ISBN: 978-3-95450-227-1 ISSN: 2673-5490 doi:10.18429/JACoW-IPAC2022-WEPOMS039

ANALYSIS OF Xcos SIMULATION MODEL FOR INTENSITY AT THIRD AND FIFTH HARMONICS UNDULATOR RADIATION

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Abstract

Xcos simulation model is analysed for the intensity of planar undulator radiation at the third and fifth harmonics. The Xcos model is designed by using the numerical approach. The results obtained from the simulation model are compared with the analytical method. The model can also be utilized for observing the effect of energy spread on radiation by numerical approach.

INTRODUCTION

Free electron laser (FEL) is a coherent and tuneable light source available with high brightness up to order of 10^{13} [1]. In FEL systems, a relativistic electron beam is undulated in presence of periodic magnetic field i.e. undulator, to bring out a coherent electromagnetic radiation. The spectral properties of out-coming coherent electromagnetic radiation and the transfer of energy from electron beam to radiation and vice versa depends on the trajectory of electrons in an undulator [2-4]. Various simulation software like Genesis, EURA, SPECTRA are used for realization of FEL systems. SCILAB is an open-source numerical computation software [5] and tool boxes available with SCILAB, are used for model-based simulation of FEL systems [6-7].

The present work corresponds to develop model based SCILAB Xcos simulation model for numerical solutions of electron trajectory equations and radiation intensity equation in Harmonic undulator. The intensity at third and fifth harmonics of harmonic undulator radiation has been computed by varying the contributions of additional harmonic field component. In this analysis, the results are compared with the reported analytical results obtained from the method of generalized Bessel Functions. The present work will be helpful to find the intensity and small signal gain of harmonics in novel scheme by numerical method.

HARMONIC UNDULATOR FIELD AND TRAJECTOTRY

Harmonic undulator magnetic field considered for simulation is

$$\overrightarrow{B_H} = [0, B_y, 0] \tag{1}$$

Where, $B_y = A_0 B_0 \sin(k_u z) + A_1 B_0 \sin(k_h z)$

With $k_u = {2\pi}/{\lambda_u}$ and $k_h = {2\pi}/{\lambda_h}$ are wave number and harmonic wave number respectively, λ_u and $\lambda_h = h\lambda_u$ are undulator wave length and harmonic undulator wave length respectively. The value of h can be 3,5,7…. corresponding to odd harmonics. A_0 and A_1 are the amplitudes of main undulator field and additional harmonic field respectively and also the ratio of additional harmonic field to main undulator field is represented as 'Δ'.

The acceleration along 'x' and 'z' directions are deduced by using Lorentz force on electron [2-4]

$$\ddot{x} = \frac{eA_0B_o}{m_0\, c\gamma} \mathrm{v}_z [\sin(\Omega_u t) + \Delta \sin(h\Omega_u t)] \tag{2}$$

$$\ddot{z} = -\frac{eA_0B_o}{m_0\, c\gamma} \mathrm{v}_x [\sin(\Omega_u t) + \Delta \sin(h\Omega_u t)] \tag{3}$$

The velocity and trajectories of relativistic free electrons along 'x' and 'z' direction can be evaluated by integrating Eq. 2 and 3 and read as,

$$\beta_x = -\frac{K}{\gamma}\left[\cos(\Omega_u t) + \frac{\Delta}{h}\cos(h\Omega_u t)\right] \tag{4}$$

$$\beta_z = \beta_* - \frac{K^2}{4\gamma^2} cos(2\Omega_u t) - \frac{\Delta^2 K^2}{4h^2\gamma^2}\cos(2h\Omega_u t) - \frac{\Delta K^2}{2\gamma^2 h}\cos((1+h)\Omega_u t) - \frac{\Delta K^2}{2\gamma^2 h}\cos((1-h)\Omega_u t) \tag{5}$$

$$x = -\frac{K}{\gamma\Omega_u}\left[\sin(\Omega_u t) + \frac{\Delta}{h}\sin(h\Omega_u t)\right] \tag{6}$$

$$z = \beta_* t - \frac{K^2}{8\gamma^2\Omega_u}\sin(2\Omega_u t) - \frac{\Delta^2 K^2}{8h^3\gamma^2\Omega_u}\sin(2h\Omega_u t) - \frac{\Delta K^2}{2\gamma^2 h\Omega_u(1+h)}\sin((1+h)\Omega_u t) - \frac{\Delta K^2}{2\gamma^2 h\Omega_u(1-h)}\sin((1-h)\Omega_u t) \tag{7}$$

Where, $\Omega_u = k_u c$ is undulator frequency, $K = \frac{eA_0B_o}{m_0\, c\Omega_u}$ is undulator parameter and $\beta_* = 1 - \frac{1}{2\gamma^2}\left(1 + \frac{K^2 + K^2\frac{\Delta^2}{h^2}}{2}\right)$

Initial values of the velocity and trajectories are

$$x_{\,t=0} = 0\,, \qquad z_{\,t=0} = 0$$

$$\beta_{x\,,t=0} = -\frac{K}{\gamma}\left(1 + \frac{\Delta}{h}\right) \text{ and}$$

$$\beta_{z\,,t=0} = \beta_* - \frac{K^2}{4\gamma^2} - \frac{\Delta^2 K^2}{4h^2\gamma^2} - \frac{\Delta K^2}{\gamma^2 h}$$

For numerical calculation of trajectories, with $z = \beta_* t$ Eq. 2 and 3 and rewritten as,

$$\ddot{x} = \frac{K}{\gamma}\Omega_u \mathrm{v}_z [\sin(\Omega_u\, \beta_* t) + \Delta \sin(h\Omega_u\, \beta_* t)] \tag{8}$$

$$\ddot{z} = -\frac{K}{\gamma}\Omega_u \mathrm{v}_x [\sin(\Omega_u\, \beta_* t) + \Delta \sin(h\Omega_u\, \beta_* t)] \tag{9}$$

The Intensity of radiation 'I' of electromagnetic radiation per unit solid angle '$d\Omega$' per unit frequency range '$d\omega$' of frequency 'ω' from accelerating electron of charge 'e' given by Lienard - Wiechart integral and read as [8],

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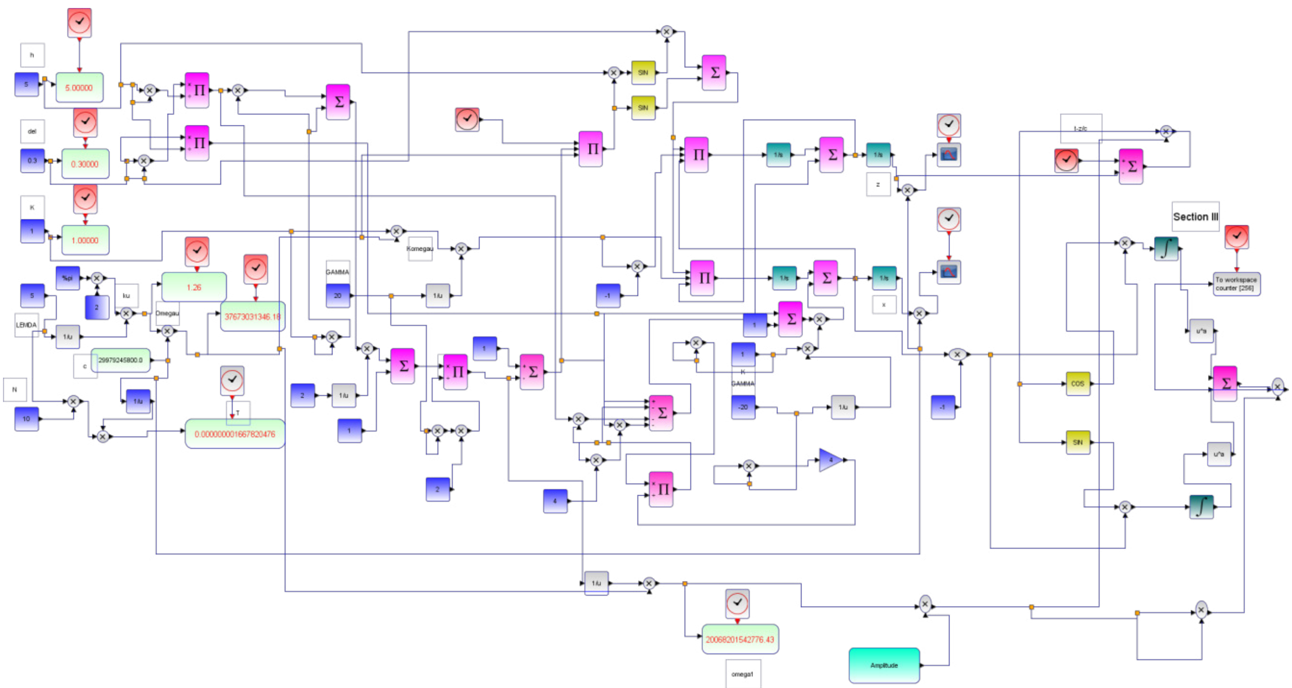

Figure 1: Xcos model for harmonic undulator radiation based on numerical approach.

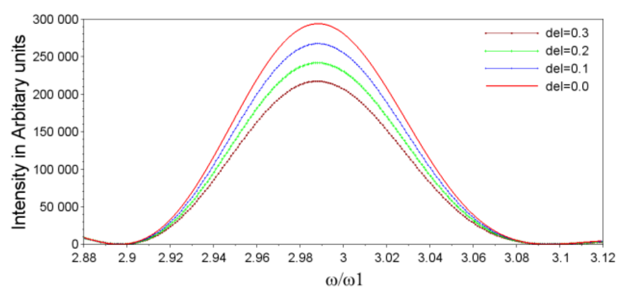

Figure 2: Intensity at third harmonics *(h=3)*

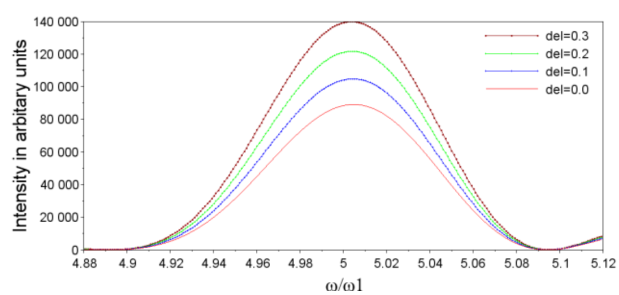

Figure 3: Intensity at fifth harmonics (*h*=5)

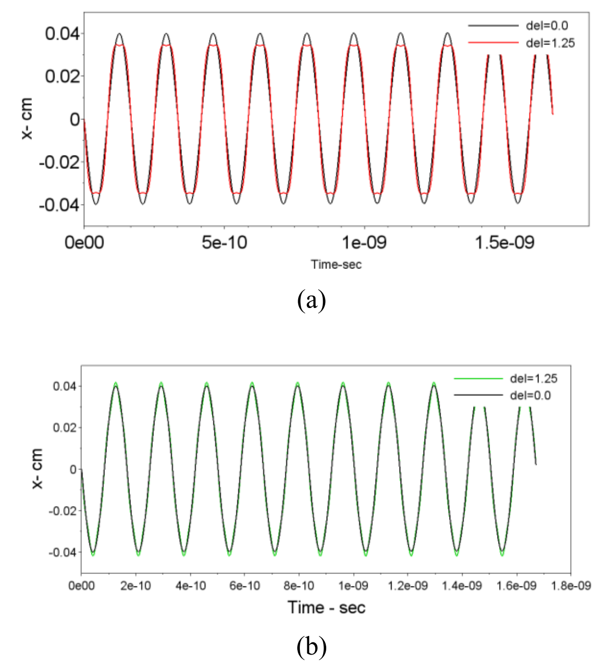

Figure 4: Trajectory of electron in ‘x’ direction in (a) third harmonic undulator and (b) fifth harmonic undulator

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$$\frac{d^2I}{d\omega d\Omega} = \frac{e^2\omega^2}{4\pi^2 c}\left|\int_{-\infty}^{\infty}\{\hat{n}\times(\hat{n}\times\hat{\beta}\}exp\left[i\omega(t-\frac{z}{c})\right]dt\right|^2 \quad (10)$$

Where, $\hat{n}$ is the unit vector in the direction of the line joining the retarded position and the point of observation. For determining the intensity of the on-axis radiation, Eq. 10 can be rewritten as:

$$\frac{d^2I}{d\omega d\Omega} = \frac{e^2\omega^2}{4\pi^2 c}\left|\int_{0}^{T}\{\widehat{\beta_x}\}exp\left[i\omega(t-\frac{z}{c})\right]dt\right|^2 \quad (11)$$

SIMULATION MODEL FOR HARMONICS

The present developed model for simulation is shown in Fig. 1 using the various blocks included in the Xcos toolboxes is an extension of work reported for intensity calculations of plannar undulator radiation by H. jeevakhan *et al* [7]. This model also consists three sections; the primary parameters along with harmonic integrer'h' and 'Δ' , that is the contribution of additional harmonic field are described in section one , section two realizes the electron's trajectory and velocity in the 'x' and 'z' directions in harmonic undulator with modified intial values as per Eq. 4 and 5, and section three integrates the output of section two, to compute the intensity of harmonic undulator radiation at third and fifth harmonics.

For present simulation, the undulator wavelength is taken as 5 cm, a short undulator is considered with number of periods (N)= 10 and relativistic parameter γ is taken as 20. The values of the velocity of electromagnetic radiation in free space is taken in the CGS unit. The total time for traversing the electron in the undulator is T decides the final integration time and steps of integration of simulation. The values of harmonic integer 'h' is selected as 3 or 5 for determining the intensity of radiation at third and fifth harmonics respectively. The value of 'Δ' in present simulation model has been changed from 0 to 0.3 with an interval of 0.1, to analyses the effect of additional harmonic field component on intensities of odd harmonics.

In section two, interdependent Eq. 8 and 9 are realised and executed. The initial values of trajectories along the 'x' and 'z' directions is taken as (0,0), which gives the intensity of on- axis radiation. The blocks for the initial value velocity along 'x' direction and 'z' as per Eq. 4 and 5 are added. The blocks for β_* is modified as per Eq. 5.

Section three, finally executes Eq. 11 and represent the numerical model for on axis harmonic undulator radiation. In numerical calculations, the values of β_x and 'z' is calculated at each instant of time and inserted in blocks realizing Eq. 11. The intensity of radiation for a particular value of "ω" is being computed in each simulation. The values of "ω" is modified by changing the value of 'h', and hence calculate the intensity at particular harmonics.

RESULTS

The simulation model described in previous section is simulated with final integration time 'T' (decided by the total number of periods) for various values of "ω". The data of intensity for different values of "ω" is stored in workspace. The Intensity v/s normalized frequency (ω/ω_1) graph is plotted though plot command on console. In the present analyses, for *h=3*, the range of "(ω/ω_1)" is kept nearby 3, to observe the effect of additional harmonic field at third harmonics and similarly kept as 5 in case of *h=5* for fifth harmonic.

Planar undulator radiation is observed at odd harmonics i.e.1,3,5,7.. and the intensity is decreased as we move from fundamental to higher harmonics. In harmonic undulators, the intensity at odd harmonics is altered with addition of harmonic component. In the present model based on numerical approach by selecting the value of *h=3* , the effect at third harmonic intensity is observed. The contribution of additional harmonic field is changed by the changing the value of 'Δ'. Fig. 2 demonstrates the intensity variation at third harmonics. The value of Δ is varied from 0 to 0.3 with an interval of 0.1. It is indicated from Fig. 2 that with the additional with harmonic component the intensity at third harmonic (calculated from numerical approach), is decreased, which is contradiction with the analytical results based on use of generalised Bessel function [2-4].

By selecting the *h=5* and varying the value of Δ from 0 to 0.3, the intensity of radiation at fifth harmonic is affected. Fig. 3 demonstrates that at fifth harmonic with increase in contribution of harmonic component, the intensity, at fifth harmonic is increased, which is correspondence with the analytical results based on use of generalised Bessel function. The intensity at fifth harmonic increases up to 55 % by changing the contribution of additional harmonic component by 30 % i.e. Δ equal to 0.3.

The effect of additional harmonic field on the intensities at third and fifth harmonics is further analysed with pattern of the trajectories. Fig. 4 (a) and (b) gives the trajectories in 'x' direction, when the additional harmonic component is 3 and 5 respectively. In planar undulator field the trajectory along 'x' direction is sinusoidal. With the addition of harmonic field, the trajectory deviates from sinusoidal nature. To demonstrate the effect, the value of additional component Δ is kept as 1.5. The effect on the trajectories is due to the superposition of main undulator field with harmonic field. The peak of the trajectory decreases, when the main undulator field superposes with the third harmonic field and its visible in Fig. 4 (a), that there is a dip at the peak (red line) in third harmonic undulator. In fifth harmonic undulator, the trajectory also deviates from sinusoidal form, but due to superposition of fifth harmonic component with main undulator field, there is sharp increase in the peak of trajectories (green line) as shown in Fig. 4 (b). These increases and decrease in trajectories, over all alters the peak of intensities at corresponding harmonics.

CONCLUSIONS

SCILAB Xcos simulation model is used to calculate the intensities of harmonic undulator radiation. The results of numerical method are compared with the analytical methods. The reason for this deviation also explained in the paper. The model described in the paper will be helpful for higher harmonic analysis of novel undulator schemes. This model can also be utilised for effect of energy spread using numerical approach by changing the relativistic parameter as per the energy distribution of electron beam.

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