

GENERALISATION AND LONGITUDINAL EXTENSION OF THE GENETIC LATTICE CONSTRUCTION (GLC) ALGORITHM

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Abstract

The GLC algorithm allows the construction of efficient transfer lines with defined imaging properties using a minimum number of quadrupole elements. This work describes a generalisation of this algorithm to make it applicable to the use of arbitrary beam optical elements. This includes an extension to longitudinal phase space.

INTRODUCTION

The design of a matching section of a particle accelerator can be formulated as an optimisation problem that can be solved algorithmically. The goal of such an optimisation is to achieve an optimal fit of a given particle distribution to an arbitrary acceptance, with minimum energy and material requirements (i.e., with the lowest possible field strengths and the fewest possible elements). The formulation of the individual problem can be reduced to the problem of minimising a single-valued fitness function over a multidimensional cube. This requires finding a suitable, unambiguous parameterisation to encode both the transfer line geometry and the field strengths used (phenotype) by a list of numbers (genotype) [1].

PARAMETRISATION

Given a general unspecific transport line of length L , in which at least one section of length L_{opt} can be equipped with new elements. If $L = L_{\text{opt}}$, then there are no constraints on the position of new elements. Furthermore, there is a set of μ freely positionable beam manipulating components $\{C_1, \dots, C_\mu\}$. These can also be meta-elements consisting of a fixed combination of components. In addition, there are those ν components whose positions are fixed from an optimisation point of view, which means that they are located outside L_{opt} . In sum, this gives $N = \mu + \nu$ components. Each of these components $C_n \in \{C_1, \dots, C_{\mu+\nu}\}$ has a certain number of λ_n free parameters $x_{n,1} \dots x_{n,\lambda_n}$. These parameters can be e.g. quadrupole strengths, accelerating gap voltages, but also the lengths of the respective elements. The total number of free parameters is then

$$\eta_p = \sum_{n=1}^N \lambda_n. \quad (1)$$

A position must now be defined for each free component from $\{C_1, \dots, C_\mu\}$. The possibility of arbitrary permutations of the components should be implicitly included. For

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this purpose, a relative position $s_n \in [0, 1]$ is defined for each component. The actual position l_n results then from

$$l_n = s_n \cdot \left(L_{\text{opt}} - \sum_{n=1}^{\mu} L_n \right), \quad (2)$$

where L_n is the length of the component C_n . Due to free positioning, the number of free parameters increases by μ to $\eta = \eta_p + \mu$.

Every possible realisation of a transport path that satisfies the given boundary conditions can be assigned to a point in an η -dimensional subspace of \mathbb{R}^7 . The parameters are normalised to the range $[-1, 1]$ with respect to the individual device limits. So this corresponds to a point

$$\mathbf{x} = \left(\tilde{x}_1 \dots \tilde{x}_{\eta_p} \tilde{s}_1 \dots \tilde{s}_\mu \right) \quad (3)$$

in the η hypercube. Here \tilde{x}_n and \tilde{s}_n are respectively the values of x_n and s_n normalised to the interval $[-1, 1]$. I want to emphasise that each design solution for such a transfer line corresponds to a point in this space. There is a distinct assignment. Each of these points can now be assigned a value of a fitness function

$$F(\mathbf{x}) = 1 - T(\mathbf{x}) + \epsilon(\mathbf{x}) \quad (4)$$

that describes the performance of the associated transfer line with respect to beam transmission T . Its minimum F_{min} corresponds to the largest transmission T_{max} . The term $\epsilon(\mathbf{x})$ represents the parameterisation of further optimisation goals, such as lowest possible field strengths or minimum component dimensions, and is defined as a sub-norm in the simplest case, e.g.

$$\epsilon(\mathbf{x}) = \|(\tilde{x}_1, \dots, \tilde{x}_{\eta_p})\| = \sqrt{\tilde{x}_1^2 + \dots + \tilde{x}_{\eta_p}^2}. \quad (5)$$

The minimum of the fitness function encodes the optimal design solution for any given problem.

Example: A Single Quadrupole

A drift section of length $L = 20$ m is given at the end of which an aperture limitation is attached. This has a diameter of 4 cm horizontally and 10 cm vertically. An asymmetric, divergent particle distribution is chosen such that most of the beam is lost on the aperture limitation. For a quadrupole of length $L_q = 1$ m, the goal is to find a position $l = s \cdot (L - L_q)$ within this distance and a gradient (kL_q) at which the transmission becomes maximum (Fig. 1).

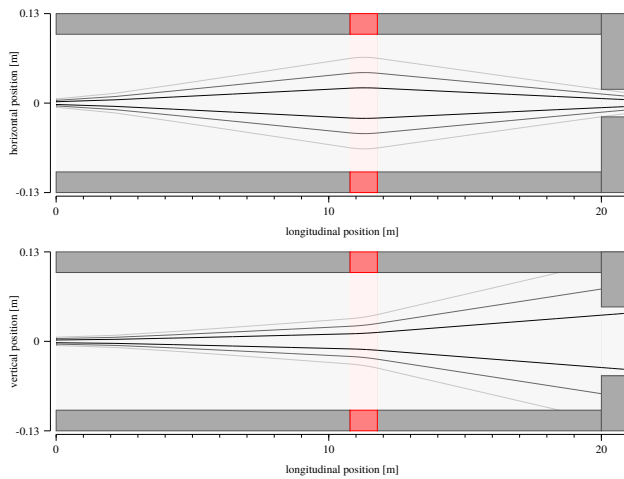


Figure 1: Representation of the solution with the maximum achievable transmission of $T = 74.5\%$ in this test case [2, page 82]. The black lines represent 1σ of the distribution, the dark grey 2σ and the light gray 3σ .

According to equation (2) and (3) the parameters

$$x_1 = (kL_q) \quad \text{and} \quad s_1 = \frac{l}{(L - L_q)} \quad (6)$$

are used for the general optimisation. The problem has $\eta = 2$ degrees of freedom and each solution here corresponds to one point

$$\mathbf{x} = \begin{pmatrix} \tilde{x}_1 \\ \tilde{s}_1 \end{pmatrix} \quad (7)$$

in the associated unit square. Since it is a 2-dimensional problem, the function $F(\mathbf{x}) = F(\tilde{x}_1, \tilde{s}_1)$ can be visualized over the entire domain of definition (Fig. 2). F has only one

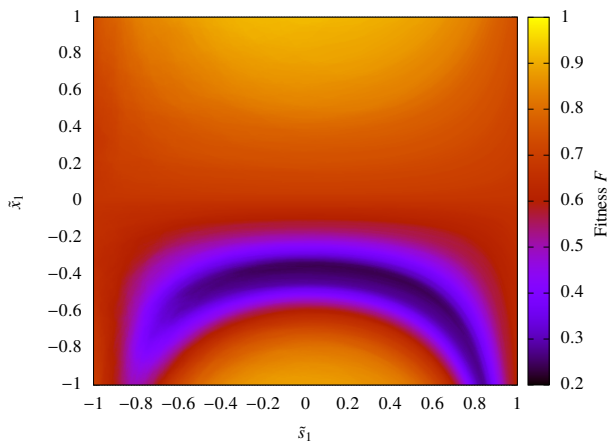


Figure 2: The result of the parameter scan [2, page 81]. Shown is the value of the fitness function depending on the normalised parameters \tilde{s}_1 and \tilde{x}_1 . The minimum is located at $\tilde{x}_1 = -0.35$, $\tilde{s}_1 = 0.135$.

minimum in this case, therefore a simple gradient method will also lead to success here. In general, however, this is not the case, because due to the quadrupole symmetry,

already from the use of a second quadrupole further secondary minima will occur and it is a priori not clear which one corresponds to the global minimum.

To determine the value of the fitness function at a certain point, a particle tracking simulation must be performed. The number of particles or the consideration of space charge and higher order effects are determined by the interested scientist. For this purpose, for example, the common matrix-based codes [3–7] can be used. However, depending on the specification of the problem and the desired accuracy, the number of necessary simulations grows rapidly. In particular, it grows exponentially with the number of dimensions of the problem space and soon exceeds all technically feasible possibilities to complete the simulation in a reasonable time.

OPTIMISATION TECHNIQUES

In cases of large multidimensional search spaces, natural-analog and metaheuristic optimisation methods have proven to be very useful, which was used also for many accelerator related optimisation studies [8–11]. For GLC, the following methods were investigated and compared (Table 1:

- genetic algorithms [12]
- particle swarm optimisation [13]
- simulated annealing [14]
- BOBYQA [15]

Table 1 shows a comparison of the optimisation methods in terms of the number of particle tracking simulations needed to find the optimal solution. Similar results are also obtained

Table 1: Comparison of a set of optimisation methods with respect to their performance. All heuristic algorithms are reliably able to find the optimum and differ only in the number of necessary optimisation cycles and therefore in the number of necessary tracking simulations. For the comparison, no special optimisation of the algorithms' hyper-parameters was performed.

optimisation method	# tracking simulations
gradient descent	63 ± 12
scan $\Delta = 0.005$	160000 ± 0
genetic algorithm	1790 ± 1048
particle swarm	7500 ± 6100
simulated annealing	8600 ± 5800
BOBYQA	26 ± 6

for other problems. The BOBYQA algorithm usually converges 2 orders of magnitude faster than all other methods. Unfortunately, it becomes increasingly unreliable in finding the optimum for more complex problems and in fractured parameter space topologies. For most examples, the genetic algorithm proved to be the best compromise between speed and probability of success. The topic of machine learning is currently very dynamic and it can be expected that new algorithms and methods will bring further improvements.

Since the GLC method is generally independent of the underlying optimisation algorithm, the algorithm can easily be replaced to directly benefit from future developments. A collection of further examples, as well as the application of GLC in 2 accelerator projects (first, the construction of an RFQ matching section in the planned HBS accelerator [16] and second, the equipping of the H4 beam line at CERN north area with Gabor lenses [17]) can be found in [2].

LONGITUDINAL EXTENSION

The extension of the algorithm to the longitudinal phase space is implicitly included in the parameterisation described above. The motion of the particles must then, necessarily be described via their path in the 6 dimensional phase space. In addition to the transverse location and momentum coordinates (x, x', y, y') at orbit position s , the longitudinal distance to the central particle and the momentum deviation $(z, \Delta p/p)$ or, equivalently phase and energy deviation $(\Delta\phi, \Delta W)$ or (z, z') must be additionally considered.

$$z = \phi(\beta\lambda), \quad z' = \frac{\Delta W}{\beta^2\gamma^3 mc^2}, \quad \frac{\Delta p}{p} = \frac{\Delta W}{\beta^2\gamma mc^2} \quad (8)$$

For all beam optical elements, the corresponding 6x6 transfer matrices must then be used in the particle tracking codes. In this way, accelerating gaps and elements based on them, such as buncher cavities, single gap resonators, but also complete RF accelerators can be used as elements. The latter can be defined, for example, as compound meta components. The free parameters are then usually the position, the integral accelerating voltage and the phase of the synchronous particle. In this way, complex matching sections for 6-dimensional acceptance targets of following accelerator sections can be designed automatically.

Example: A Single Gap

Given is a matching section of length $L = 10$ m in which an accelerating gap is to be positioned to match a beam to a given target acceptance ellipsoid. The main constraint is the phase acceptance of $\pm 30^\circ$. All other limiting quantities were chosen to be no real constraint. Calculations were performed with the Accelerator Construction Set [2] (3dKV space charge, no compensation, 1 mA protons, $\frac{\Delta p}{p} = 0.001$, gaussian beam $\sigma_\phi = 25^\circ$). There are 3 degrees of freedom in this example: the position of the gap mid s_{gap} , the gap voltage U_{gap} and the phase ϕ_{gap} . Thus each solution corresponds to one point

$$\mathbf{x} = \begin{pmatrix} \tilde{U}_{gap} \\ \tilde{\phi}_{gap} \\ \tilde{s}_{gap} \end{pmatrix}. \quad (9)$$

After an average of 40 generations (corresponding to 4000 simulation runs), the GLC solver converged. The result is shown in Fig. 3. Scanning voltage and phase with the gap position held fixed gives a projection of the fitness function onto the U - ϕ -plane (Fig. 4).

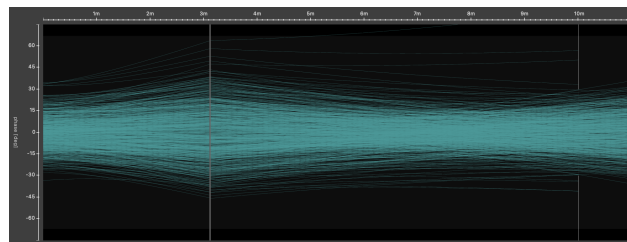


Figure 3: Solution with the maximum achievable transmission of $T = 91\%$ in this test case. The gap position is 3.12 m.

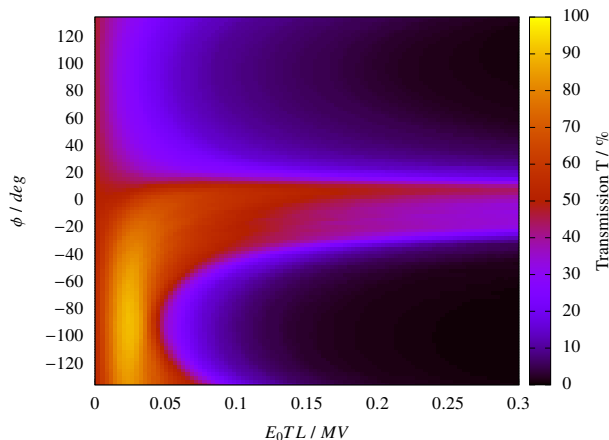


Figure 4: The result of the parameter scan with fixed gap position. Shown is the value of the transmission depending on the parameters ϕ and $U = E_0TL$. The optimum is located at $U = 25.3$ kV, $\phi = -92^\circ$.

ERROR STUDIES

In all cases where positioning errors were induced already during the phenotype construction within the optimisation process, there was evidence [2] that genetic lattice construction favours solutions that are robust to positioning errors. In this way, error tolerance could be implemented early in the design phase and the need for dedicated error studies and subsequent iterations of the design process could be eliminated.

CONCLUSION

The GLC algorithm is applicable to a large class of design problems and, together with precise particle tracking codes, GLC provides an excellent tool for the ion-optical design of future linear accelerators or for the optimisation of beam transport lines and matching sections [2].

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