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STUDIES OF TRANSVERSE COUPLED-BUNCH INSTABILITIES FROM RESISTIVE-WALL AND CAVITY HIGHER ORDER MODES FOR DIAMOND-II

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Abstract

The transverse coupled-bunch instabilities from resistive-wall impedance and main cavity higher order modes (HOMs) are studied for the Diamond-II storage ring. The growth rates of all the coupled-bunch modes are calculated using both the results from tracking simulations and analytic formula, which show a good consistency. The instability threshold from the resistive-wall impedance is estimated and verified by simulation. The impact of the main cavity HOMs is studied in a similar way, and the results show instabilities from HOMs are much smaller than that from resistive-wall impedance.

INTRODUCTION

Diamond-II is an upgrade for the Diamond Light Source to provide higher brightness and more beamlines for the users. Some of the main parameters of the Diamond-II storage ring can be found in [1].

The coupled-bunch instability is one of the main sources of transverse collective instabilities in storage rings. It is important to know the instability threshold for a storage ring to maintain stable operation. Two sources of transverse coupled-bunch instability, the resistive-wall impedance and the transverse main cavity HOMs, are studied for the Diamond-II storage ring.

GROWTH RATE OF COUPLED-BUNCH MODES

The transverse motion of centroid of bunch n in an evenly filled beam affected by a dipolar long-range wake field is described by [2]

$$\ddot{y_n}(t) + \omega_\beta^2 y_n(t) = A\sum_k \sum_{m=0}^{M-1} W(-kC - \frac{m-n}{M}C) \times y_m(t - kT_0 - \frac{m-n}{M}T_0), \tag{1}$$

where ω_β is the betatron angular frequency, C is the circumference, T_0 is the revolution period, M is the number of bunches, W is the wake function, and A is some constant. The equation has M eigenvalues, each corresponding to a coupled-bunch mode with a certain phase difference between each bunch, the solution for mode μ can be expressed by:

$$y_n^{(\mu)}(t) = \mathrm{Re}(Be^{2\pi i\mu n/M}e^{-i\Omega_\mu t}), \tag{2}$$

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where Ω_μ is the complex frequency of mode μ, and B is some constant. In the case of a small deviation from the betatron frequency, the complex frequency shift can be expressed by [2–4]:

$$\Delta\Omega_\mu = \frac{ecI_0}{4\pi E_0 \nu_\beta}\sum_p Z_{\perp,\mathrm{ELEGANT}}(\omega_p), \tag{3}$$

where c is the speed of light, I_0 the beam current, E_0/e the beam energy in eV, and $Z_{\perp,\mathrm{ELEGANT}}$ the lumped impedance in the ELEGANT convention, which is directly the Fourier transform of the wake function [5]. ν_β should be evaluated by $\nu_\beta = C/2\pi\bar{\beta}$, where $\bar{\beta}$ is already included in the impedance normalised by the local beta function. The impedance is evaluated at the frequencies:

$$\omega_p = (\mu + Mp + \nu_\beta)\omega_0,$$

where μ is the mode index (ranging from 0 to $M-1$), and ν_β is the betatron tune. The summation is made over the index p to get the total contribution to mode μ. The real part of the complex frequency shift gives the tune shift whereas the imaginary part gives the growth rate of the mode.

INSTABILITY FROM RESISTIVE-WALL IMPEDANCE

Due to the narrow vacuum chamber and in-vacuum IDs, it is expected that the resistive-wall impedance will cause the most significant transverse coupled-bunch instability. The vacuum vessel of the Diamond-II storage ring will be mostly composed of a NEG-coated copper circular pipe 10 mm in radius. For the simulations, a thickness of 1 μm has been assumed for the NEG coating with a conductivity of 10^5 S/m as a worst-case scenario. The long-range resistive-wall wake is generated for each element of the Diamond-II storage ring according to the chamber size and shape using the code ImpedanceWake2D [6] to include multilayered pipes with NEG coating. A lumped wake function is calculated by normalisation with the local beta function at each element:

$$W_\perp(t) = \frac{1}{\beta_{0x,y}}\sum_i \beta_{x,y,i}W_i(t), \tag{4}$$

where $\beta_{0x,y}$ represents the local horizontal or vertical beta function at the position where the lumped wake element is located. The step size of long-range resistive-wall wake function is taken to be half the bunch distance, and the wake function decreases monotonically as the time increases. It is estimated that after 1 turn the wake function decreases

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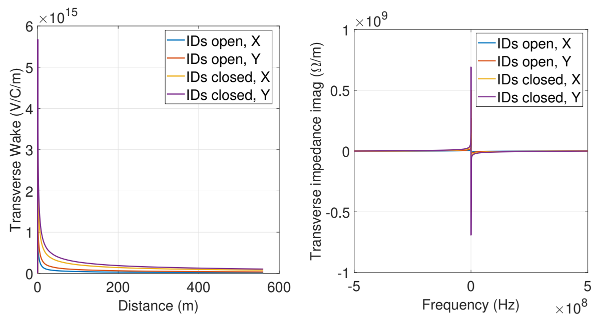
to 2% of the largest value, and after 10 turns it decreases to less than 1% of the largest value. This shows that the coupled-bunch instability is most dependent on the first few turns, and it is sufficient to set the 'turns_to_keep' option to 10 in the LRWAKE element in ELEGANT. When taking the Fourier transform of the long-range wake function, the impedance will be in the range from 0 to the RF frequency, the growth rate of different modes will be calculated by the imaginary part of the impedance according to Eq. (2). The wake function and the imaginary part of the impedance is shown in Fig. 1.



(a) (b)

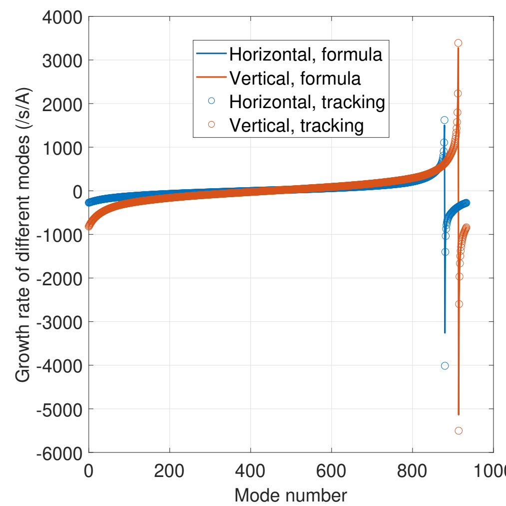
Figure 1: Transverse long-range resistive-wall wake and impedance. (a) The wake function in about one turn, it will continue to decrease afterwards. (b) The imaginary part of impedance in ELEGANT convention. The highest frequency is the RF frequency.

Tracking is carried out in ELEGANT using a linear one-turn map (ILMATRIX) together with the lumped long-range resistive-wall wake (LRWAKE), without cavity and radiation, and in a uniform fill pattern. The tracking simulations are carried out using a single particle to represent each bunch, with 50 mA beam current and 1000 turns, which is sufficient to get enough data for fitting the growth and damping of all the modes before the eigen modes get mixed. A transverse offset is set to the leading bunch to give the same initial amplitude for all the modes. This setting is inspired by the discrete Fourier transform of [1,0,0,…,0] being [1,1,1,…,1], so only the first bunch is given a transverse offset and the centroid of other bunches remain zero. From Eq. (2), a discrete Fourier transform can be performed on the normalised complex action-angle coordinates to view the behaviour of all the modes. Since the simulation is carried out with turn-by-turn tracking, the betatron phase difference between bunches is not included in the tracking result directly, so an additional betatron phase term should be added to the complex coordinates, which leads to a series [7]:

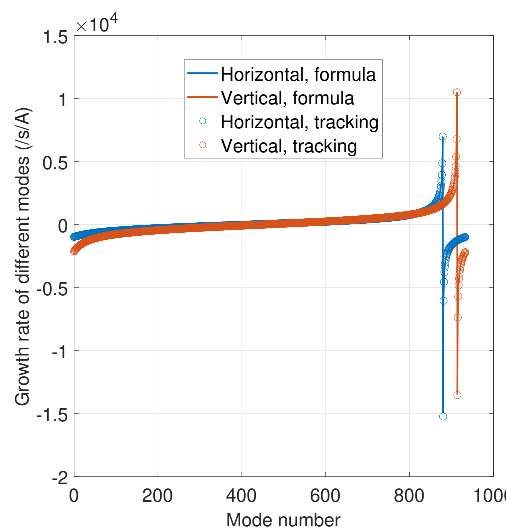
$$z_n = \left(\frac{x_n}{\sqrt{\beta_n}} - i\left(\sqrt{\beta_n}x'_n + \frac{\alpha_n}{\sqrt{\beta_n}}x_n\right)\right)e^{-\frac{2\pi i \nu_\beta (n-1)}{M}}. \quad (5)$$

For every bunch in each turn, n is the bunch index ranging from 1 to M. The minus sign in the betatron phase factor comes from the fact that bunch 1 is the leading one, and bunch M is the last one. If there are multiple particles in each bunch, then the x_n and x'_n will be the centroid of the bunch. After taking the discrete Fourier transform on $\{z_n\}$ every turn, the growth rate of each mode is then fitted by an exponential function and normalized to the beam current at 1A.

Figure 2 shows a comparison between the growth rates obtained from tracking and that from Eq. (3) at zero chromaticity. The growth rates calculated from tracking agree well with the analytic formula. The growth rate of the fastest mode is compared with the radiation damping time to estimate the instability threshold, which is shown in Table 1.



(a)



(b)

Figure 2: Growth rate of different coupled-bunch modes with resistive-wall impedance. (a) The bare lattice. (b) The lattice with all IDs closed.

To verify the estimated beam current threshold, a simulation with 10,000 particles per bunch was carried out in PELEGANT [8] for the lattice with closed IDs, which tracks the beam for 50,000 turns at 10 mA, 20 mA and 30 mA, the synchrotron radiation damping and main cavity is also included. In the horizontal direction, the beam is stable at

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Table 1: The estimated beam current threshold of coupled-bunch instability from resistive-wall impedance at zero chromaticity for a uniformly filled beam. τ_{SR} is the synchrotron radiation damping time. λ_g is the largest growth rate from formula. I_{th} is the estimated beam current threshold. X,Y stands for the horizontal and vertical direction, respectively.

	τ_{SR} (ms)	λ_g (/A/s)	I_{th} (mA)
X, IDs open	9.66	1622	63.8
Y, IDs open	18.1	3390	16.3
X, IDs closed	5.66	6993	25.3
Y, IDs closed	7.78	10505	12.2

10 mA and 20 mA, but unstable at 30 mA. In the vertical direction, the beam is stable at 10 mA, and unstable at 20 mA and 30 mA. The result shows consistency with the estimated beam thresholds given in Table 1.

A similar study was also carried out for the standard filling pattern, which has 5 gaps of 7 buckets. The results show a similar threshold as the uniform filling pattern at 0 chromaticity [1]. Chromaticity can help to stabilise the beam a little. The motion of the first bunch centroid is shown in Fig. 3 using the absolute value of Eq. (5).

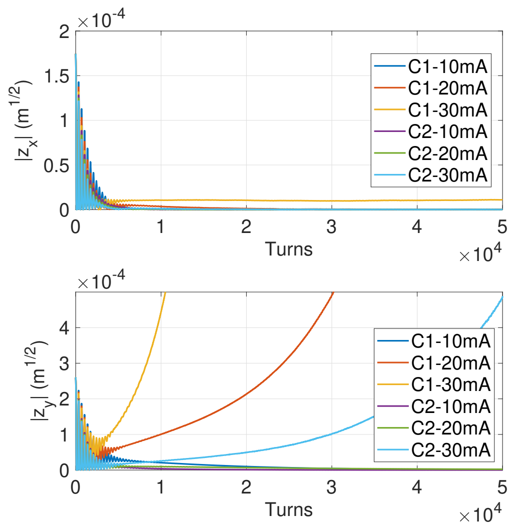

Figure 3: Electron bunch motion as a function of current and chromaticity for the Diamond-II lattice with closed IDs for the horizontal (upper plot) and vertical (lower plot) directions. The chromaticity is set to either 1 (C1) or 2 (C2).

INSTABILITY FROM THE MAIN CAVITY HOMS

The Diamond-II upgrade will use EU HOM-damped normal conducting (NC) cavities [1]. The transverse coupled-bunch instability from the main cavity HOMs is studied in a similar way as the resistive-wall impedance. In simulation, the element FTRFMODE in ELEGANT is used, which contains shunt impedance, resonant frequency and Q factor for different resonator modes. The growth rates for all the modes are calculated from the tracking data. To compare with the analytic formula, the impedance of the transverse HOMs can be calculated by a summation of different resonator modes [2]:

$$Z_\perp(f) = -i\sum_k \frac{f_{r,k}}{f}\frac{R_{s,k}}{1+iQ_k(\frac{f}{f_{r,k}}-\frac{f_{r,k}}{f})}, \qquad (6)$$

where $R_{s,k}$, Q_k, $f_{r,k}$ stands for the shunt impedance, Q factor, and resonant frequency, respectively. The growth rate is calculated by Eq. (3) using the imaginary part of this resonator impedance. A comparison between the tracking results and analytic formula is plotted in Fig. 4. The result shows that the transverse coupled-bunch instability from the main cavity HOMs is much smaller than that from resistive-wall impedance.

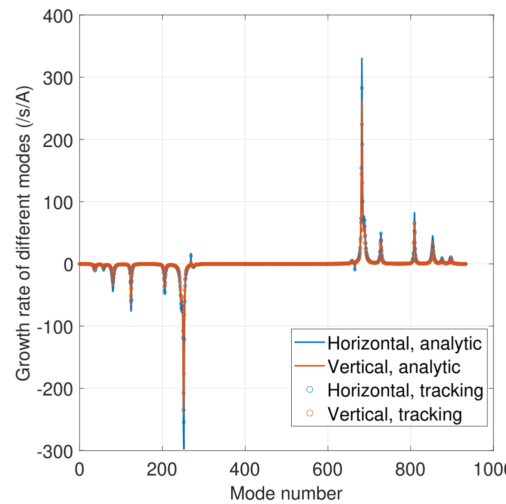

Figure 4: Growth rate of different coupled-bunch modes with main cavity higher order modes.

CONCLUSIONS AND FUTURE WORK

This paper uses a method to calculate the growth rate of all the coupled-bunch modes with only one simulation. The growth rates are compared with the analytic formula and have good consistency. The result shows that the fastest growth rate from resistive-wall impedance is much larger than that from main cavity HOMs. The growth rate of the fastest growing mode of resistive-wall impedance is used to estimate the instability threshold, and is verified by the simulation with multiple particles per bunch.

Studies of beam instabilities in other conditions have begun. Preliminary results show that the beam is stable at 300 mA with harmonic cavity and chromaticity 2 in both horizontal and vertical direction, and a transverse multi-bunch feedback can also effectively stabilize the beam in various conditions. More detailed studies are on-going.

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