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DETECTOR SOLENOID COMPENSATION IN THE EIC ELECTRON STORAGE RING*

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Abstract

The Electron-Ion Collider (EIC) uses crab cavities to restore the geometrical luminosity loss associated with the large crossing angle. Due to space limitations, the detector solenoid cannot be compensated locally. This paper presents the lattice design to compensate the detector solenoid effects without interfering with the crab cavities. Skew quadrupoles are employed to avoid additional crab cavities. The correction scheme is checked by beam-beam simulation.

INTRODUCTION

A large crossing angle in the interaction region (IR) is necessary for fast separation of two colliding beams in ring-ring type colliders to achieve high collision rates. Crab cavities, first proposed for linear colliders [1], can compensate for the geometrical luminosity loss induced by crossing angle. This idea was later expanded to include circular colliders [2].

The Electron Ion Collider (EIC) adopts the local crabbing scheme to achieve the desired luminosity [3], as shown in Fig. 1. In the local crabbing scheme, a pair of crab cavities

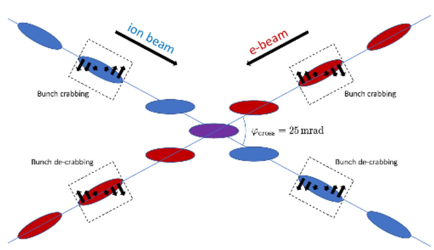

Figure 1: EIC local crabbing compensation scheme [3].

are installed at both sides of the IP. The upstream crab cavity tilts the beam in $x-z$ plane, and the downstream crab cavity rotates the beam back. In the rest of the rings, both planes stay unaffected.

In the ideal local crabbing scheme, the two crab cavities, located at the location with the betatron phase advance of $\pm\pi/2$ from IP, create desired crab "bump" between them. However, the detector solenoid, a 4 m long solenoid with an integrated strength of up to 12 T · m in EIC, would destroy the ideal configuration. Due to the IR space limitation, the anti-solenoid between the crab cavity and the detector solenoid, is not an option. Therefore, the vertical crabbing — the coupling of the vertical and longitudinal motion — is introduced, and has to be corrected in case of a significant luminosity loss.

This paper will first introduce the concept of crab dispersion. With the help of the crab dispersion, two methods are proposed to correct the vertical crabbing caused by the detector solenoid. The beam-beam simulations are performed to check the correction effect.

CRAB DISPERSION

The Hamiltonian of a thin crab cavity is

$$H=\left(\frac{x\lambda_x}{\Lambda_x}+\frac{y\lambda_y}{\Lambda_y}\right)\frac{\sin\left(k_c z\right)}{k_c} \tag{1}$$

where x, y, z are the horizontal, vertical, and longitudinal coordinates when a test particle pass through the thin crab cavity, $\lambda_{x,y}/\Lambda_{x,y}$ denoting the horizontal and vertical kick strength with $\Lambda_x=\sqrt{\beta_x^*\beta_{x,c}}$, $\Lambda_y=\sqrt{\beta_y^*\beta_{y,c}}$, and k_c the wave number. Here we use the lattice related parameters $\Lambda_{x,y}$ to normalize the crab cavity strength for simplicity.

Following the definition in [4], the crab dispersion is defined as the transverse dependence on longitudinal coordinate z,

$$\zeta=\left(\frac{\partial x}{\partial z},\frac{\partial p_x}{\partial z},\frac{\partial y}{\partial z},\frac{\partial p_y}{\partial z}\right)^{\mathrm{T}} \tag{2}$$

Assuming the lattice is symmetrical around IP, the crab dispersion from the upstream and downstream crab cavities is,

$$\begin{aligned}\zeta_b&=\left(0,\lambda_{b,x}/\Lambda_x,0,\lambda_{b,y}/\Lambda_y\right)^{\mathrm{T}}\\ \zeta_a&=\left(0,\lambda_{a,x}/\Lambda_x,0,\lambda_{a,y}/\Lambda_y\right)^{\mathrm{T}}\end{aligned} \tag{3}$$

where the subscript "b" denotes before IP, and "a" denotes after IP. Projecting the crab dispersion back to IP, the crab dispersion before and after the collision is

$$\begin{aligned}\zeta_b^*&=\begin{bmatrix}\lambda_{b,x}\sin\Psi_x\\ \lambda_{b,x}\cos\Psi_x/\beta_x^*\\ \lambda_{b,y}\sin\Psi_y\\ \lambda_{b,y}\cos\Psi_y/\beta_y^*\end{bmatrix}\\ \zeta_a^*&=\begin{bmatrix}\left(\lambda_{b,x}-\lambda_{a,x}\right)\sin\Psi_x\\ \left(\lambda_{b,x}+\lambda_{a,x}\right)\cos\Psi_x/\beta_x^*\\ \left(\lambda_{b,y}-\lambda_{a,y}\right)\sin\Psi_y\\ \left(\lambda_{b,y}+\lambda_{a,y}\right)\cos\Psi_y/\beta_y^*\end{bmatrix}\end{aligned} \tag{4}$$

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where $\Psi_{x,y}$ is the horizontal/vertical phase advance from the upstream crab cavity (IP) to IP (downstream crab cavity).

To create an effective head-on collision in the boost frame and avoid the crab dispersion leaking out of IR, ζ_b^* and ζ_a^* should fulfill the following conditions,

$$\zeta_b^* = (\theta_c, 0, 0, 0)^{\mathrm{T}}, \qquad \zeta_a^* = (0, 0, 0, 0)^{\mathrm{T}} \tag{5}$$

where θ_c is the half crossing angle.

A possible solution to Eq. (4) is

$$\begin{gathered} \Psi_x = \pi/2 \\ \lambda_{b,x} = \lambda_{a,x} = \theta_c, \qquad \lambda_{b,y} = \lambda_{a,y} = 0 \end{gathered} \tag{6}$$

Taking the detector solenoid into consideration, and projecting the crab dispersion at IP,

$$\zeta_b^* = \begin{bmatrix} \lambda_{b,x}\cos\phi\sin\Psi_x + \lambda_{b,y}\sin\phi\sin\Psi_y \\ \lambda_{b,x}\cos\phi\cos\Psi_x/\beta_x^* + \lambda_{b,y}\sin\phi\cos\Psi_y/\beta_y^* \\ \lambda_{b,y}\cos\phi\sin\Psi_y - \lambda_{b,x}\sin\phi\sin\Psi_x \\ \lambda_{b,y}\cos\phi\cos\Psi_y/\beta_y^* - \lambda_{b,x}\sin\phi\cos\Psi_x/\beta_x^* \end{bmatrix}$$

$$\zeta_a^* = \begin{bmatrix} \left(\lambda_{b,x}\cos(2\phi) - \lambda_{a,x}\right)\sin\Psi_x \\ +\lambda_{b,y}\sin(2\phi)\sin\Psi_y \\ \left(\lambda_{b,x}\cos(2\phi) + \lambda_{a,x}\right)\cos\Psi_x/\beta_x^* \\ +\lambda_{b,y}\sin(2\phi)\cos\Psi_y/\beta_y^* \\ \left(\lambda_{b,y}\cos(2\phi) - \lambda_{a,y}\right)\sin\Psi_y \\ -\lambda_{b,x}\sin(2\phi)\sin\Psi_x \\ \left(\lambda_{b,y}\cos(2\phi) + \lambda_{a,y}\right)\cos\Psi_y/\beta_y^* \\ -\lambda_{b,x}\sin(2\phi)\cos\Psi_x/\beta_x^* \end{bmatrix} \tag{7}$$

where 2ϕ is the rotation angle by the detector solenoid. As a demonstration, the solenoid focusing effect, which will be considered in the following sections, is ignored here. Substituting Eq. (7) into Eq. (5), a possible solution is

$$\begin{gathered} \Psi_x = \Psi_y = \pi/2 \\ \lambda_{b,x} = \lambda_{a,x} = \theta_c\cos\phi \\ \lambda_{b,y} = -\lambda_{a,y} = \theta_c\sin\phi \end{gathered} \tag{8}$$

However, the conditions in Eq. (8) are very hard to match. The required vertical crab cavity voltage is extremely large due to the small β_y^*. Multiple crab cavities may conflict with the impedance budget. As a result, the Eq. (8) is unrealistic in the actual lattice design.

CORRECTION BY SKEW QUADRUPOLES

The Eq. (5) gives 8 constraints. Without introducing aditional crab cavities, there are only 2 knobs provided by the crab cavity voltage. The other 6 knobs can be found from the appropriate transfer matrix between the crab cavities and IP.

Let M be the 4-by-4 transfer matrix from the upstream crab cavity to IP. If M has the following form

$$M = \begin{bmatrix} m_{11} & m_{12} & m_{13} & m_{14} \\ m_{21} & 0 & m_{23} & m_{24} \\ m_{31} & 0 & m_{33} & m_{34} \\ m_{41} & 0 & m_{43} & m_{44} \end{bmatrix} \tag{9}$$

tuning the crab cavity voltage so that

$$\zeta_b = (0, \theta_c/m_{12}, 0, 0)^{\mathrm{T}}$$

it is easy to show

$$\zeta_b^* = M\zeta_b = (\theta_c, 0, 0, 0)^{\mathrm{T}}$$

i.e. the beam is correctly crabbed for crab crossing collision.

Let N be the 4-by-4 transfer matrix from the upstream crab cavity to the downstream crab cavity. If N has the following form

$$N = \begin{bmatrix} n_{11} & 0 & n_{13} & n_{14} \\ n_{21} & n_{22} & n_{23} & n_{24} \\ n_{31} & 0 & n_{33} & n_{34} \\ n_{41} & 0 & n_{43} & n_{44} \end{bmatrix} \tag{10}$$

tuning the downstream crab cavity so that

$$\zeta_a = (0, -\theta_c n_{22}/m_{12}, 0, 0)^{\mathrm{T}}$$

Then we have

$$\zeta_a^* = N\zeta_b + \zeta_a = (0, 0, 0, 0)^{\mathrm{T}}$$

i.e. there is no crab dispersion leaking out of IR.

We can put skew quadrupoles between the crab cavities and IP to adjust the linear transfer matrix. Noticing that M and N are $x-y$ coupled, more skew quadrupoles are needed out of the crab bump to finish the decoupling. The $x-y$ decoupling scheme for EIC can be found in [5].

MATCHING FOR EIC ELECTRON STORAGE RING

We added skew components to the existing straight quadrupoles in IR6, and matched the linear transfer matrix to Eq. (9) and Eq. (10). Correction results are shown in Fig. 2. After compensation, the linear transfer matrix from the upstream crab cavity to IP is

$$M = \begin{bmatrix} -0.001553 & 9.394 & -0.06912 & 2.809 \\ -0.1064 & -0.3939 & 0.002898 & -0.1178 \\ 0.03014 & -2.790\mathrm{e}{-10} & -0.1585 & 2.347 \\ 0.03214 & -4.362\mathrm{e}{-11} & 0.07506 & -7.419 \end{bmatrix}$$

Except m_{22}, m_{32} and m_{42} are well matched to 0. The non-zero m_{22} is acceptable as the beam-beam has large tolerance on $\partial p_x/\partial z$ (see beam-beam simulations below). The linear transfer matrix from the upstream crab cavity to the downstream crab cavity is

$$N = \begin{bmatrix} 0.8591 & -3.662\mathrm{e}{-9} & -4.449\mathrm{e}{-10} & 6.804\mathrm{e}{-9} \\ 0.0002619 & 1.164 & -0.006533 & 0.2299 \\ -0.1832 & 2.289\mathrm{e}{-9} & 1.495 & -19.97 \\ 0.02776 & -3.066\mathrm{e}{-11} & -0.2572 & 4.104 \end{bmatrix}$$

The N matrix is well matched.

The maximum skew component strength used in our compensation is 1.2 T/m which is below the limitation of 1.5 T/m from the magnet design group.

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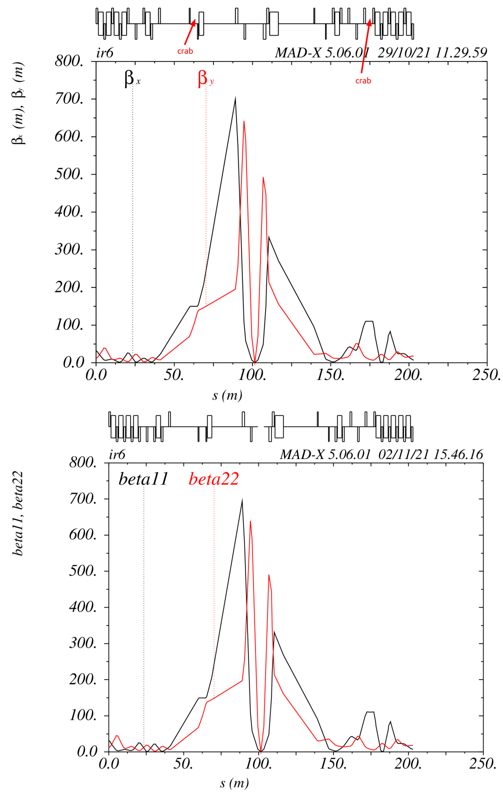

Figure 2: Ripken coupled β functions after coupling compensation for ESR detector solenoid, top: with detector solenoid off, bottom: with detector solenoid on and compensation by skew quadrupoles.

The chromatic effect may harm the compensation effect. To check the chromatic effect, we express the transfer matrix as a polynomial of momentum spread δ,

$$M = M_0 + M_1\delta + M_2\delta^2 \cdots \tag{11}$$

and the inverse matrix is expressed as

$$\begin{aligned} M^{-1} = &M_0^{-1} - M_0^{-1}M_1M_0^{-1}\delta \\ &- M_0^{-1}\left(M_2 - M_1M_0^{-1}M_1\right)M_0^{-1}\delta^2 + \cdots \end{aligned} \tag{12}$$

Weak-strong simulation is a widely used approach in beam-beam study [6, 7]. We use a self-written weak-strong code to check the correction result. In the simulation, the ion beam is rigid with a horizontal centroid as

$$x_i = -\theta_c\left[\frac{4}{3}\frac{\sin(k_{c,i}z)}{k_{c,i}} - \frac{1}{3}\frac{\sin(2k_{c,i}z)}{2k_{c,i}} - z\right] \tag{13}$$

where $k_{c,i}$ is the wave number of the crab cavities in the ion ring. A second order harmonic crab cavity is used to flatten the ion bunch [8]. The ion bunch is cut into multiple slices. Each slice is represented by a 2D Gaussian distribution in $x-y$ plane. The weak electron beam are simulated by a number of macro particles. The one-turn map at IP is represented by the linear betatron map. The transfer maps between crab cavities and IP, including the detector solenoid and quadrupoles, are expressed by Eq. (11) truncated upto different orders. The crab cavity kick is from the Hamiltonian in Eq. (1). The beam-beam kick is calculated by the Bassetti and Erskine formula [9]. The effects of radiation damping and quantum excitation are represented by a lumped element [10].

The simulation results are shown in Fig. 3. The orange curve shows the beam size evolution after the detector solenoid is compensated, while the blue curve displays the tracking data with the solenoid off. Both of them overlap with each other. It turns out that our compensation scheme works successfully. The red and green curves show the tracking when Eq. (11) and Eq. (12) are truncated upto δ and δ^2. As M and M^{-1} are symplectic upto the truncated order, it is understandable the green cuve reaches a different equilibrium size in the vertical plane. The red curve overlaps with the reference line too, which turns out the chromatic effect is negligible.

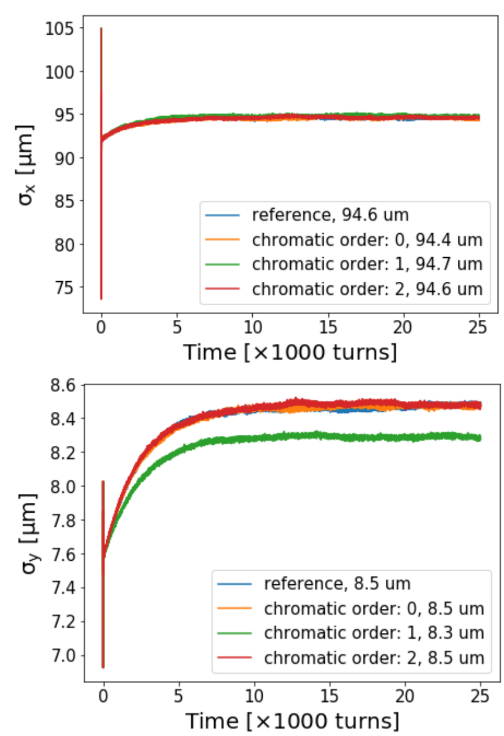

Figure 3: Weak-strong simulation with detector solenoid compensated. The blue curve is a reference with detector solenoid off. The linear transfer matrix is truncated at different orders of δ, as shown in Eq. (11) and Eq. (12).

SUMMARY

The detector solenoid in EIC Electron Storage Ring introduces vertical crabbing which has to be compensated. This paper proposes two different methods to compensate the detector solenoid with the help of crab dispersion. The second method is applied to the ESR lattice, and the weak-strong simulation validates the compensation effect.

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