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DESIGNING LINEAR LATTICES FOR ROUND BEAM IN ELECTRON STORAGE RINGS USING SLIM*

Yongjun Li†, Robert Rainer, Brookhaven National Laboratory, Upton 11973, New York, USA

Abstract

For some synchrotron light source beamline applications, a round beam is preferable to a flat one. A conventional method of obtaining round beam in an electron storage ring is to shift its tune close to a linear difference resonance. The linearly coupled beam dynamics is analyzed with perturbation theories, which have certain limitations. In this paper, we adopt the Solution by LInear Matrices (SLIM) analysis to calculate exact beam sizes to design round beam lattices. The SLIM analysis can deal with a generally linearly coupled accelerator lattice. The effects of various coupling sources on beam emittances and sizes can be studied within a self-consistent frame. Both the on- and off-resonance schemes to obtain round beams are explained with examples. Commonly used radiator devices, such as planar wigglers and undulators, can be incorporated.

INTRODUCTION

Round beam rather than a flat one is preferable for some beamline applications in the synchrotron light source community. Concurrently, an increased vertical beam size can significantly improve beam lifetime as well, particularly in extremely low emittance rings. Therefore, some future diffraction-limited light sources, such as ALS-U [1] and APS-U [2], are planning to operate with a round beam mode. Most of light source rings only have horizontal bending magnets, which leads to an intrinsically flat beam. Either dedicated devices, such as skew quadrupoles, or some imperfections in magnets, such as normal quadrupole roll errors, can couple the beam motion transversely. Conventionally, a geometric round beam in an electron machine is obtained by: (1) equally distributing the natural horizontal emittance into the horizontal and vertical planes $\epsilon_x = \epsilon_y$ through shifting the machine's tune close to a linear difference resonance $\nu_x - \nu_y = n$, with n an integer, (2) adjusting the envelop Twiss functions so that $\beta_x = \beta_y$ at the locations of radiators. Here we also assume that radiators are located at non-dispersive sections, because achromat lattices are often adopted for light source rings. The beam emittances and sizes for this on-resonance coupling case were often analyzed with perturbation theories, such as [3–5] etc. However, when the linear coupling is sufficiently strong, such perturbation analyses might not be accurate any longer and a more accurate analysis might be considered necessary.

In the presence of linear coupling, the uncoupled 2-dimensional Courant-Synder parameterization [6] can be generalized to the 4-dimensional coupled motion. Such parameterizations, proposed by Ripken and his colleagues [7,8] and further developed by Lebedev and Bogacz [9] are already available. There are also some other exact parameterizations [10–12]. These analyses only deal with linear Hamiltonian systems, the radiation damping and quantum excitation diffusion for electron beams are not considered. Therefore, the equilibrium emittance for electron storage rings has not been derived here. Instead, the following emittance re-distribution approximation [4],

$$\epsilon_x = \frac{1+2k^2}{1+4k^2}\epsilon_{x,0},\ \ \epsilon_y = \frac{2k^2}{1+4k^2}\epsilon_{x,0} \qquad (1)$$

is often used. Here $k = \frac{|\kappa|}{\Delta\nu}$, κ is the well-known coupling coefficient given in ref. [4, 13], $\Delta\nu = \nu_x - \nu_y - p$ is the distance from the resonance, $\epsilon_{x,0}$ is the horizontal emittance for the uncoupled motion, and the natural vertical emittance $\epsilon_{y,0}$ is negligible. Eq. (1) is only valid by assuming: (1) coupling coefficient κ are sufficiently weak to be considered as perturbations, (2) the total transverse emittance remains as a constant, and (3) the coupling is caused by a single isolated resonance, (4) the vertical dispersion is negligible. Exact computations as shown later in this paper indicate that the approximation in Eq. (1) can break down when these assumptions are violated, particularly when vertical dispersion is blown up.

In this paper, to design round beam lattices for light source rings, we adopt an exact and self-consistent analysis – the Solution by LInear Matrices (SLIM) technique, developed by Chao back in the 1970–1980s [14–16]. This analysis can yield fruitful results such as the trajectory of the electron distribution center and the beam sizes and shapes in phase space. Linear coupling effects among the horizontal, vertical, and longitudinal motions are included in a straightforward manner even without introducing the auxiliary Twiss functions. Alternate, and also exact approaches, such as [17, 18] have been implemented in the code SAD [19], AT [20] and OPA [21] which could also be used for this purpose. We used AT and SLIM to compute a same coupled NSLS-II lattice and confirmed that their emittance computations are equivalent.

SLIM AND TWISS FUNCTIONS

The detailed SLIM formalism can be found in the references [14–16, 22]. It deals with the motion of a charged particle in a linear electromagnetic device by purely using their transport matrices. First, symplectic one-turn linear matrices for a storage ring are used to compute the eigenvalues and eigenvectors. The eigenvalues indicate whether the linear motion is stable or not, and provides the fractional parts of the tunes when the motion is stable. The eigenvectors evolving along the ring provide information about closed

* Work supported by US DOE Office of Science operated by BNL under Contract No. DE-SC0012704
† yli@bnl.gov

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orbit, beam size, etc. For electron rings, non-symplectic one-turn matrices including the radiation damping are used to compute the damping rates. Then equilibrium emittances are obtained by balancing the quantum diffusion and radiation damping in all radiating magnets around the ring. The particle distributions within a bunch along the ring can be given with 21 independent second moments. In the presence of linear couplings, no approximation is needed and therefore, the computations are exact. The ring's global emittances and the local s-dependent beam sizes are derived within a self-consistent frame. When there is no linear coupling, two SLIM's second moments $\langle xx \rangle$, $\langle yy \rangle$ were confirmed to agree with the beam sizes obtained with Sands's formalism [23] using the Courant-Snyder parameterization.

No auxiliary Twiss functions are needed in the SLIM analysis. However, it is worth noting are these coupled Twiss functions parameterized with Ripken method [7, 8] are still useful in interpreting the same physics meanings. Given an equilibrium emittances $\epsilon_{x,y}$ and energy spread σ_δ, the beam size along the ring can also be computed with the following formula [24],

$$\sigma_{x,y}^2 = \beta_{I,(x,y)}\epsilon_I + \beta_{II,(x,y)}\epsilon_{II} + \eta_{x,y}^2\sigma_\delta^2. \qquad (2)$$

Using Eq. (2), we can further understand the blow-up of vertical beam size by decomposing it into three components as shown in Fig. 1. When the dispersion is coupled from the horizontal plane to the vertical plane, it generates a considerable amount of mode II emittance ϵ_{II} and introduces local vertical energy oscillation $\eta_y\sigma_\delta$ as well. In the meantime, the coupled $\beta_{I,y}$ function can also generate a contribution $\beta_{I,y}\epsilon_I$. When the vertical dispersion is sufficiently large, even no significant beam size change is observed in the horizontal plane, a new equilibrium state is formed in the vertical plane.

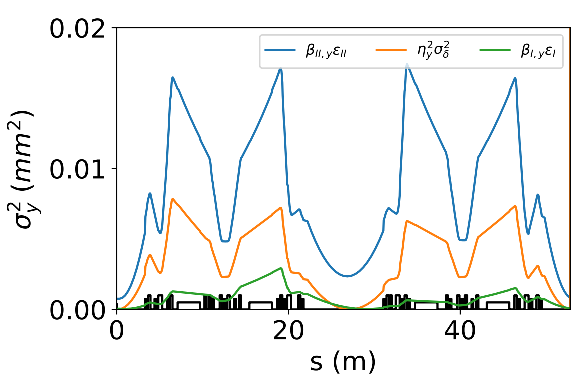

Figure 1: Vertical beam size decomposition.

COUPLING DUE TO RANDOM ERRORS

Linear coupling can be from not only dedicated magnets, such as skew quadrupoles and solenoids, but some random errors. In this section, the effects of two primary sources, the vertical closed orbit through sextupoles and the roll errors of normal quadrupoles, are discussed.

Closed Orbit Errors

In reality, small misalignments and magnet imperfections are unavoidable. Therefore, when circulating beam reaches an equilibrium distribution around a closed orbit, it always differs, at least slightly, from the design orbit. Closed orbit, if it exists, can be accurately obtained by performing an iterative one-turn 6-dimensional tracking till a convergence is reached. This method is widely used in many existing lattice codes, such as ELEGANT [25] and MAD [26]. Alternatively, the SLIM analysis adds a seventh component, which is always a unitary constant 1, to expand one-turn transport matrices to a 7×7 format. The closed orbit corresponds to the eigenvectors with eigenvalues of 1. The nonlinear kicks from nonlinear multipoles can also be accounted for by iteratively updating the transport matrices around the local closed orbit to reach a convergence.

A vertical offset through sextupoles introduces linear coupling (non-zero m_{23} and m_{41}). In modern high brightness light source rings, strong sextupoles are needed to correct chromaticity and enlarge dynamic aperture. Small closed orbit errors might introduce some coupling which can blow up the vertical emittance. When the uncoupled tune is close to the difference resonance $\nu_x - \nu_y = n$, the vertical beam emittance can be easily increased.

Normal Quadrupole Roll Errors

Another linear coupling source is from the random normal quadrupole roll errors. Quadrupoles can be aligned within several hundreds of microradians roll angles using the modern alignment techniques. When the linear tune is sufficiently separated from resonances, even though the total beam emittances are only slightly increased (about 1-2%), a significant part of the transverse emittance can be gradually redistributed to the vertical plane. While the machine's tune is sufficiently close to a difference resonance, even small roll errors can easily couple the transverse motion. This is the most common way to obtain an approximately round beam.

TWO ROUND BEAM SCHEMES

To obtain round beam in an electron storage ring, its vertical emittance needs to be blown up with either dedicated coupling elements (skew quadrupoles, solenoids), or by shifting the machine's tunes close to a difference resonance, and letting random coupling errors (such as quadrupole roll errors) to couple the emittances transversely. Below we quantitatively investigate two schemes.

Round Beam with On-Resonance Tune

Currently, the most common method to obtain round beam is by shifting the machine's tune close to a difference resonance. With this on-resonance scheme, random imperfections are usually sufficient to couple the transverse motion. Because orbit displacements through sextupoles centers can

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be well controlled using the beam-based-alignment technique [27,28], quadrupoles' roll errors are regarded as the primary coupling sources, which are often at a level of several hundred microradians.

Although this on-resonance scheme can be explained with the perturbation theory, we re-investigated it with the exact SLIM analysis. Our design goal is to make the beam to be round at the short straight centers (SSC). First, the local quadrupoles QL_{1-3} there were re-tuned to let the local eigenvectors absolute values to be close (or the coupled $\beta_{(x,y),(I,II)}$ functions to be close if one prefers to use Eq. (2) instead.) Then the quadrupoles in the long straight sections QH_{1-3} were tuned to shift the fractional tune close to a difference resonance. Here we assumed the RMS roll angles for quadrupoles is 500 μrad, which can be easily achieved with the current alignment technique. The beam sizes for one supercell with one specific random seed is illustrated in Fig. 2, which can be used for the beam lifetime estimation. If quadrupoles are aligned accurately with smaller roll angles, eigen-emittances might not be equally distributed by the resonance coupling, i.e., $\epsilon_I > \epsilon_{II}$. In this case, we can tune the local quadrupoles (QL_{1-3}) to make $\beta_{(I,II),x} < \beta_{(I,II),y}$ correspondingly to get a geometric round beam.

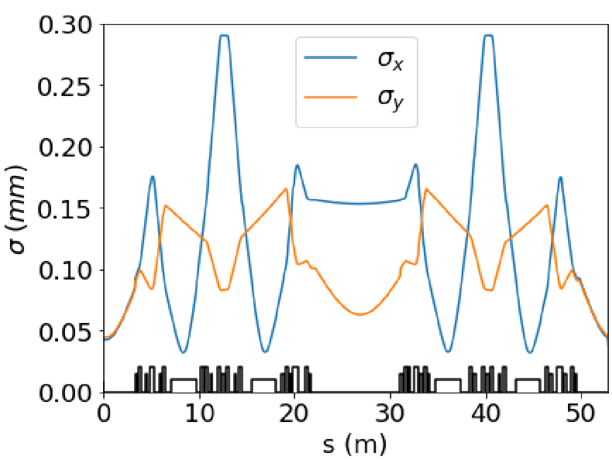

Figure 2: On-resonance scheme: horizontal and vertical beam sizes of one supercell for one specific random seed with 500μrad quadrupole roll errors.

In a realistic situation, the skew quadrupole components due to the vertical offsets through sextupoles need to be taken into account. A simulation shows that the round beam profiles can still be maintained when the closed orbit errors are within a quite wide range. Usually the closed orbit can be well controlled after the beam-based alignment technique is implemented, this error shouldn't be a concern.

Round Beam with Off-Resonance Tune

The second scheme would be to use some strong skew/tilted quadrupoles (or solenoids), which would allow the machine's tune to stay off resonances. In this case, the perturbation theory is not applicable. To obtain round beam at specific locations, the linear lattice design can be summarized as an optimization problem: given normal/skew magnet's focusing strengths K_1, and/or tilt angles ϕ as knobs, to simultaneously minimize eigen-emittances and get same beam transverse sizes at specific locations; subject to the some constraints, such as, keeping the fractional tunes are sufficiently away from low order resonances, etc.

CONCLUSION

Two schemes to obtain round beam, i.e., with machine's tune sitting on- or off- difference resonance, are studied with the SLIM analysis. The on-resonance scheme is easy to implement in a real machine, however, beam profiles, coupled optics functions, and dispersions etc. have quite large and uncontrollable fluctuations. The off-resonance scheme can provide a more controllable and robust round beam, but needs to integrate dedicated magnets into the lattice to generate strong coupling. Some more complicate schemes, such as, a hybrid flat-round beam scheme [29] is being under investigation for a steady-state microbunching [30] ring. A similar idea has been studied using a different method [31] for a diffraction-limited light source ring.

The linear lattice design eventually needs to be optimized iteratively after taking the dynamic aperture and energy acceptance into account, but was not covered in this paper. Other related topics, such as, orbit and linear optics characterization and correction, are different from weakly coupled lattices, which need to considered in machine commissioning and operation. A more details can be founded in [32].

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