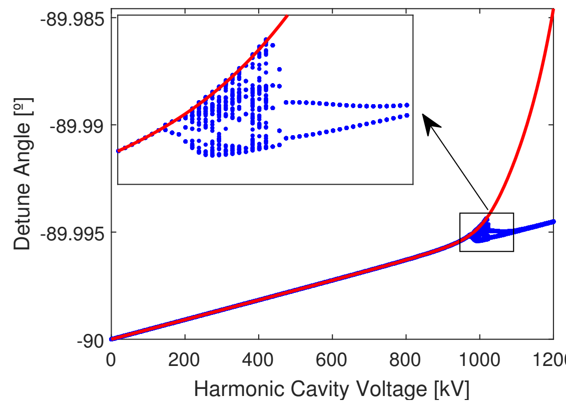

Figure 1: Detune angle as a function of the harmonic cavity voltage for the cases without (blue) and with (red) Anderson acceleration. In the upper left corner, a detail of the region close to the threshold voltage $V_{th} = 969$ kV is also shown.

Figure 4 shows the equilibrium bunch density for the flat potential voltage[1], which occurs at a harmonic voltage of 951 kV for Sirius's parameters. The lifetime increase ratio and the peak current for this regime are 4.66 and 6.693 A, respectively.

Also shown in Fig. 4 are the bunch densities that provide the maximum lifetime increase and the minimum current peak. The maximum lifetime increase is 5.93 at a harmonic voltage of 1.005 MV and the minimum peak current is 6.058 A at 984 kV. It can be seen that both cases lie in the overstretching regime, where the longitudinal bunch density starts to show two peaks and a strong deformation. Table 2 summarizes the results discussed.

Table 2: Results for Flat Potential Voltage (I), Minimum Current Peak (II) and Maximum Lifetime Increase (III)

Parameter	Case I	Case II	Case III
Harmonic voltage [kV]	951	984	1005
Peak current [A]	6.693	6.058	6.318
Lifetime increase	4.661	5.659	5.934

Care must be taken while trying to achieve these results in practice since overstretching implies in a smaller harmonic cavity detune. Passive harmonic cavities are inherently Robinson unstable because they operate on the growing side of the impedance curve and thus the smaller detune could compromise stability [1].

Besides the advantages of enforcing the convergence of the self-consistent algorithm, Anderson acceleration method also reduces the computational effort in calculating the equilibrium longitudinal bunch density. The voltage sweep presented in this paper was carried on using 401 different voltage values, which took only about 12 seconds with Anderson

[1] Since for a passive cavity the magnitude and phase of the voltage cannot be controlled independently, operation at the flat potential voltage does not necessarily imply in operation at the flat potential phase, hence the bunch distortion.

ISBN: 978-3-95450-227-1 ISSN: 2673-5490 doi:10.18429/JACoW-IPAC2022-MOPOST046

acceleration. For comparison, below the threshold voltage, where the self-consistent approach converges without Anderson acceleration, the bunch density calculation for a single voltage value takes about 2 seconds. On average, Anderson acceleration makes the self-consistent calculation 70 times faster.

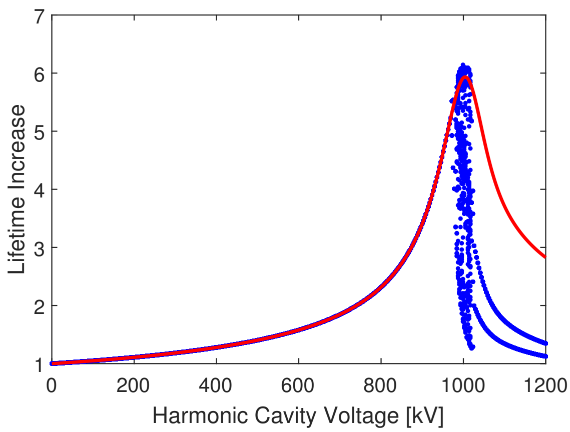

Figure 2: Beam lifetime increase as a function of the harmonic cavity voltage for the cases without (blue) and with (red) Anderson acceleration.

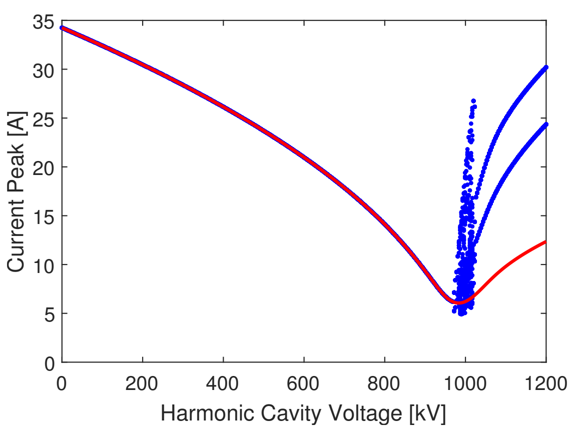

Figure 3: Peak current value as a function of the harmonic cavity voltage for the cases without (blue) and with (red) Anderson acceleration.

CONCLUSION

A passive superconducting third harmonic cavity is planned to be installed at Sirius synchrotron light source in order to provide beam lifetime increase through longitudinal bunch lengthening. It was shown that Anderson acceleration method enforces the convergence of the self-consistent algorithm for calculation of the equilibrium longitudinal bunch density while enhancing its computational performance. It was then possible to identify the required harmonic cavity detune and the corresponding bunch densities for the cases where the peak current per bunch is minimum and where the lifetime increase ratio is maximum.

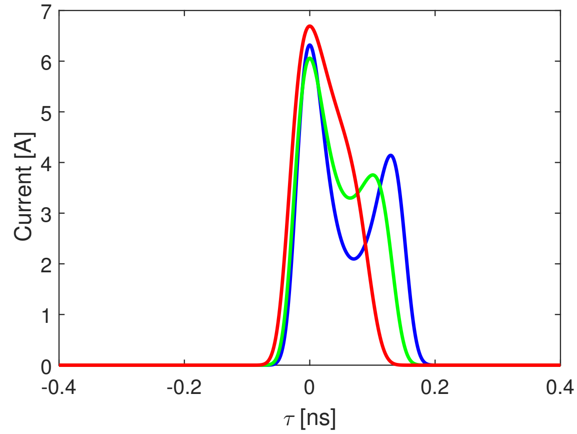

Figure 4: Equilibrium longitudinal bunch densities for maximum lifetime (blue), minimum current peak (green) and flat potential voltage (red) cases.

ACKNOWLEDGEMENTS

The authors would like to thanks Fernando H. de Sá and Murilo B. Alves from Sirius's Accelerator Physics Group for the fruitful conversations regarding numerical methods to increase the performance of self-consistent algorithms.

REFERENCES

[1] J.M. Byrd, M.Georgsson, "Lifetime Increase Using Passive Harmonic Cavities in Synchrotron Light Sources", *Phys. Rev. ST Accel. Beams*, vol. 4, p. 030701, Jun. 2001.

[2] G. Penco, M. Svandrlik, "Experimental Studies on Transient Beam Loading Effects in the Presence of a Superconducting Third Harmonic Cavity", *Phys. Rev. ST Accel. Beams*, vol. 9, p. 044401, Apr. 2006.

[3] I.C. Almeida, A.P.B. Lima, M.H. Wallner, "Third Harmonic Superconductive Cavity for Bunch Lengthening and Beam Lifetime Increase of Sirius Synchrotron Light Source", in *Proc. 20th Int. Conf. RF Superconductivity (SRF'21)*, East Lansing, MI, USA, Jun.-Jul. 2021, pp. 37-41.

[4] P.F. Tavares *et al.*, "Equilibrium Bunch Density Distribution with Passive Harmonic Cavities in a Storage Ring", *Phys. Rev. ST Accel. Beams*, vol. 17, p. 064401, Jun. 2014.

[5] R. Warnock, "Equilibrium of an Arbitrary Bunch Train with Cavity Resonators and Short Range Wake: Enhanced Iterative Solution with Anderson Acceleration", in *Phys. Rev. Accel. Beams*, vol. 24, p. 104402, Oct. 2021.

[6] Z.K. Liu *et al.*, "Performance Simulation for a Prototype 1.5 GHz Superconducting Harmonic Cavity", in *IEEE Transactions on Applied Superconductivity*, vol. 29, pp. 1-5, Aug. 2019.

[7] A. Anghel *et al.*, "The SUPER-3HC Project: an Idle Superconducting Harmonic Cavity for Bunch Length Manipulation", in *Proc. 7th European Particle Accelerator Conf. (EPAC'00)*, Vienna, Austria, Jun. 2000, paper THP7B10, pp. 2052-2054.

[8] J. Rose, N. A. Towne, C. H. Boulware, and T. L. Grimm, "Design of a 1500 MHz Bunch Lengthening Cavity for NSLS-II", in *Proc. 15th Int. Conf. RF Superconductivity (SRF'11)*, Chicago, IL, USA, Jul. 2011, paper MOPO050, pp. 210-212.