ISBN: 978-3-95450-227-1 ISSN: 2673-5490 doi:10.18429/JACoW-IPAC2022-WEPOMS001

EFFECT OF BETATRON COUPLING ON TRANSVERSE MODE-COUPLING AND HEAD-TAIL INSTABILITIES

W. Foosang*, A. Gamelin, R. Nagaoka, Synchrotron SOLEIL, Gif-sur-Yvette, France

Abstract

In the context of SOLEIL Upgrade, the 4th generation storage ring project of SOLEIL, several methods are pursued to extend the beam lifetime and limit the emittance growth by reducing the Touschek effect and intra-beam scattering. Betatron coupling is one of the potential techniques to achieve this objective as it can increase the beam volume by transforming a flat beam into a round beam. However, the effect of the coupling on the collective effects is not fully comprehended, but some studies have shown an improvement in transverse instability thresholds. It was, therefore, crucial to investigate the impact of coupling on beam instability for SOLEIL Upgrade. This work presents numerical studies on the impact of coupling on the transverse mode-coupling and the head-tail instabilities. The results showed that coupling could be not only beneficial, but also detrimental.

INTRODUCTION

SOLEIL is moving towards a 4th generation light source in which beam emittance will be reduced below 100 pm rad [1–3]. In order to obtain this challenging emittance, the vacuum chamber radius needs to be reduced to about 6 mm - approximately half the size of the current one - to accommodate focusing magnets which are required to be as close to the beam as possible. This much smaller aperture will lead to stronger collective effects compared to the present SOLEIL ring. A consequence of the low-emittance design of the upgraded ring is the increased Touschek and intra-beam scattering rates due to the high electron density. For a betatron coupling corrected to 1%, the vertical emittance in the new ring can be as small as 0.8 pm.rad. The full parameter list of the SOLEIL Upgrade CDR lattice can be consulted in the conceptual design report [1].

Betatron coupling is one of the possible methods that can increase the beam volume, thus reducing the electron density, by going from a flat beam to a round beam. The main objective of employing this technique is to reduce the Touschek and intra-beam scattering rates. The smaller scattering rates given by a larger beam volume would extend the beam lifetime and limit the emittance growth. However, betatron coupling impacts several aspects of the beam dynamics and its effect on the collective effects has not been widely explored. Among a few papers on the topic, it has been observed that coupling could reduce instability threshold in some machines such as the CERN PS and SPS [4–6]. It is then of special interest to investigate this effect in the SOLEIL Upgrade case.

This article presents the effect of coupling on two single bunch transverse instabilities: the Transverse Mode-Coupling Instability (TMCI) and the Head-Tail Instabilty (HTI). All the simulations presented were done using the mbtrack2 tracking code [7].

* watanyu.foosang@synchrotron-soleil.fr

BETATRON COUPLING

Betatron coupling can transfer quantities related to betatron oscillation between the two transverse planes. The maximum coupling effect can be achieved when the ring is operated on the linear coupling resonance, which is when the fractional tune of the two transverse planes are equal, with a skew quadrupole component. This is the reason why the tunes were set at 54.2 and 18.2 in this study. The result of full coupling is an equipartition of beam qualities in both planes, this includes emittance and radiation damping times as shown in Fig 1. Note that Courant-Snyder invariant of a single particle is shown in the figure, not beam emittance. They are however interchangeable in case of multi-particle. It can be seen that the coupled Courant-Snyder invariant and the coupled damping time are the average of the non-coupled values. Here and throughout this work, the coupling was introduced by using a skew quadrupole with the integrated normalized strength $k_s l = 0.001\ \mathrm{m}^{-1}$.

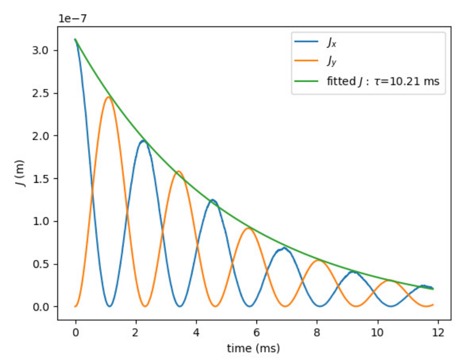

Figure 1: Courant-Snyder invariant of a single particle at full coupling. The non-coupled horizontal and vertical radiation damping times are 7.3 and 13.1 ms, respectively.

EFFECT ON HEAD-TAIL INSTABILITY

Head-tail instability (HTI) is a transverse single bunch instability commonly found in circular accelerators. It occurs in case of nonzero chromaticity and manifests itself in the form of local transverse coherence within a bunch, whose oscillation amplitude could, even though relatively slowly, constantly build up over beam revolutions around the ring. The rise time of HTI is inversely proportional to the transverse effective impedance. The coupling could affect this rise time as it can share the impedance from one plane to another. The plane that has a larger impedance will profit from

ISBN: 978-3-95450-227-1 ISSN: 2673-5490 doi:10.18429/JACoW-IPAC2022-WEPOMS001

this effect as the impedance will be lowered. On the other hand, the plane with a lower impedance will experience a detrimental effect due to the transferred impedance.

The effect of impedance sharing is illustrated in Fig. 2 as the A cases on the left of the plot. In these cases, a broadband resonator with the shunt impedance $R_s = 10\,\mathrm{M\Omega/m}$, the resonance frequency $f_r = 1\,\mathrm{GHz}$, and the quality factor $Q = 10$ was used. These parameters were chosen based on the fact that they give only one unstable head-tail mode (mode -1 in this case) and its analytic growth rate (computed from MOSES [8]) is linear to avoid any possible complexity due to higher order modes and non-linearity. The chromaticities were 1.6 in both planes. The impedance was set only in the vertical plane, and not in the horizontal one. This case can be seen as the one of a machine with highly asymmetric vacuum chamber whose impedance in one plane is much higher than in the other. The synchrotron radiation was also excluded from these two cases in order to demonstrate the pure effect of impedance sharing. It can be seen that, by adding coupling, the rise time in the vertical plane is raised by approximately 50%. Now, there is also a horizontal rise time, which did not exist in case A1, at the same value due to the same partition of impedance that was transferred from the horizontal plane.

In addition to the impedance sharing effect, coupling also results in sharing the damping times of both transverse planes as shown in the B cases of the same figure. In this study, the same impedance described previously for A cases was also set to the horizontal plane to exclude the impact of impedance transfer. Synchrotron radiation was added to the model with the horizontal and vertical damping time of 7.1 and 13.2 ms, respectively. It is worth mentioning that, due to the presence of synchrotron radiation, the rise time shown in this case is composed of the instability rise time τ_i and the radiation damping time τ_r as

$$\tau = \frac{1}{1/\tau_i - 1/\tau_r}\,. \tag{1}$$

Without coupling, the two planes had different rise times according to the different damping times. However, when coupling was added, the rise time of the two planes became equivalent. Given that both planes had the same τ_i due to the same impedance, one can see from Eq. (1) that this result implies an identical τ_r. In this case, the coupled damping time was 10.2 ms which is the average of the non-coupled values. The decreased vertical damping time then resulted in the augmentation of the rise time in the vertical plane.

The effect of impedance sharing and damping time sharing constructively superimpose each other as can be seen in the C case. In this simulation, the horizontal impedance was removed again, but the synchrotron radiation was retained. This configuration allows both aforementioned effects to be effective due to the asymmetry between the two planes. In this setting, the rise time is increased by more than 125% compared to the vertical rise time of the B1 case with the horizontal and the vertical rise time were again equal, relatively.

In conclusion, after these three case studies, it shows that coupling balances the HTI by transferring impedance and damping times from one plane to another, which can lead to increased rise times. The same conclusion should also be true for other weak transverse instability types, for instance resistive-wall instability, whose growth rate is directly proportional to current and does not have a threshold behavior by itself.

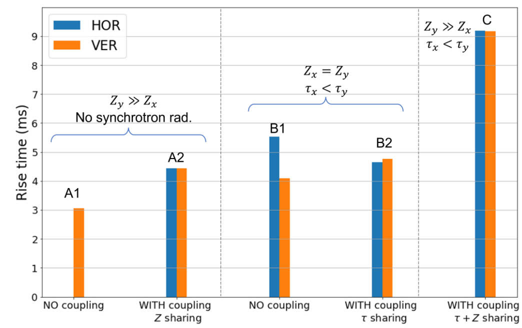

Figure 2: Rise time of head-tail instability (mode -1) at 10 mA under the influence of impedance sharing (A1,A2), damping time sharing (B1,B2), and both combined (C).

EFFECT ON TMCI

Another well-known transverse single bunch instability is the TMCI. Unlike HTI, this instability occurs at zero chromaticity and has a well-defined threshold to emerge. Its rise time is generally much shorter than that of HTI at the onset of the instability. Its threshold is determined by a merging between two adjacent coherent betatron frequency modes, which are orginally separated by one unit of incoherent synchrotron frequency. Each mode frequency can be shifted as the current is increased, but mode 0 is the most susceptible one since its oscillation spectrum overlaps the most with the impedance spectrum at zero chromaticity [9, 10].

The mechanism of TMCI is shown in Fig. 3a. Here, the resistive wall impedance of a circular copper (resistivity $\rho = 1.68 \times 10^{-8}\,\Omega\mathrm{m}$) beam pipe with the length $l = 336\,\mathrm{m}$ and the radius $r = 5\,\mathrm{mm}$ was used in the absence of coupling. The fractional tune was set to 0.2 and the synchrotron tune was 0.0018. The mode 0 frequency was then found at $\nu_x = 0.2$ at zero current and mode ± 1 were therefore at 0.1982 and 0.2018. The instability was seen at 2.35 mA which is where the mode 0 and -1 frequencies coincide.

Coupling was then added to the simulation in case of a highly asymmetric impedance on the coupling resonance ($\nu_x = \nu_y = 0.2$), a shift in the threshold current was observed. This shift was however not in the direction that we had anticipated as the threshold went from 2.35 mA to 2.10 mA. The coherent tune spectrum of the bunch in Fig. 3b allows us to see the possible origin of this surprising result that is the presence of the initial downward shift of mode 0 at zero current. Provided that mode -1 remains immobilized and the shift rate of mode 0 with respect to the current is identical to case (a), this initial shift reduces the distance

ISBN: 978-3-95450-227-1 ISSN: 2673-5490 doi:10.18429/JACoW-IPAC2022-WEPOMS001

between mode 0 and -1. As a consequence, they collide sooner, hence the lower threshold.

Coupling also brings about the appearance of the vertical tune into the horizontal frequency spectrum. This visual enables us to understand that the initial shift of mode 0 is in fact the result of a tune split induced by coupling. The width of the split that is seen here is equal to 5×10^{-4} and is strictly equal to the minimum tune approach $\Delta\nu_{\mathrm{min}}$ [11] which indicates the minimum achievable separation of the transverse tunes. In other words, the transverse tune can never be exactly the same value. $\Delta\nu_{\mathrm{min}}$ is proportional to the skew quadrupole strength k_s, therefore the initial shift will be larger if k_s is increased and the threshold should then be reduced even further.

However, in case of a symmetric impedance with coupling, the threshold was again found at 2.35 mA which is identical to the non-coupled case. The explanation to this finding is not yet clear. In any case, the spectrum in Fig. 3c shows that mode -1 is actually displaced from its supposed location of 0.1982 to slightly below 0.198. In fact, its location is now 0.19795 which means it has been shifted by 2.5×10^{-4} which coincides with half of $\Delta\nu_{\mathrm{min}}$. This can explain the threshold lift-up compared to case (b) despite the same initial tune split, but the reason why mode -1 in case (b) did not experience the shift is still unknown.

For the highly asymmetric impedance case (only Z_x while $Z_y = 0$), coupling could actually yield a beneficial effect, but it was necessary to leave some distance between the two transverse tunes. A scan of TMCI threshold current with respect to the tune separation is shown in Fig. 4. One can see that at the coupling resonance and around this point, the threshold evidently drops from the normal value of 2.35 mA. It was only when $\nu_x - \nu_y = -0.001, 0.002$ that we found a threshold lift-up. A remarkable feature about this result is the asymmetric profile about the zero point. Strictly speaking, not only the tune separation is essential to escalate the threshold, but the sign is also important in order to get the biggest threshold improvement [6, 12].

As expected because the TMCI has a very short rise time past the threshold, adding synchrotron radiation damping to the simulations did not change the TMCI threshold in presence or absence of coupling.

CONCLUSION

It has been shown that betatron coupling can share impedances and synchrotron radiation damping times between the two transverse planes. Concerning the transverse single bunch instabilities, it seems that coupling can be beneficial but not always. For HTI, the overall effect of coupling results in an escalation of the threshold. Both damping time sharing and impedance sharing have a significant contribution in this favorable outcome. Whereas for TMCI, the picture is more complex. On machines with very asymmetric vacuum chambers, coupling could lower the threshold at the coupling resonance and its immediate vicinity. This effect seems to be linked with the minimum tune approach effect.

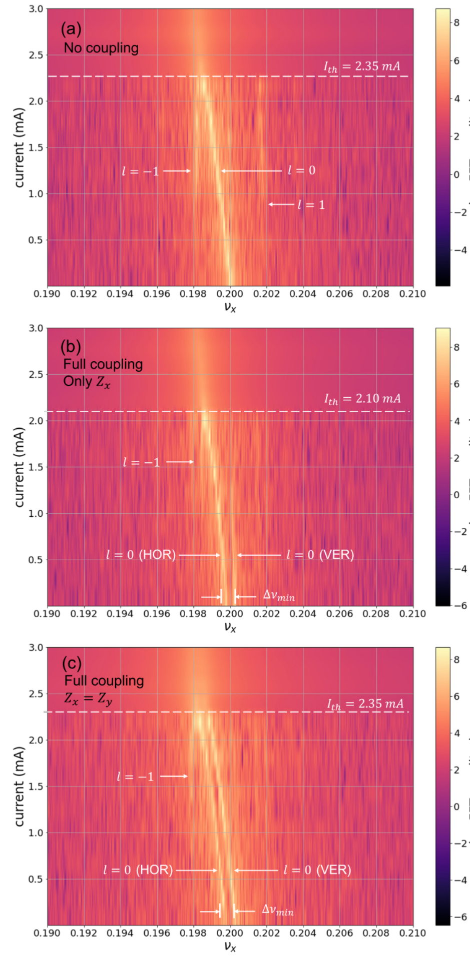

Figure 3: Coherent tune spectrum in the horizontal plane as a function of current showing the evolution of the tune ($l = 0$) in case of (a) no coupling and (b) highly asymmetric impedance, (c) symmetric impedance. The fractional unperturbed tune is 0.2 in both transverse planes.

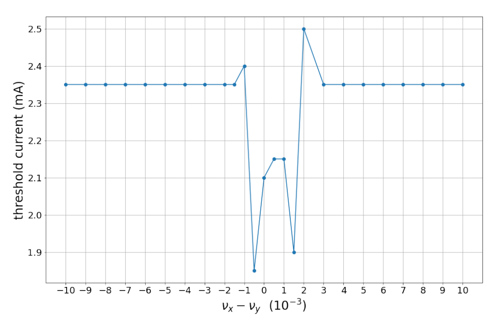

Figure 4: Threshold current as a function of fractional tune difference. The full coupling is when $\nu_x - \nu_y = 0$. The larger the tune difference, the weaker the coupling.

However, at lower coupling strength, which corresponds to larger tune separation values, the threshold is lifted. Further investigation remains to be done to better understand these effects.

ISBN: 978-3-95450-227-1 ISSN: 2673-5490 doi:10.18429/JACoW-IPAC2022-WEPOMS001

REFERENCES

[1] “Conceptual Design Report for SOLEIL Upgrade”, Synchrotron SOLEIL, Gif-sur-Yvette, France. Accessed: May 31, 2022. [Online]. Available: https://www.synchrotron-soleil.fr/fr/file/13803/download?token=0Uzsp46P

[2] A. Loulergue *et al.*, “CDR BASELINE LATTICE FOR THE UPGRADE OF SOLEIL”, in *Proc. IPAC’21*, Campinas, Brazil, May 2021, pp. 1485–1488. doi:10.18429/JACoW-IPAC2021-TUPAB054

[3] A. Loulergue *et al.*, “TDR Baseline Lattice for the SOLEIL Upgrade”, presented at IPAC’22, Bangkok, Thailand, Jun. 2022, paper TUPOMS004, this conference.

[4] E. Métral, “Coupled Landau Damping of Transverse Coherent Instabilities in Particle Accelerators”, Ph.D. thesis, Univ. Joseph Fourier - Grenoble 1, Grenoble, France, 1999. [Online]. Available: https://emetral.web.cern.ch/PHDReport_EliasMetral.pdf

[5] R. Cappi, E. Métral, and D. Möhl, “Transverse Coherent Instabilities in the Presence of Linear Coupling”, in *Proc. EPAC’00*, Vienna, Austria, Jun. 2000, paper WEP4A08, pp. 1155-1157.

[6] E. Métral and G. Rumolo, “Simulation Study on the Beneficial Effect of Linear Coupling for the Transverse Mode Coupling Instability in the CERN Super Proton Synchrotron”, in *Proc. EPAC’06*, Edinburgh, UK, Jun. 2006, paper THPCH058, pp. 2916–2919.

[7] A. Gamelin, W. Foosang, and R. Nagaoka, “mbtrack2, a Collective Effect Library in Python”, in *Proc. IPAC’21*, Campinas, Brazil, May 2021, pp. 282–285. doi:10.18429/JACoW-IPAC2021-MOPAB070

[8] Y. H. Chin, “User’s Guide for New MOSES Version 2.0 (MOde-coupling Single bunch instability in an Electron Storage ring)”, CERN/LEP-TH/88-05, 1988.

[9] D. Amorim, “Study of the transverse mode coupling instability in the CERN Large Hadron Collider”, Ph.D. thesis, Univ. Grenoble Alpes, Grenoble, France, 2019. [Online]. Available: https://tel.archives-ouvertes.fr/tel-02493227

[10] E. Métral, G. H. Hoffstaetter, and F. Willeke, “Destabilising Effect of Linear Coupling in the HERA Proton Ring”, in *Proc. EPAC’02*, Paris, France, Jun. 2002, paper WEPRI045, pp. 1535-1537.

[11] L. R. Carver, X. Buffat, K. Li, E. Métral, and M. Schenk, “Transverse beam instabilities in the presence of linear coupling in the Large Hadron Collider”, in *Phys. Rev. Accel. Beams*, vol. 21, no. 4, pp. 044401, Apr. 2018. doi:10.1103/PhysRevAccelBeams.21.044401

[12] E. Métral, “Effect of bunch length, chromaticity, and linear coupling on the transverse mode-coupling instability due to the electron cloud”, in *Proc. ECLOUD’02*, Geneva, Switzerland, Apr. 2002, pp. 211–218. doi:10.5170/CERN-2002-001.211