BEAM MATCHING IN AN ELLIPTICAL PLASMA BLOWOUT DRIVEN BY HIGHLY ASYMMETRIC FLAT BEAMS

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Abstract

Particle beams with highly asymmetric emittance ratios, or flat beams, are employed at accelerator facilities such as the AWA and foreseen at FACET-II. Flat beams have been used to drive wakefields in dielectric structures and are an ideal candidate for high-gradient wakefields in plasmas. The high aspect ratio produces a blowout region that is elliptical in cross section and this asymmetry in the ion column structure creates asymmetric focusing in the two transverse planes. The ellipticity of the plasma blowout decreases as the normalized peak current increases, and gradually approaches an axisymmetric column. An appropriate matching condition for the beam envelope inside the elliptical blowout is introduced. Simulations are performed to investigate the ellipticity of the resultant wakefield based on the initial drive beam parameters, and are compared to analytical calculations. The parameter space for two cases at the AWA and FACET facilities, with requirements for plasma profile and achievable fields, is presented.

INTRODUCTION

Particle beams with highly asymmetric emittances can be created at facilities like AWA [1] and are proposed at FACET-II [2]. These beams reduce beamstrahhlung at the interaction point [3] and have possible applications in future particle colliders and advanced accelerators. Asymmetric beams can also drive stable wakefields that are both focusing and accelerating in a hollow plasma channel which makes them good candidates for positron acceleration [4]. Flat beams can be employed to drive asymmetric wakefields in a plasma accelerator and the blowout created by these beams is transversely elliptical [5] when the density perturbation to the plasma is not large. In this paper, we aim to study the beam dynamics of the particles in the elliptical blowout that is created by beams that have transversely asymmetric spot sizes with asymmetric emittances. The importance of the ellipsoid distributions comes from the uniformity of the ion distribution within the bubble (neglecting the extreme case of ion motion), since uniformly charged ellipsoids have well-behaved linear self fields [6]. The linear focusing forces provided by the ion column can be used to match the divergence of the beam [7], which results in no oscillations of the beam envelope. A mismatched beam will undergo characteristic beating due to betatron oscillations, which can lead to an enlarged phase space or emittance dilution. The ellipticities of the ion column depends on the asymmetry of the beam and decreases with an increase in the beam density of the beam with respect to the plasma. Larger normalized beam densities lead to stronger blowouts, which concomitantly reduce the higher order moments and the blowout tends to be more axisymmetric. It is important to understand the dynamics of asymmetric drive beams and the necessary conditions to match these beams in a plasma wakefield accelerator (PWFA). Simulations were performed using QuickPIC [8], a 3D quasi-static particle-in-cell code, to investigate the plasma structures and beam dynamics resulting from the flat beam plasma interaction. Practical considerations for developing nascent experiments at the AWA and FACET-II facilities are also discussed.

TRANSVERSE FOCUSING

The transverse fields in the elliptical blowout structure can be calculated analytically by approximating it as an infinitely long cylinder of ions, similar to the axisymmetric case but with an elliptical cross section and is given by:

$$E_{x,p} = \frac{en_0 b_p x}{\epsilon_0(a_p + b_p)} = \frac{en_0 x}{\epsilon_0(1 + \alpha_p)} \tag{1}$$

$$E_{y,p} = \frac{en_0 a_p y}{\epsilon_0(a_p + b_p)} = \frac{en_0 \alpha_p y}{\epsilon_0(1 + \alpha_p)} \tag{2}$$

where a_p and b_p are the elliptical cross section's semimajor and semiminor axes respectively, and the ellipticity α_p is defined as the ratio a_p/b_p.

The matched beta functions for the beam envelope can be calculated from the transverse focusing forces of the elliptical column:

$$k_{\beta,x} = \sqrt{\frac{1}{\gamma m_e c^2}\frac{eE_x}{x}} = \sqrt{\frac{n_0 e^2}{\gamma \epsilon_0 m_e c^2 (1 + \alpha_p)}} \tag{3}$$

$$k_{\beta,y} = \sqrt{\frac{1}{\gamma m_e c^2}\frac{eE_y}{y}} = \sqrt{\frac{n_0 \alpha_p e^2}{\gamma \epsilon_0 m_e c^2 (1 + \alpha_p)}} \tag{4}$$

The spot size of the beam is then given by $\sigma_{x,m} = \sqrt{\beta_x \epsilon_x}$ and $\sigma_{y,m} = \sqrt{\beta_y \epsilon_y}$. The ellipticity of the matched beam, $\alpha_{b,m}$, is then given by:

$$\alpha_{b,m} = \frac{\sigma_{x,m}}{\sigma_{y,m}} = \sqrt{\frac{\epsilon_x}{\epsilon_y}\frac{\beta_x}{\beta_y}} = \sqrt{\frac{\epsilon_x}{\epsilon_y}\frac{k_{\beta,y}}{k_{\beta,x}}} = \sqrt{\frac{\epsilon_x}{\epsilon_y}}\sqrt{\alpha_{p,m}} \tag{5}$$

The ellipticity equation shows that the emittance needed to match the beam envelope to the focusing forces needs to account for the ellipticity of the plasma column.

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Table 1: Parameters for AWA

Parameter	Value	Unit
Beam		
Charge, Q	2	nC
Energy, E_b	50	MeV
σ_z	674	μm
σ_x, σ_y	113, 11.3	μm
Plasma		
Species	H^+	-
Density, n_0	4.2×10^{13}	cm^{-3}
Simulation		
Simulation window (x,y,z)	(8, 8, 14)	k_p^{-1}
Grid	$(1024)^3$	-
Particles per cell	4	-
Timestep	0.25	ω_p^{-1}
Beam particles	1.68×10^7	-

SIMULATIONS

The simulations were carried out using the beam and plasma parameters expected at the AWA shown in Table 1 but keeping the emittances arbitrary to match the beam into the elliptical plasma column. This was done to ensure that the beam fields and asymmetry are constant while observing the asymmetry of the plasma column.

Drive Beam Matching and Head Erosion

To generate the properly matched emittances for a fixed drive beam, the plasma blowout ellipticity is needed as per Eq. 5. An estimate for this value is obtained by first simulating a non-evolving drive beam of the same size propagating through the plasma. Once the ion column stabilizes, the ellipticity at its largest transverse slice is taken to be the needed $\alpha_{p,m}$. Specifically, the asymmetrical ellipse is obtained from a least squares fit performed on the curve defined by the border between the blowout and the surrounding neutral plasma. This border is defined where the normalized plasma density falls below $1/e$. The radius and angle of each such cell is taken as a "data point," and the fit is performed with the polar equation of an ellipse as shown in Fig. 1. The extracted α_p is used to generate the needed emittances for the matched drive beam, and the simulation for an evolving beam can then be run. The evolving beam and plasma interaction is shown in Fig. 3. The elliptical matching conditions are shown to reduce beam oscillations in the yz plane.

The weak blowout that is necessary for the formation of the elliptical blowout cavity leads to severe head erosion in the drive beam. The part of the beam that is outside the plasma ion column will continue to expand in vacuum as $\sigma(s) = \sigma(s_0)\sqrt{1 + ((s - s_0)/\beta)^2}$, which reduces the normalized beam density further. Consequentially, the blowout wake is reduced, which leads to an increase in the ellipticity of the plasma cavity with time. This dynamically changes the matching condition inside the blowout cavity, making it hard to match the beams for the entirety of the interaction. (There would also be small effects due to energy decrease and energy spread, but we consider those to be negligible based on our parameter space.) To see differences in matching in spite of this head erosion, spot sizes and emittances are calculated using the back 50% of the beam.

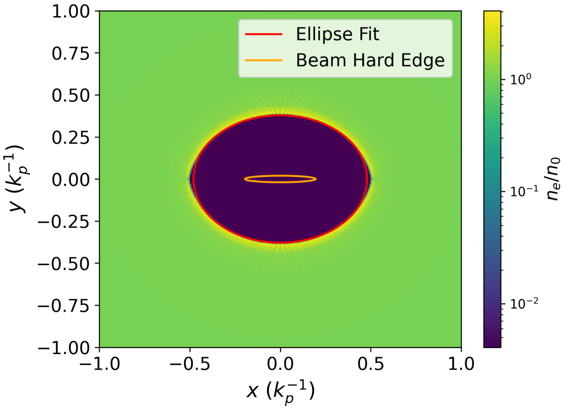

Figure 1: The plasma column elliptical fit used to match the drive beam. The plasma density is 4.2×10^{13} cm^{-3} and beam evolution is turned off. An ellipse is fit to the border of the blowout as described below, and the hard edge of the beam ($\sqrt{2}\sigma$) is shown as well.

Plasma Density Variation

The plasma density was varied keeping the beam parameters constant in position space but varying the beam in momentum space to asymmetrically match the beam based on the new conditions. Figure 2 demonstrates how increasing the plasma density and thus weakening the blowout leads to a more asymmetric blowout. The effect of head erosion is shown in Fig. 2 by calculating the ellipticities after a propagation distance of 100 k_p^{-1}. As expected, head erosion is seen to cause an increase in ellipticity as the beam propagates.

Witness Beam Matching

To isolate the effects of the asymmetric focusing fields and test our matching conditions, a witness beam with an

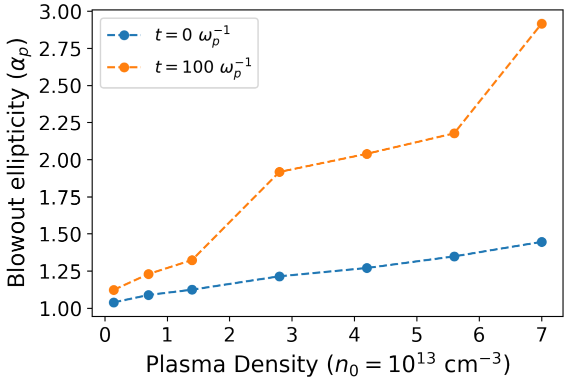

Figure 2: Blowout ellipticity for a fixed spot size beam, asymmetrically matched, as a function of the plasma density. Ellipticities are calculated by fitting ellipses to the transverse slice of largest initial blowout, typically around $\zeta = -2\ k_p^{-1}$.

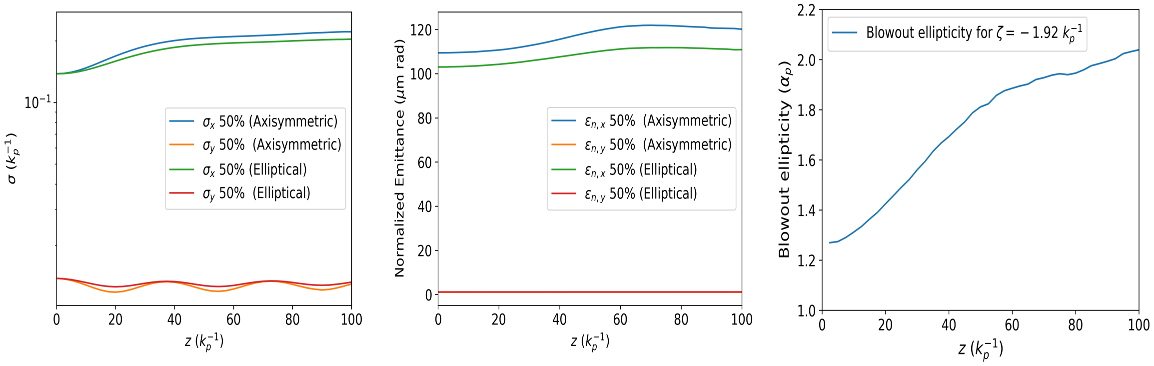

Figure 3: The evolution of the driver rms spot sizes (left), emittances (center), and ellipticity of the blowout cavity (right). The elliptical matching condition shows reduced emittance and spot size growth. Only the back 50% of the beam is examined here to separate head erosion effects from matching.

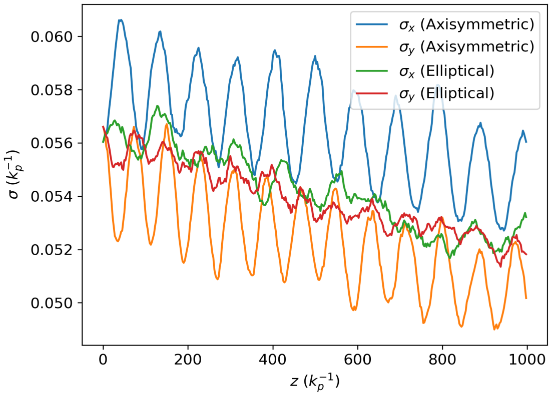

Figure 4: Matched emittances assuming an axisymmetric focusing force compared to matched emittances using the asymmetric focusing forces determined from the elliptical cavity. The gradual decrease in size is due to the acceleration of the witness while the envelope oscillations characterize the mismatched beam envelope. The driver evolution was turned off to maintain the ellipticity of ion cavity.

energy spread of 2% was introduced inside the ion cavity at $\zeta = -5.2\ k_p^{-1}$ and the driver beam evolution was turned off. Using the ellipticity of the plasma wake at this ζ as determined by simulations, the asymmetric emittance of the witness was determined. The reduction of oscillations in the beam envelope shown in Fig. 4 verifies our matching model, while the small oscillations that still exist might correspond to the finite width of the plasma sheath. The beam envelope reduces in size with propagation due to the increase in energy as it gets accelerated in the wake. It is interesting to note that the emittance of the mismatched beam shows no appreciable emittance growth over the course of this simulation when compared to the matched case.

EXPERIMENTAL INVESTIGATIONS

Understanding the plasma columns formed by elliptical beams, and the beam dynamics associated with the focusing forces, is important for future experiments. The proposed experiment at AWA would involve using the asymmetric beam to drive a weak blowout in a plasma created by a capillary discharge. The plasma density can be varied, which will allow exploration of the elliptical bubble regime investigated in this paper. A working model for the elliptical wake is being developed to predict the ellipticity without the help of simulations, and will aid in informing the matching conditions for the beam on a rapid basis. The limiting case for the flat beam (very wide beam limit) has been studied [9] in theoretical context to show the different regimes based on available plasma density. The condition of weak blowout creates head erosion in the drive beam and for this reason, beams with different longitudinal slice emittances would need to be used to match the beam. The AWA case was sucessfully benchmarked using OSIRIS [10], a three dimensional full relativistic PIC code.

In the other scenario, beams that will be available at FACET-II would have significantly higher beam densities at the final focus. The extreme beam density will generate an axisymmetric wake from an asymmetric beam. Simulations with the asymmetric FACET-II beam show the bifurcation of the beam after long propagation distances, in which the beam splits into two equal halves. The source of this bifurcation could be a potential violation of the quasi-static assumption used by QuickPIC. The extreme beam density case would need to be benchmarked with a full 3D PIC code, however the asymmetric bunch shape makes it computationally intensive.

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