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SIMULATING QUASI-INTEGRABLE OPTICS WITH SPACE CHARGE IN THE IBEX PAUL TRAP*

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Abstract

The intensity frontier has called for new initiatives in hadron accelerator design in order to accommodate space charge dominated beams. Octupoles are often used to damp beam instabilities caused by space charge, however the insertion of octupole magnets leads to a nonintegrable lattice which reduces the area of stable particle motion. One proposed solution is Quasi-Integrable optics (QIO), where the octupoles are inserted between sections of a specific lattice insertion called a T-insert. An octupole with a strength that scales as $1/\beta^3(s)$ is applied in the drift region, where the horizontal and vertical beta functions are equal, to create a time independent octupole field. This leads to a lattice with a time-independent Hamiltonian which is robust to small perturbations. IBEX is a Paul trap which allows the transverse dynamics of a collection of trapped particles to be studied, mimicking the propagation through multiple quadrupole lattice periods, whilst remaining stationary in the laboratory frame. In order to test QIO at the IBEX experiment, it has recently undergone an upgrade to allow for the creation of octupole fields. We present our design of the IBEX experiment upgrade along with simulation results of our proposed experiment to test QIO with space charge.

INTRODUCTION

Space charge forces are the result of Couloumb interactions between charged particles in a beam. As higher intensity accelerators are being designed and built, resonances and instabilities caused by space charge can become the limiting factor in the intensities these machines can reach. Building test accelerators to study high intensity beams can be costly and, when carrying out beam loss studies, accelerator components can be activated and damaged. Therefore, simulations are a vital part of the accelerator design process, however they are computationally intensive when reproducing space charge forces over long timescales (tens of thousands of turns) and can never be a replacement for experimental verification.

These challenges led to the design and construction of linear Paul traps to investigate transverse beam dynamics more efficiently at Hiroshima University, Japan [1], Princeton University, US [2] and, most recently, the Intense Beams Experiment (IBEX), at the Rutherford Appleton Laboratories (RAL), UK [3]. IBEX is a table-top sized experiment that can replicate the transverse betatron motion in alternating gradient accelerators in a dispersion- and chromaticity-free environment.

This paper aims to simulate a lattice proposed by the theory of Nonlinear Integrable Optics (NIO) to damp coherent resonances created by space charge.

THE IBEX PAUL TRAP AND UPGRADE

The IBEX trap was recently upgraded to allow for the excitation of sextupole and octupole fields. The trap consists of two sections, an ionisation region (IR) and an experimental region (ER) as seen in Fig. 1. The design was adapted from the Hiroshima group's nonlinear trap [4]. Argon gas is introduced into the trap vessel and is ionised in the IR with an electron gun. Typically, a sinusoidal RF voltage is applied to the central rods with a maximum peak-to-peak of 300 V and frequency of 1 MHz. Voltages of the same form but opposite polarity are applied to the blue and red outlined rods in Fig. 1 to provide transverse confinement of the ions. Longitudinal confinement of the ions is achieved by applying a DC offset to the end caps and gate electrodes. Once the argon gas is ionised, the DC voltage on the gate is dropped to allow ions to pass into the experimental region.

Adjusting the peak voltage applied to the central rods is analogous to changing the quadrupole strength in an accelerator, which in turn changes the betatron tune in both the horizontal and vertical planes. Ions can be stored for around 1 s in IBEX, corresponding to 10^6 RF periods. Octupole and sextupole fields can be created by applying voltages to the plate electrodes in the ER. The DC voltage on the end caps is then dropped and the ions are directed onto a Micro-Channel Plate (MCP) detector. The number of ions stored in the trap is controlled by adjusting the length of time that the electron gun is on. This allows for a wide range of intensities to be studied within the trap. Due to the low energy of the ions (< 1 eV), high intensity beam loss studies can be carried out in the trap without damaging or activating components.

NONLINEAR INTEGRABLE OPTICS

Current synchrotrons and linacs utilize a system of alternating focusing and defocusing quadrupole magnets that are used to confine a beam of charged particles. The use of linear magnets leads to a Hamiltonian of the system that can be made time-independent and hence integrable. However, in reality these linear lattices are susceptible to perturbations such as magnet errors and space charge forces, and so components such as sextupoles and octupoles are often used to apply higher order corrections. In general, the addition of these nonlinear components creates a non-integrable system,

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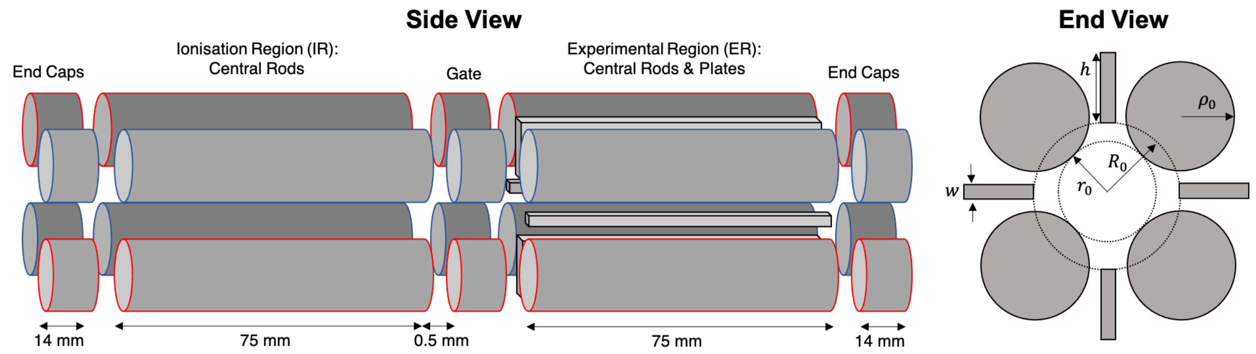

Figure 1: Schematic of the IBEX trap nonlinear upgrade. Opposing RF voltages are applied to the red and blue outlined rods for transverse confinement of ions. A DC voltage is applied to the end caps and gate electrodes to provide longitudinal trapping. In IBEX $r_0 = 5$ mm and $\rho_0 = 5.75$ mm. Four additional plates between the rods are present in the nonlinear trap at an inscribed radius $R_0 = 8.5$ mm to enable the creation of octupole fields.

which limits the available area of phase-space where the particle motion is non-chaotic. Therefore, it is desirable to design a lattice which includes nonlinear components in such a way that it remains integrable. If an integrable, nonlinear lattice exists, then the system will always be close to an integrable solution, even when effects such as magnet misalignment and space charge are included.

The theory of NIO [5] proposed an integrable, nonlinear lattice consisting of a linear T-insert and a drift region for a nonlinear magnet insert. The three conditions of the T-insert lattice are (1) $n\pi$ (where n is an integer) phase advance over the linear section to provide quasi-periodic motion through the drift region. (2) Equal beta functions in the drift region ($\beta_x = \beta_y$). (3) For Quasi-Integrable optics, the octupole strength will scale with $1/\beta^3(s)$.

The Integrable Optics Test Accelerator (IOTA), Fermilab [6] will test the fully integrable solution, which requires a complex elliptical potential in the drift region. However, the elliptical potential is a challenge to create experimentally. Therefore, IBEX will first test the quasi-integrable case, which uses an octupole potential of the form

$$V(x,y,s) = \frac{\kappa}{\beta^3(s)}\left(\frac{x^4}{4} + \frac{y^4}{4} - \frac{3x^2y^2}{2}\right), \qquad (1)$$

where κ is a constant and $\kappa/\beta^3(s)$ defines the strength of the octupole field. Assuming equal beta functions in the horizontal and vertical direction ($\beta_x(s) = \beta_y(s) = \beta(s)$), the octupole potential creates the time-independent Hamiltonian

$$H_N = \frac{p_{xN}^2 + p_{yN}^2}{2} + \frac{x_N^2 + y_N^2}{2} + U(x_N, y_N) \qquad (2)$$

where $U(x_N, y_N) = \beta(s)V(x,y,s)$ and the coordinate transform, $z_N = z/\sqrt{\beta(s)}$ is used. The time-independent Hamiltonian becomes an integrable of motion, creating a quasi-integrable lattice which should be robust to small perturbations. In this work, a two cell T-insert lattice close to the 1/4 integer resonance was created. The lattice was tested in the presence of space charge and the benefits of applying a quasi-integrable octupole were studied.

SIMULATION RESULTS

The particle-in-cell (PIC) code VSim 11.0 [7] was used to create the T-insert lattice in a 2D simulation of the IBEX trap. Figure 2 plots the voltage waveform applied to the rod electrodes in IBEX to create the two cell T-insert lattice. The lattice was designed to have a π phase advance through the T-insert region and equal horizontal and vertical beta functions within the drift region. Two distinct cells with mirror symmetry were designed to create a super-period with tunes of $Q_x = 1.276$ and $Q_y = 1.277$, close to the 1/4 integer resonance. On the left of Fig. 2 is the T-insert lattice with no octupole applied in the drift region. In the center of Fig. 2 is the Quasi-Integrable (QI) lattice where the octupole is turned on in the drift region and varies as $1/\beta^3(s)$. On the right of Fig. 2, a comparison lattice is used where the octupole is turned off every other cell. The octupole strength is twice as large in this lattice to make the integrated octupole strength the same as the QI lattice, however this lattice is non-integrable.

A 2D Gaussian distribution of 25,000 Ar^+ macro particles was tracked in a VSim 11 model of IBEX for 1000 T-insert lattice super-periods (2000 cell periods). The charge ratio of macro particles to physical particles was 1:1000. The distribution of 2.5×10^7 physical particles created an incoherent space charge tune shift of $\Delta Q_{rms} = -0.036$. This tune shift was chosen to excite coherent oscillations which satisfied the following resonant condition [8]

$$Q_0 + C_2\Delta Q_{rms} = \frac{1}{2}\left(\frac{5}{2}\right) \qquad (3)$$

where $C_2 = 3/4$ [9].

The number of particles surviving over 1000 T-insert super-periods is plotted against time in Fig. 3. The linear T-insert lattice with no octupoles applied is plotted in blue and can be seen to lose 36.5(3) %. When the octupoles are turned on and they meet the quasi-integrability condition (orange), the particle loss is reduced to 12.6(2) %. If we compare the QI lattice (Fig. 2, center) to a nonlinear lattice

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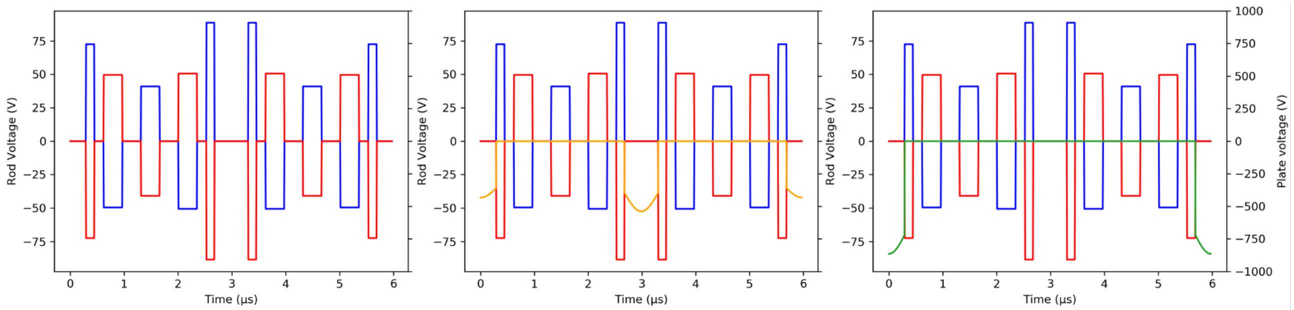

Figure 2: Voltage waveform, applied to the rods in IBEX (red and blue) to create the two-cell T-insert super-period. **Left:** No octupoles applied in the drift region. **Center:** Octupole pulse with $1/\beta^3$ strength scaling (orange) applied in the drift region to create Quasi-Integrable lattice. **Right:** Octupole pulse with $1/\beta^3$ strength scaling (green) with twice the strength but applied only once per super-cell.

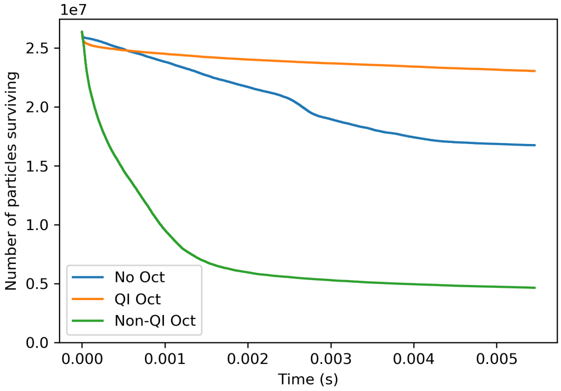

Figure 3: Rate of particle loss over 1000 super-periods of the T-insert lattice. (Blue) No octupoles applied in the drift region. (Orange) Octupoles turned on in the drift region which meet the quasi-integrability condition. (Green) Octupoles turned on with twice the strength but applied only once per super-period.

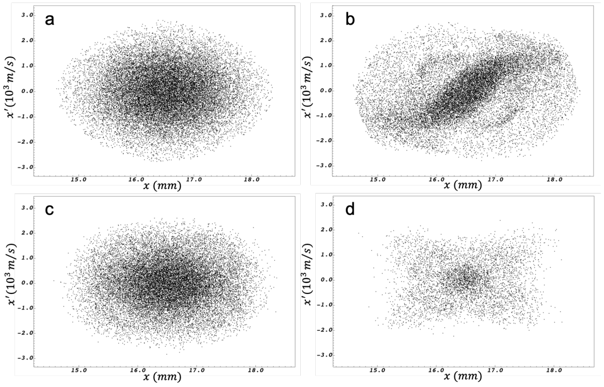

Figure 4: (x,x’) phase space plotted after 500 super-periods of the T-insert lattice. a) Octupoles off in the drift region (no space charge). b) Octupoles off in the drift region. c) Octupoles on and QI conditions met. d) Octupoles on but QI conditions broken.

which breaks the integrability condition (Fig. 2, right) we get 82.4(2) % particle loss, which is plotted in green in Fig. 3.

The mechanisms behind the particle loss can be observed from the phase space plotted in Fig. 4. Figure 4a is the (x,x’) phase space of a matched Gaussian distribution of 25,000 macro particles with no space charge (ratio of macro particles to physical particles was 1:1 where space charge is negligible). The phase space is plotted after 500 super-periods of the linear lattice (Fig. 2, left) and resembles the initial Gaussian distribution it started with.

In Fig. 4b the phase space is plotted after 500 super-periods through the linear lattice but with space charge. Due to the resonant condition in Eq. (3) being met, the 2nd order coherent resonance is excited. This can be seen from the two arms forming in the phase space of Fig. 4b.

Figure 4c compares the phase space of the lattice when the octupoles are turned on and the quasi-integrability condition is met. The coherent resonance caused by the space charge tune shift has been damped by the octupole field and the distribution is returned to what resembles a Gaussian distribution. It is comparable to the phase space in Fig. 4a when no space charge was present.

In order to show the benefits of Quasi-Integrable Optics (QIO) over the use of ordinary octupoles to dampen this coherent resonance, we also compared a lattice which broke the integrability condition (Fig. 2, right). Figure 4d shows the phase space when just one octupole of twice the strength is turned on per two cell super-period. The four arms that appear in the phase space show that the 4th order incoherent resonance is being excited and is responsible for the significant particle loss seen in Fig. 3.

CONCLUSION

These simulation results show that a coherent resonance can be damped with octupoles when arranged in a lattice prescribed by QIO. The results also show that deviating from the conditions of quasi-integrability can result in the octupole driving it’s own set of 4th order resonances. The IBEX Paul trap has undergone an upgrade to allow for the creation of octupole fields within the trap with the aim to confirm these simulations with detailed experiments.

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