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A FLEXIBLE NONLINEAR RESONANCE DRIVING TERM BASED CORRECTION ALGORITHM WITH FEED-DOWN

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Abstract

The optics in the insertion regions of the LHC and its upgrade project the High Luminosity LHC are very sensitive to local magnetic errors, due to the extremely high beta-functions. In collision optics, the non-zero closed orbit in the same region leads to a "feed-down" of high-order errors to lower orders, causing additional effects detrimental to beam lifetime. An extension to the well-established method for correcting these errors by locally suppressing resonance driving terms has been undertaken, not only taking this feed-down into account, but also adding the possibility of utilizing it such that the powering of higher-order correctors will compensate for lower order errors. Existing correction schemes have also operated on the assumption of (anti-)symmetric beta-functions of the optics in the two rings. This assumption can fail for a multitude of reasons, such as inherently asymmetric optics and unevenly distributed errors. In this respect, an extension of this correction scheme has been developed, removing the need for symmetry by operating on the two separate optics of the beams simultaneously. Unlike earlier implementations, the resonance driving terms to be corrected can also be changed flexibly. The mathematical background as well as some implementation details of this new enhancement are presented.

INTRODUCTION

The sensitivity of accelerator beam optics to magnetic errors depends directly on the β-function, which is highest in the Insertion Regions (IR) around the Interaction Points (IP) with the lowest β^* (the value of the β-function at the location of the IP). Hence, correcting the non-linear magnetic errors in these regions has been of significant importance in optimizing the LHC machine performance [1–6]. Installation of stronger magnets in the IR and the decrease of β^* in operation in the High Luminosity upgrade of the LHC (HL-LHC) [7, 8], is foreseen to result in even tighter constraints on residual errors.

At the same time, the influence of feed-down has been observed and investigated in the IRs of the LHC as well, where the crossing-angle scheme of the collision optics creates a large orbit bump. For both, LHC and HL-LHC, the need to correct this feed-down has been established [2, 4–6, 9–12].

To estimate the powering of the corrector magnets, a local correction scheme based on the Resonance Driving Terms (RDTs) in the IRs has been utilized [13]. Up to now, the implementation of this scheme calculated the correction based on the input from a single optics, for either Beam 1 or Beam 2, and made use of symmetries between the beams to optimize the correction for both. Cases can occur in which this symmetry does not hold, e.g. through the introduction of feed-down, or the use of inherently asymmetric optics. An example of the latter are flat optics [14, 15], in which the β^* in the two transversal planes no longer has identical values. These optics allow for a more distributed radiation deposition in the LHC magnets as well as an increase in luminosity [15]. Their feasibility has been studied during machine developments in the LHC [16] and preliminary analysis regarding their influence on corrections and amplitude detuning has been conducted [17].

A new and flexible version of the correction principle has been implemented [18], taking both optics into account and hence not relying on symmetry assumptions, allowing to target RDTs freely, as well as including feed-down into the calculations. The implementation allows for the feed-down from higher orders to the RDT to be corrected, as well as using the feed-down from higher order corrector magnets to correct for lower order errors.

CORRECTOR PACKAGES

To compensate for errors locally, both sides of the LHC IRs hosting experiments (ATLAS in IR1, ALICE in IR2, CMS in IR5 and LHCb in IR8) are equipped with linear and non-linear corrector packages. As shown in the schematics for HL-LHC in Fig. 1, these packages are located within the common aperture region of the machines, between Q3 and the separation dipoles D1, and hence contain common magnets for the two beams. Any correction should therefore take the optics of both beams into account. In the experimental IRs of the LHC and in HL-LHC IR2 and IR8, nonlinear correctors for skew and normal sextupoles (a_3, b_3), skew and normal octupoles (a_4, b_4) and normal dodecapoles (b_6) are available. In IR1 and IR5 of HL-LHC on the other hand, the corrector package will be upgraded to also include skew and normal decapoles (a_4, b_5) as well as skew dodecapoles (a_6) and offer therefore a wider range of field errors to correct, to account for the increase in the β-function in this high-performance machine [8, 12, 19].

CORRECTION PRINCIPLE

The algorithm aims to minimize the RDT locally at the entrance of the IR, with some additional simplifications on the calculation of the RDT value (compared to e.g. [20]) as outlined in [13]:

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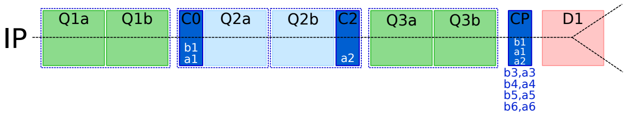

Figure 1: Schematic of the right hand side of HL-LHC IR1 and IR5. Q1a/b, Q2a/b and Q3a/b are the triplet quadrupoles. C0, C1 and CP show the corrector packages with the field order to be corrected indicated. The blue lines mark common cryostats. The non-linear correctors are included in CP.

- Only the contribution from elements in a single IR to the RDTs are minimized at a time, i.e. IRs are treated independently.
- Constants between all contributing elements are ignored (not needed for minimization).
- The phase per side of the IP is assumed constant, as $\Delta\Phi(a,b) = \int_a^b \frac{1}{\beta(s)} ds$ and $\beta(s)$ is very large in the triplets.
- The phase-advance between the left and right side of the IP is π.

With these approximations the *effective* RDT of order $n = j+k+l+m$ to minimize is:

$$f^{\mathrm{IR}}_{jklm} = \int_{\mathrm{IR}} \Re\left[i^{l+m}\left(K_n(s) + iJ_n(s)\right)\right] \cdot \beta_x(s)^{\frac{j+k}{2}}\,\beta_y(s)^{\frac{l+m}{2}}\,e^{i\pi n\theta(s-s_{\mathrm{IP}})}\mathrm{d}s\,, \tag{1}$$

with $K_n(s)$ and $J_n(s)$ being the field strength of normal and skew magnetic multipole fields of order n (starting with $n=1$ for dipole fields), $\theta(x)$ the heaviside step function and s_{IP} the location of the IP within the IR. The n in the exponent replaces $j-k+l-m = n-2k-2l$, as for even (odd) values of n, this value will also be even (odd), independent of the particular choices for j,k,l,m. The main concept of the correction is to find the $K_n(s)$ and $J_n(s)$ of the corrector magnets, which minimize a set of given f^{IR}_{jklm} based on given optics.

As there are usually two correctors per multipole field available (one on each side of the IP), two combinations of $l+m$ and $j+k$ (the exponents of the β function) can be corrected. As the correctors are responsible for the correction of both beams, the exponents need to be chosen, such that the correction is valid for both beams, which means that either only a single RDT per beam can be corrected or two RDTs if they are compatible with the symmetry of the optics [21].

Equation System

In our simulations, the input to the correction algorithm will be the output of `TWISS` and `ESAVE` functions from `MAD-X` [22]. These are tables in which $K_n(s)$ and $J_n(s)$ are not continuous functions, but given as already integrated values K_nL_w, J_nL_w ($\mathtt{K}_{n-1}\mathtt{L}$, $\mathtt{K}_{n-1}\mathtt{SL}$ in the terminology of `MAD-X`) for each element w. Values for the longitudinal position s_w, $\beta_{x,w}$, $\beta_{x,w}$ and the transversal orbit x_w, y_w, which will be important when calculating feed-down (see below), are also provided.

To assure an accurate estimate for the integral in Eq. (1), the lattice is *sliced* in `MAD-X`, i.e. all magnets are approximated by single kicks surrounded by drift-spaces. Long magnets are cut into multiple slices to increase accuracy. Corrector magnets on the other hand, which are in any case short compared to e.g. dipoles, can be represented by a single slice.

In this thin-lens approximation, Eq. (1) transforms into a sum over all elements (slices) w in the IR, which needs to be set to zero to suppress the RDT:

$$f^{\mathrm{IR}}_{jklm} = \sum_{w\in\mathrm{IR}} \Re\left[i^{l+m}\left(K_nL_w + iJ_nL_w\right)\right] \cdot \beta_{x,w}^{\frac{j+k}{2}}\beta_{y,w}^{\frac{l+m}{2}}\,e^{i\pi n\theta(s_w-s_{\mathrm{IP}})} \stackrel{!}{=} 0\,. \tag{2}$$

Splitting the elements into corrector elements $\mathscr{C}$ and non-corrector elements IR $\mathscr{C}$, Eq. (2) transforms into:

$$\sum_{w\in\mathscr{C}} \Re\left[i^{l+m}\left(K_nL_w + iJ_nL_w\right)\right] \beta_{x,w}^{\frac{j+k}{2}}\beta_{y,w}^{\frac{l+m}{2}}\,e^{i\pi n\theta(s_w-s_{\mathrm{IP}})}$$
$$= -\sum_{w\in\mathrm{IR}\mathscr{C}} \Re\left[i^{l+m}\left(K_nL_w + iJ_nL_w\right)\right] \beta_{x,w}^{\frac{j+k}{2}}\beta_{y,w}^{\frac{l+m}{2}}\,e^{i\pi n\theta(s_w-s_{\mathrm{IP}})}\,. \tag{3}$$

It is important to note, that each corrector is defined by either K_nL or J_nL, so that per order n and orientation (normal, skew) only a limited set of correctors is left. As there are two of these correctors per IR in the LHC/HL-LHC, $\mathscr{C} = \{cl, cr\}$, a left (cl) and a right (cr) corrector element. Defining the sum over IR$\mathscr{C}$ in Eq. (3) as I_{jklm} and

$$\begin{aligned} b^{(cl)}_{jklm} &= \, i^{l+m}\,\beta_{x,cl}^{\frac{j+k}{2}}\beta_{y,cl}^{\frac{l+m}{2}} \\ b^{(cr)}_{jklm} &= (-1)^n\, i^{l+m}\,\beta_{x,cr}^{\frac{j+k}{2}}\beta_{y,cl}^{\frac{l+m}{2}}\,, \end{aligned} \tag{4}$$

Eq. (3) can be split into a two equation system

$$\begin{aligned} \begin{pmatrix} b^{(cl)}_{jklm} & b^{(cr)}_{jklm} \end{pmatrix} \begin{pmatrix} K_nL_{cl} \\ K_nL_{cr} \end{pmatrix} &= -I_{jklm} && \text{if } l+m \text{ even,} \\ \begin{pmatrix} b^{(cl)}_{jklm} & b^{(cr)}_{jklm} \end{pmatrix} \begin{pmatrix} J_nL_{cl} \\ J_nL_{cr} \end{pmatrix} &= iI_{jklm} && \text{if } l+m \text{ odd,} \end{aligned} \tag{5}$$

each of which can be easily extended to include multiple RDTs, e.g.:

$$\begin{pmatrix} b^{(cl)}_{jklm} & b^{(cr)}_{jklm} \\ b^{(cl)}_{j'k'l'm'} & b^{(cr)}_{j'k'l'm'} \end{pmatrix} \begin{pmatrix} K_nL_{cl} \\ K_nL_{cr} \end{pmatrix} = -\begin{pmatrix} I_{jklm} \\ I_{j'k'l'm'} \end{pmatrix} \tag{6}$$

with $j+k+l+m = j'+k'+l'+m' = n$ and $l+m$ and $l'+m'$ both even. Including multiple beam optics, e.g. from LHC Beam 1 (B1) and Beam 2 (B2), can be done in a similar manner:

$$\begin{pmatrix} b^{(cl,\mathrm{B1})}_{jklm} & b^{(cr,\mathrm{B1})}_{jklm} \\ b^{(cl,\mathrm{B2})}_{jklm} & b^{(cr,\mathrm{B2})}_{jklm} \end{pmatrix} \begin{pmatrix} K_nL_{cl} \\ K_nL_{cr} \end{pmatrix} = -\begin{pmatrix} I^{(\mathrm{B1})}_{jklm} \\ I^{(\mathrm{B2})}_{jklm} \end{pmatrix}\,. \tag{7}$$

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Feed-Down

The effect of feed-down occurs whenever a particle beam is passing off-center through a magnet, due to either a transverse misalignment of the magnet or an off-center closed orbit of the beam itself. In these cases, the magnetic field can be described via Taylor expansion as a composition of magnetic field components with identical geometry as lower order fields in addition to the current field of higher order. These components therefore cause the same effects on the beam as lower order sources would [23].

Feed-down to field order $n \geq 2$ from fields up to $n + Q$ can be included by:

$$K_n + iJ_n \overset{\text{w/ feeddown}}{\rightarrow} \sum_{q=0}^{Q} (K_{n+q} + iJ_{n+q}) \frac{(\Delta x + i\Delta y)^q}{q!}. \quad (8)$$

Not only can feed-down be used to calculate the influence of field errors of orders $\geq n$ on the RDT, i.e. by contributing to I_{jklm}, but it can also be used to calculate the strengths of correctors of orders $n_C > n$ to counteract the RDT via feed-down. The matrix elements of the corrector coefficients in Eq. (4) will then contain the feed-down coefficient

$$z_p = \frac{(\Delta x + i\Delta y)^p}{p!}, \quad \text{with } p = n_C - n. \quad (9)$$

As $z_p \in \mathbb{C}$, this makes the evaluation of the real part in Eq. (3) - needed to separate the correctors as in Eq. (5) - less straightforward and yields the equation system Eq. (10).

$$\begin{aligned}
&\begin{pmatrix} \Re\left[z_p\right] \cdot b_{jklm}^{(cl)} & \Re\left[z_p\right] \cdot b_{jklm}^{(cr)} & -\Im\left[z_p\right] \cdot b_{jklm}^{(cl)} & -\Im\left[z_p\right] \cdot b_{jklm}^{(cr)} \end{pmatrix} \begin{pmatrix} K_{n+p}L_{cl} \\ K_{n+p}L_{cr} \\ J_{n+p}L_{cl} \\ J_{n+p}L_{cr} \end{pmatrix} = -I_{jklm} \quad \text{for even } l+m \\
&\begin{pmatrix} \Im\left[z_p\right] \cdot b_{jklm}^{(cl)} & \Im\left[z_p\right] \cdot b_{jklm}^{(cr)} & \Re\left[z_p\right] \cdot b_{jklm}^{(cl)} & \Re\left[z_p\right] \cdot b_{jklm}^{(cr)} \end{pmatrix} \begin{pmatrix} K_{n+p}L_{cl} \\ K_{n+p}L_{cr} \\ J_{n+p}L_{cl} \\ J_{n+p}L_{cr} \end{pmatrix} = iI_{jklm} \quad \text{for odd } l+m
\end{aligned} \quad (10)$$

Full Equation System

The "extensions" to the linear equation systems Eqs. (5) to (7) and (10) can be flexibly combined and the resulting equation system will incorporate arbitrary RDTs, multiple optics, correcting for and via feed-down and can therefore also include correctors of various orders for a single RDT. It can be solved or optimized for $K_nL_{cl,cr}$ or $J_nL_{cl,cr}$ by standard algorithms.

IMPLEMENTATION

A correction based on the algorithm described in the previous section has been released as a `python3` package `irnl-rdt-correction` [18]. The implementation allows for arbitrary RDTs as input, which can also be specified to be corrected via feed-down from higher order correctors. In addition, up to two beam optics can be given and corrected for at the same time.

The implemented algorithm performs a multitude of sanity checks on the user input and separates the correctors, to build independent equation systems when possible, i.e. per IP and for RDTs that do not share correctors. The resulting linear equation systems are solved via `numpy`'s [24] linear least-squares algorithm, which allows to optimize under-, well-, or over-determined equation systems. In case of complex correction schemes, an iterative approach is also possible. A detailed description of the implementation can be found in [25].

CONCLUSION AND OUTLOOK

An improved algorithm to correct nonlinear errors by locally compensating effective RDTs in the IRs has been outlined, overcoming the rigidness of previous implementations and giving the user more control over the correction. Its main features include the option to target arbitrary RDTs, include more than one beam optics, and either include feed-down into the RDTs to be corrected, or using the feed-down from the corrector magnets themselves for compensation.

The new correction package has since been extensively used for studies, investigating the influence of feed-down [26], the correctability of asymmetric optics [21] and the feasibility to correct systematic normal decapole errors in the separation and recombination dipoles of the HL-LHC [27, 28]. The ease of use and availability allows to utilize the new package with little effort in future studies of non-linear IR corrections.

The algorithm has been received with interest and additional features have been suggested. Among these is the inclusion of the phase-advance between elements, to further approach (and correct) the exact value of the RDT, instead of the *effective* RDTs targeted in the current implementation.

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