ISBN: 978-3-95450-227-1 ISSN: 2673-5490 doi:10.18429/JACoW-IPAC2022-WEPOMS019

BEAM-BEAM RESONANCE WIDTHS IN THE HL-LHC, AND REDUCTION BY PHASING OF INTERACTION POINTS

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Abstract

Beam-beam interactions are a limiting factor in the planned high luminosity (HL) upgrade to the Large Hadron Collider (HL-LHC). Over the two main interaction regions of the LHC, a particle experiences two head-on and many long-range beam-beam interactions which drive betatron resonances in the system. Each resonance line in the space of horizontal and vertical tunes has a finite (non-zero) lock-on width. If particles' tunes fall within this width, they will eventually lock on to the resonance and be driven to large amplitude. We show that it is possible to reduce the resonance widths of a given order by using specific values of the phase advance between interaction points. This paper presents the derivation of resonance width for the weak-strong beam-beam effect, as an extension of A. Chao's width formulae for magnetic sextupoles. (A Lie-algebraic approach is used to combine the effect of the individual beam-beam impulses.) The paper then studies the lock-on width arising from two interaction regions containing 70 beam-beam impulses, and shows the cancellation of specific resonances by relative phasing of interaction points in the HL-LHC lattice.

INTRODUCTION

Beam-beam interactions drive betatron resonances and are a limiting factor for the HL-LHC. These resonances can occur when the vertical and horizontal betatron tunes are related by an integer equation. This is represented by straight lines (Fig. 1)in tune space:

$$m\nu_x + n\nu_y = q \tag{1}$$
$$m\mu_x + n\mu_y = 2\pi q \ , \tag{2}$$

where m, n, and q are integers, and ν, μ are tune and phase advances. Depending on the properties of the interaction(s) driving resonances, some of these lines can represent dangerous resonances with large lock-on widths. However, not all such lines represent an active resonance of the system and not all resonances are dangerous.

The model used in this paper is a 70 impulse Lie algebraic weak-strong model. The impulses are spread over two interaction regions (IR1 and IR5), including both head-on and long-range interactions. The separations and phasing for each bunch was calculated with MadX using the suggested HL-LHC lattice. The details can be found in Ref. [1]. The resonances of this model are analysed using a lock-on width formula. It is shown resonances of any order can be weakened or removed entirely by the relative phasing of interaction points.

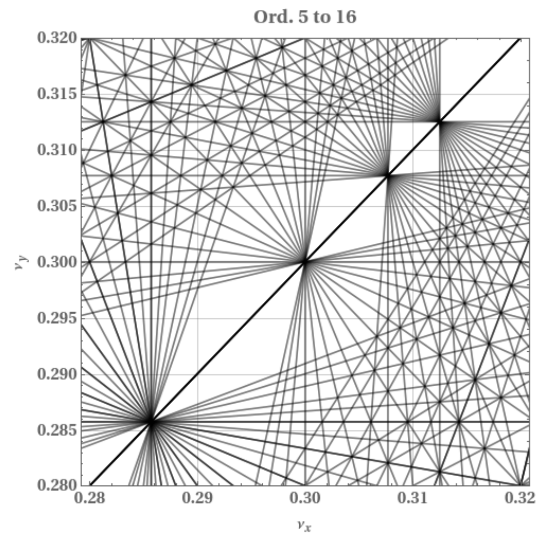

Figure 1: Potential resonance lines of order 5 to 16 near suggested working point (0.31,0.32)

RESONANCE WIDTH

The lock-on width of a one dimensional resonance line (Fig. 2) is defined as the range of tune oscillation amplitude (of a particle close to resonance) within which a particle will eventually lock-on to the precise resonance condition. A particle whose tune is inside the lock-on width will eventually land exactly on the resonant tune. The particle's motion then becomes co-periodic with the driving interaction; this allows the effects of the driving perturbation to accumulate.

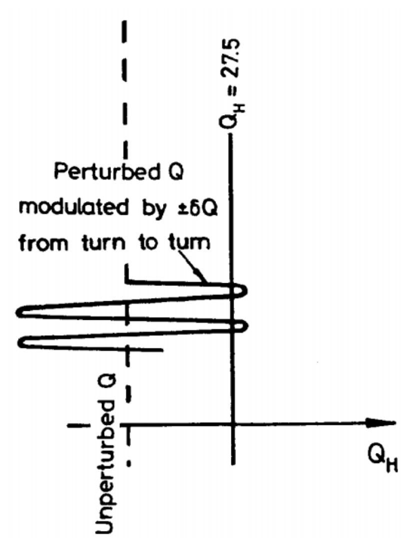

Figure 2: 1D Resonance width in tune space ($Q_H = \nu_x$), (Fig. 5 in Ref. [3])

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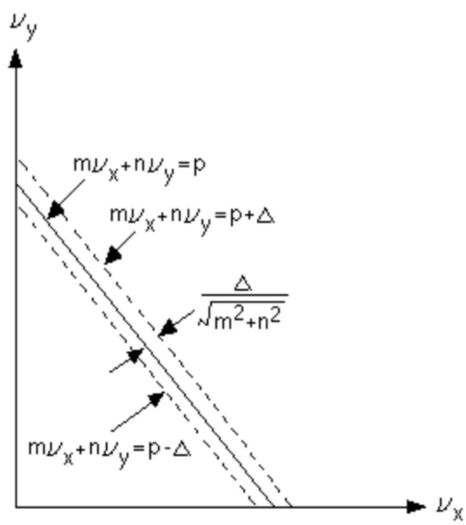

Figure 3: Resonance width in tune space (Fig. 9.7 in Ref. [2])

In two dimensionsline (Fig. 3), one can define an analogous resonance width as Chao did [2]. Repeating his procedure but with beam-beam interactions in mind, consider a one turn map with beam-beam perturbations $e^{-:H_{eff}:} = e^{-:H_2:}e^{-:H_{bb}:}$. We find the effective Hamiltonian to first order can be written as:

$$H_{\text{eff}} = -\mu_x A_x - \mu_y A_y + [\sum_j \sum_{n,m=-\infty}^{\infty} \frac{(m\mu_x + n\mu_y)}{2\sin(m\frac{\mu_x}{2} + n\frac{\mu_y}{2})}] \times [c_{mn}^{(j)} e^{im(\frac{\mu_x}{2}+\phi_x+\Delta\phi_x^{(j)})} e^{in(\frac{\mu_y}{2}+\phi_y+\Delta\phi_y^{(j)})}]. \quad (3)$$

c_{mn}^j are the Fourier coefficients of the j^{th} beam-beam impulse Hamiltonian, which can be computed using the method in Ref. [4]. $\Delta\phi_x^{(j)}$ and $\Delta\phi_y^{(j)}$ are the horizontal and vertical phases of the j^{th} beam-beam impulse.

Let $m\mu_{r_x} + n\mu_{r_y} = 2\pi q$ be a resonance. Consider a system near resonance at frequencies μ_x and μ_y such that $m\mu_x + n\mu_y = 2\pi q + \epsilon$ for some small ϵ. We move to a co-rotating coordinate system,

$$\phi_x' = \phi_x + k\mu_{r_x}, \quad \phi_y' = \phi_y + k\mu_{r_y} \quad (4)$$

where k indexes turn number. In normal form (see Section 9.8 in Ref. [2]) near resonance, one can write $e^{:H_{bb}:}$ as $e^{:H_0+H_r:}$. H_0 is the non-oscillatory part, and H_r is the resonant part. In this coordinate system, in normal form , the effective Hamiltonian takes the form of

$$\begin{aligned} H' =&(\mu_x - \mu_{r_x})A_x + (\mu_y - \mu_{r_y})A_y + H_0 \\ &+ \sum_j c_{mn}^{(j)} \frac{i\epsilon}{1-e^{-i\epsilon}} e^{im\phi_x'+\Delta\phi_x^{(j)}} e^{in\phi_y'+\Delta\phi_y^{(j)}} \\ \approx&(\mu_x - \mu_{r_x})A_x + (\mu_y - \mu_{r_y})A_y + H_0 \\ &+ \sum_j c_{mn}^{(j)} e^{im\phi_x'+\Delta\phi_x^{(j)}} e^{in\phi_y'+\Delta\phi_y^{(j)}} \end{aligned} \quad (5)$$

for small ϵ.

Hamilton's equations can be used to find the change in phase of the reference particle with respect to turn number k, which is the particle tune (in the co-rotating coordinates).

$$\frac{d\phi_x'}{dk} = \mu_x - \mu_{r_x} + \frac{\partial H_0}{\partial A_x} + \sum_j \frac{\partial c_{mn}^{(j)}}{\partial A_x} e^{im\phi_x'+\Delta\phi_x} e^{in\phi_y'+\Delta\phi_y}.$$

$$\frac{d\phi_y'}{dk} = \mu_y - \mu_{r_y} + \frac{\partial H_0}{\partial A_y} + \sum_j \frac{\partial c_{mn}^{(j)}}{\partial A_y} e^{im\phi_x'+\Delta\phi_x} e^{in\phi_y'+\Delta\phi_y}.$$

For a one dimensional resonance, the resonance width is the oscillation amplitude of the tune. For a two dimensional resonance (Fig. 4), we define the resonance width Δ as the oscillation amplitude of the quantity $\frac{1}{\sqrt{m^2+n^2}}(m\frac{d\phi_x'}{dk} + n\frac{d\phi_y'}{dk})$. We note that we have absorbed $\frac{1}{\sqrt{m^2+n^2}}$ into the definition of Δ.

$$\Delta = \sum_j \frac{1}{\sqrt{m^2+n^2}} \left(m\frac{\partial c_{mn}^{(j)}}{\partial A_x} + n\frac{\partial c_{mn}^{(j)}}{\partial A_y} \right) e^{im\Delta\phi_x^{(j)}} e^{in\Delta\phi_y^{(j)}}. \quad (6)$$

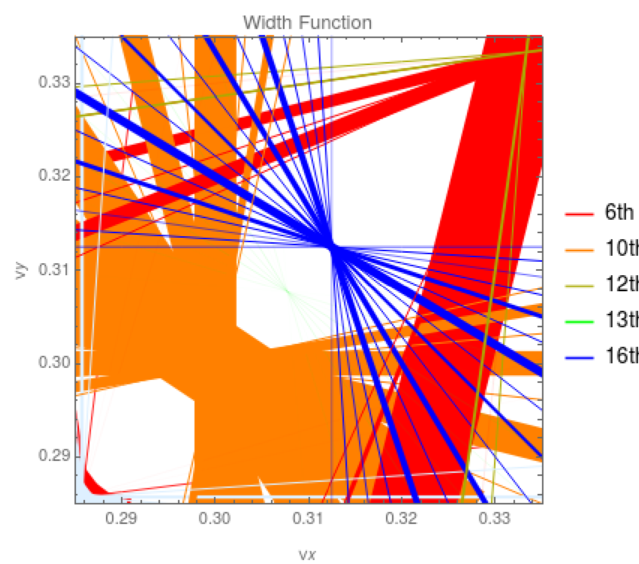

Figure 4: 70 impulse IR1-IR5 model width function Δ with no phasing: order 6(red), 10(orange), 13(green), 16(blue). Fig. 13 in Ref. [1].

RESONANCE WIDTH REDUCTION BY INTERACTION POINT PHASING

At resonance, the effective Hamiltonian (3) is singular; the sine term in the denominator approaches zero as $m\mu_x + n\mu_y$ approaches $2\pi q$ for integers m, n and q. However, for such integers, it is possible to choose values of $\Delta\phi_x, \Delta\phi_y$ such that the effective Hamiltonian is no longer singular. This should in theory remove the resonant behaviour. This was first seen in Ref. [5].

Consider the effective Hamiltonian (3) for a two beam-beam impulse model. Let the first be located at phase $(0, 0)$ and the second one at $(\Delta\phi_x, \Delta\phi_y)$. Since the beam-beam

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potential H_{bb} is a real function, its Fourier coefficients have the following symmetries: $c_{-mn} = c_{mn}$, $c_{m-n} = c_{mn}$.

Assuming that the two impulses are identical except for a phase difference,

$$H_{\text{eff}} = -\mu_x A_x - \mu_y A_y + \sum_{m,n=1}^{\infty} \frac{4c_{mn}}{2\sin\left(\frac{m\mu_x}{2} + \frac{n\mu_y}{2}\right)} \Big(\cos\left[m\left(\frac{\mu_x}{2} + \phi_x\right) + n\left(\frac{\mu_y}{2} + \phi_y\right)\right] + \cos\left[m\left(\frac{\mu_x}{2} + \phi_x + \Delta\phi_x\right)\right) + n\left(\frac{\mu_y}{2} + \phi_y + \Delta\phi_y\right)\right]\Big).$$

Using the sum of cosine identity, this can be rewritten as

$$H_{\text{eff}} = -\mu_x A_y - \mu_y A_y + \sum_{m,n=1}^{\infty} \frac{4c_{mn}}{\sin\left(\frac{m\mu_x}{2} + \frac{n\mu_y}{2}\right)} \Big(\cos\left[m\left(\frac{\mu_x}{2} + \frac{\Delta\phi_x}{2} + \phi_x\right) + n\left(\frac{\mu_y}{2} + \frac{\Delta\phi_y}{2} + \phi_y\right)\right] \times \cos\left[m\frac{\Delta\phi_x}{2} + n\frac{\Delta\phi_y}{2}\right]\Big).$$

Given the tunes, and for any m, n such that $\sin\left(\frac{m\mu_x}{2} + \frac{n\mu_y}{2}\right) = 0$, it suffices to find $\Delta\phi_x, \Delta\phi_y$ such that $\cos\left(\frac{m\Delta\phi_x}{2} + \frac{n\Delta\phi_y}{2}\right) = 0$. Therefore, if $m\mu_x + n\mu_y = 2\pi q$ is a resonance, then choosing $\Delta\phi_x, \Delta\phi_y$ such that $m\Delta\phi_x + n\Delta\phi_y = \pi p$ will cancel it for odd p.

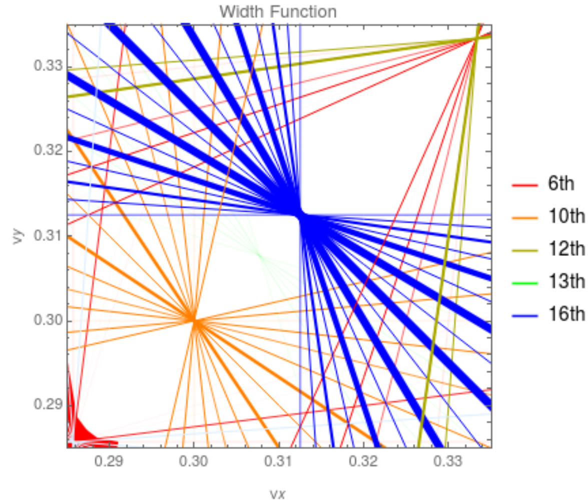
For a resonance line of order $m + n = k$, a phase shift between IP of $\Delta\phi_x = \frac{\pi}{k} = \Delta\phi_y$ satisfies the resonance canceling condition.

In our model with 70 impulses in two interaction regions, each impulse in the first region must be phased appropriately with an impulse in the second region. Furthermore, the Fourier coefficients of these pairs are sufficiently symmetric. The 16th order resonances lie close to the suggested working point of (0.31, 0.32). The width of these resonances can be reduced by applying an additional phase shift of $2\pi \times (-0.17535)$ in $\Delta\phi_x$ and $2\pi \times (-0.33835)$ in $\Delta\phi_y$ to the standard HL-LHC bunch phasing, bringing the the phase advance from IP to IP to $2n\pi + \frac{\pi}{16}$. See Fig. 16 in Ref. [1].

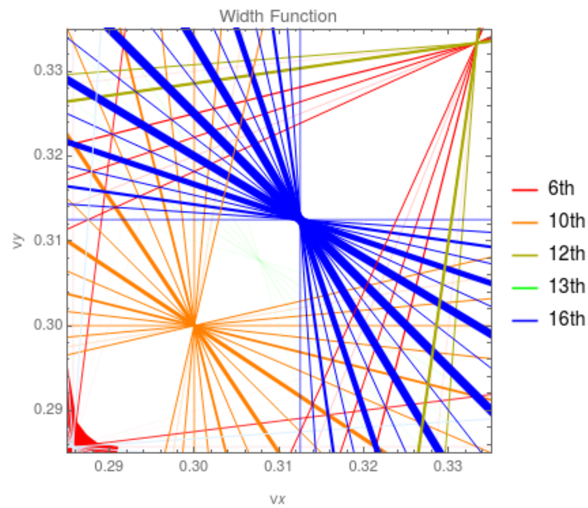
For 10th order resonances, one finds that a phase advance of $\frac{\pi}{2}$ offers cancellation. This phasing needs not be exact (Fig. 5). One finds that being within $2\pi \times 10^{-3}$ offers reasonable resonance width reduction.

CONCLUSION

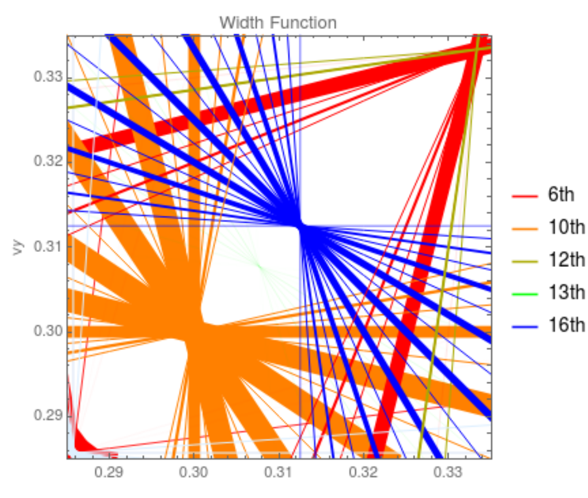
The resonances driven by beam-beam interactions are a limiting factor for the High Luminosity LHC. By examining the lock-on width of resonances, it is possible to identify whether a suggested working point is in a dangerous region in tune space. Furthermore, it is possible to reduce the width of resonances of any order via an appropriate phasing.



(a) Resonances order 5-16 at $(32.25 + 10^{-4}) \times 2\pi$ phasing



(b) Resonances order 5-16 at $(32.25 + 10^{-3}) \times 2\pi$ phasing



(c) Resonances order 5-16 at $(32.25 + 10^{-2}) \times 2\pi$ phasing

Figure 5: 70 impulse IR1-IR5 model 10th order (orange) resonance cancellation by the phasing of 2 IRs: 6th order(red), 10th order(orange), 13th order(green), 16th order(blue). Fig. G25 in Ref. [1]

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