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SUMMARY OF NUMERICAL NOISE STUDIES FOR ELECTRON-ION COLLIDER STRONG-STRONG BEAM-BEAM SIMULATION *

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Abstract

The Electron-Ion Collider (EIC) presently under construction at Brookhaven National Laboratory will collide polarized high energy electron beams with hadron beams with design luminosities up to $1\times10^{34}\mathrm{cm}^{-2}\mathrm{s}^{-1}$ in center mass energy range of 20-140 GeV. We studied the planned electron-proton collisions using a Particle-In-Cell (PIC) based Poisson solver in strong-strong beam-beam simulation. We observed a much larger proton emittance growth rate than in weak-strong simulation. To understand the numerical noise and its impact on strong-strong simulation results, we carried out extensive studies to identify all possible causes for artificial emittance growth and quantify their contributions. In this article, we summarize our study activities and findings. This work will help us better understand the simulated emittance growth and the limits of the PIC based strong-strong beam-beam simulation.

INTRODUCTION

The Electron-Ion Collider (EIC) presently under construction at Brookhaven National Laboratory will collide polarized high energy electron beams with hadron beams with design liminosities up to $1 \times 10^{34}\mathrm{cm}^{-2}\mathrm{s}^{-1}$ [1]. Both weak-strong and strong-strong models are used for the EIC beam-beam interaction simulation studies [2, 3]. For the weak-strong model, the electron bunch is assumed as a rigid 6-d Gaussian charge distribution. The proton bunch is represented with macro-particles. The beam-beam kick to proton macro-particles are calculated analytically. For the strong-strong model, each bunch is represented with typically 0.5 to 1 million macro-particles. Particle-in-cell (PIC) method and Fast Fourier Transformation (FFT) are used to solve 2-d Poisson equation on rectangle grids. The charge of each macro-particle is deposited onto nearest 9 grids and the beam-beam kick to each macro-particle is interpolated from the potentials on those 9 nearest grids.

In the following, we present simulation results for the collision between 275 GeV protons and 10 GeV electrons [4]. At this collision mode, the horizontal and vertical beam sizes are 95 μm and 8.5 μm at IP. The beam-beam parameter is 0.012 for the protons and 0.1 for the electrons, both reach their highest values in the EIC. We pay more attention on the proton bunch's emittance growth, especially in the vertical plane. We normally extrapolate beam size growth rate from a linear fitting in the tracking turns to %/hour. From strong-strong simulation, the simulated proton's vertical beam size growth rate is normally larger than 1000%/hour. However, from weak-strong simulation, it is less than 5%/hour.

Strong-strong simulation is subject to numerical noises due to limited macro-particle numbers, transverse grids, longitudinal slices, and the algorithm itself. To better understand the simulated growth rates and the numerical noises in the EIC strong-strong beam-beam simulation, we carried out extensive studies to identify all possible causes for the artificial emittance growth and quantify their contributions.

CONVERGING STUDY

Analytically, the numerical noise introduced artificial emittance growth rate in the strong-strong beam-beam simulation is inversely proportional to the number of macro-particles and proportional to the square of the beam-beam parameter. Figure 1 shows one example of simulated proton's vertical beam size growth rate versus the number of macro-particles of electron bunch. Increasing the macro-particles of electron bunch will reduce the simulated proton's emittance growth rate. If fitting the simulated growth rates with a function a/M_e^p, where M_e is the number of macro-particles of electron bunch, we found p is close 0.5 instead of 1 as predicted. We also found that fitting with a function $a/M_e + b$ can better match the simulation results. For the CDR design parameters, b is about 300%/hour.

Figure 2 shows the simulated proton beam size growth rates as function of proton's beam-beam parameter. In this study, we adjusted proton's beam-beam parameter by scaling down electron beam's bunch intensity while keeping the tunes of proton bunch center unchanged. The simulated proton's beam size growth rates can be fitted well with a function $a \times \xi_{bb}^q$, with q about 2.75 for both planes. With a lower beam-beam parameter about 0.008, we re-scanned the growth rate's dependence on the electron bunch's macro-particle number and obtained b between 50-100%/hour.

Sufficient number of longitudinal slices of electron bunch in strong-strong simulation is also important to reduce the

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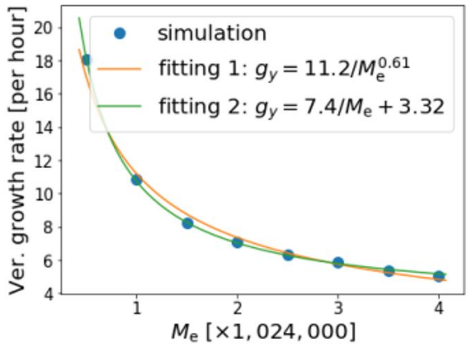

Figure 1: Simulated growth rate of proton vertical beam size versus number of macro-particles of electron bunch.

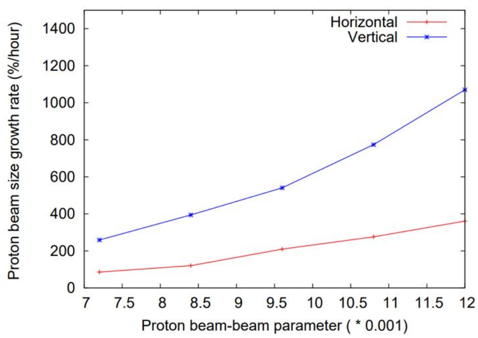

Figure 2: Simulated growth rate of proton transverse beam size versus proton's beam-beam parameter.

simulated proton beam size growth rate. For EIC, due to electron beam's large beam-beam parameter and near-integer tunes, its vertical beam size vaies dramatically near IP.

We normally use 128×128 transverse grids for PIC Poisson solver. With 256×256 grids, the simulated proton growth rates will be smaller too. However, with increased grids, we need to increase the number of macro-particles at the same time and the tracking time will be significantly increased.

TURN-BY-TURN DIPOLE MOTION

In strong-strong model, the distribution of macro-particles varies turn by turn (TBT). With typically 0.5 to 1 million macro-particles per bunch, the RMS variations of electron bunch center and transverse beam sizes are about 0.2-0.3% of their RMS beam sizes at IP.

In the spectrum of proton bunch's center motion, the electron's imprints at electron's tunes are clearly seen. The amplitudes of electron tune peaks scale with $1/\sqrt{M_e}$. The 'noise' of other frequencies in the spectrum also drops with increased macro-particle number but does not scale as $1/\sqrt{M_e}$.

The impact of dipole moment of electron bunch can be eliminated by introducing virtual symmetric macro-particles in the PIC Poisson solver. For example, for one macro-particle at (x, y), we can introduce virtual macro-particles at $(-x, -y)$, or at $(x, -y)$ and $(-x, -y)$. Strong-strong simulation shows that this approach can eliminate the dipole moment and moderately reduce the proton's beam size growth rate. The growth rates with this method are comparable to directly increasing electron bunch's macro-particle number by a same amount of virtual macro-particles.

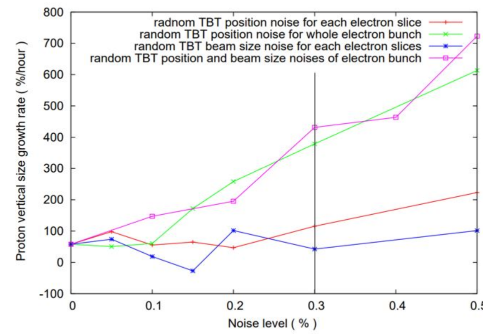

Figure 3: Simulated growth rates from weak-strong simulation with TBT noises added to the electron bunch's centroid and beam sizes.

Knowing the levels of RMS TBT variations in electron bunch's center position and beam sizes, we could estimate their contributions by introducing the same levels of artificial random noises into weak-strong simulation. Figure 3 shows the simulated proton's vertical beam size growth rates with different levels of random noises. With a typical relative 0.3% TBT variations in the weak-strong simulation, the proton vertical beam size growth rate due to the noises is between 100-400%hour.

From above studies, we learned that the TBT variation in the electron bunch's center position can generate artificial proton beam size growth. However, it can not explain majority of the simulated proton beam size growth we observed in the strong-strong simulation which is typically more than 1000%/hour.

CONTRIBUTION TO GROWTH

To identify the sources of proton beam size growth in the strong-strong simulation, we study the dependence of beam size growth rate on the amplitudes of macro-particles [5]. We group macro-particles according to their initial longitudinal action and transverse action in units of σ. We use calculated geometric beam size growth rate as an observable to measure stability of each group of macro-particles.

Figure 4 shows the proton's vertical beam size growth rate as function of longitudinal amplitude of macro-particles. Three models are used. Frozen model is an extended weak-strong simulation after a strong-strong simulation with electron's charge distribution frozen. From the plot, the vertical beam size growth rate increases with the longitudinal action for all models. However, the strong-strong model gives a much faster growth rate than other two models and its beam size growth starts at a very small longitudinal action.

Figure 5 shows the vertical beam size growth rate as function of transverse amplitude of macro-particles. From the plot, the growth rates from strong-strong model are much larger than other two models. Strangely, the macro-particles

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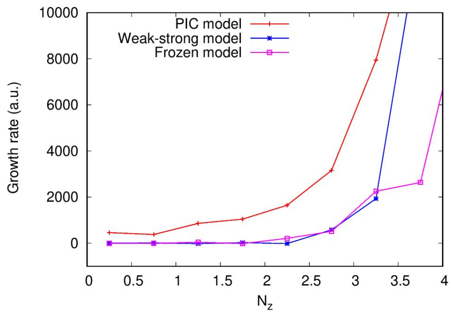

Figure 4: Comparison of growth rates of macro-particles versus their longitudinal actions for three models.

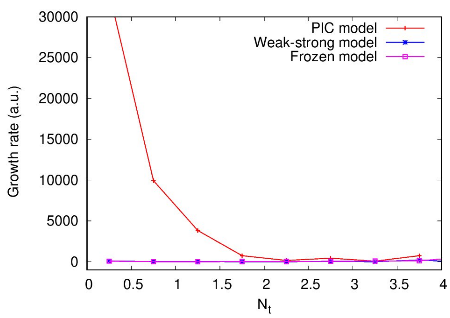

Figure 5: Comparison of growth rates of macro-particles versus their transverse actions for three models.

with a transverse amplitude less than 1.5σ has a very big growth rate in the strong-strong model, which is not seen in the weak-strong models. This means that macro-particles in the transverse bunch core contribute most of the artificial emittance growth rate in the strong-strong beam-beam simulation.

NOISE FROM PIC SOLVER

In the strong-strong simulation, the beam-beam kick is calculated through PIC based Poisson solver. Here we calculate RMS variation of beam-beam kick for a 4-D Gaussian macro-particle distribution [6]. As we know, the emittance growth rate of macro-particles are proportional to the square of the variation in the beam-beam kick. In the following, we use 1000 distributions of macro-particles with known beam sizes to estimate the variation in beam-beam kick in PIC method.

Both round and flat beams are used for comparison. The round beam's transverse sizes are (77 μm, 77 μm), which is close to the RHIC case. The flat beam's transverse sizes are (77 μm, 7.7 μm), which is close to the EIC case. Figure 6 shows the RMS variation of vertical beam-beam kick for both beams. Horizontal axis is radial amplitude in σ. From the plot, the variation of beam-beam kick is larger for small transverse amplitudes for both cases. However, the variation is about 2-3 times larger for the flat beam than the round beam.

Figure 7 shows the beam-beam kick variation on the horizontal and vertical axes for the flat beam. The horizontal axis

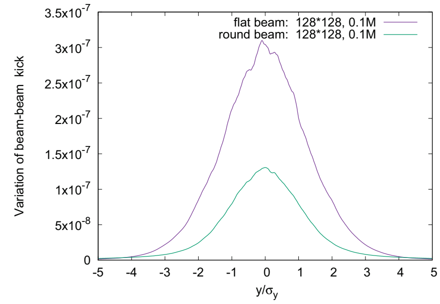

Figure 6: Comparison of beam-beam kick variations for a round beam and a flat beam.

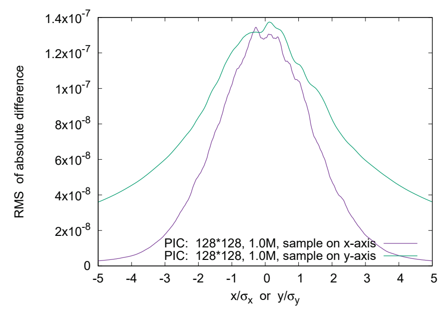

Figure 7: Comparison of beam-beam kick variations on horizontal and vertical axes for a flat beam.

is x/σ_x or y/σ_y in units of σ. From the plot, the beam-beam kick variation is always larger in the vertical direction than in the horizontal direction. And the variation in the vertical direction stays at a high level even for large amplitudes.

From these studies, we learn that PIC based Poisson solver tends to generate a larger variation (noise) in the beam-beam kick for macro-particles in the bunch core, especially in the vertical plane for a flat beam. This explained why we observed much faster beam size growth rates for macro-particles in the bunch core in the strong-strong simulation.

SUMMARY

Strong-strong beam-beam simulation based on PIC Poisson solver is subject to large numerical noises, which can be reduced by increased macro-particle number, transverse grids, and longitudinal slices. Turn-by-turn variation of bunch center can increase the simulated proton emittance growth rate but not the main contributor to the artificial emittance growth. We found that macro-particles in bunch core contribute most of the artificial emittance growth in the strong-strong simulation. The reason is found that PIC based Poisson solver generates a larger numerical variation in the beam-beam force calculation for macro-particles in the bunch core, especially in the vertical plane for a flat beam.

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