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# MODELLING GROWTH AND ASYMMETRY IN SEEDED SELF-MODULATION OF ELLIPTICAL BEAMS IN PLASMA

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*Abstract*

The seeded self-modulation (SSM) of long particle bunches for the generation of gigavolts-per-meter wakefields that can accelerate witness electron beams was first shown using the SPS beam as a driver by the AWAKE experiment. The stability of the produced microbunch trains over tens or hundreds of meters is crucial for extrapolating this scheme as proposed for use in several high energy plasma-based linear colliders. However, aside from the competing hosing instability, which has been shown to be suppressible by SSM when that process saturates, few works have examined other effects of transverse asymmetry in this process. Here, we use analytical modelling and 3D particle-in-cell simulations with *QuickPIC* to characterise the impact on the SSM growth process due to transverse asymmetry in the beam. A metric is constructed for asymmetry in simulation results, showing that the initial azimuthal complexity changes only slightly during SSM growth. Further, we show quantitative agreement between simulations and analytical predictions for the scaling of the reduction SSM growth rate with unequal aspect ratio of the initial beam profile. These results serve to inform planning and tolerances for both AWAKE and other SSM-based novel acceleration methods in the future.

## INTRODUCTION

Proton-driven plasma wakefield acceleration (PDPWFA), has been proposed to overcome the problem of energy depletion of drivers in previous experiments, with the view of application towards a new generation of plasma-wakefield-based colliders for high energy physics research. However, current high-energy-content bunches, such as those of the Super Proton Synchrotron (SPS) used in AWAKE are too long by two orders of magnitude to efficiently drive a wakefield in plasma of suitable density . Therefore, the concept relies on the self-modulation of the long proton bunch in plasma due to an initial weak ‘seed’ wakefield driven by the unmodulated bunch which causes the bunch to compress and diverge at periodic intervals along its length. The resulting train of shorter micro-bunches, if formed so that they are positioned correctly within the wakefield [1], can then resonantly excite much stronger accelerating gradients in the plasma to accelerate a witness beam [2].

Seeding the self-modulation process requires an initial wakefield with a sufficiently strong longitudinal component at the plasma wavelength, $\lambda_p = 2\pi c\sqrt{\frac{m_e \epsilon_0}{n_p e^2}}$, where $n_p$ is the plasma density, $\epsilon_0$ is the permittivity of free space, $c$ is the speed of light, and $m_e$ and $e$ are the electron mass and charge, respectively. This may be achieved by a smaller preceding bunch as proposed in AWAKE Run 2 [3] or by ionising the plasma with a co-propagating laser pulse placed at the midpoint of the Gaussian proton beam to create a discontinuity in beam longitudinal profile as seen by the plasma [4]. Such seeding is required to control the initial phase of the modulation process along the longitudinal beam profile, to ensure an efficient resultant microbunch arrangement upon saturation of the SSM growth [5].

It has been shown by numerical investigations in previous works that the seeded self-modulation (SSM) process may be sensitive to beam parameters such as emittance and radial spot size [6]. However, such works have almost consistently considered only transversely round bunches. Previously, it was shown that even slightly unequal aspect ratio of the driving beam leads to strong asymmetric profiles of the resultant microbunches [7] which is reflected in the transverse profiles of the resultant wakefields [8]. More recently, [9] has demonstrated the variation of the event-to-event aspect ratio of the SPS beam by as much as 15%. Here we present a metric constructed to represent the asymmetry of the beam transverse profile beyond parameterisation with only root-mean-square sizes. The examination of its evolution during the SSM growth is used to justify the model we derived in [10]. Finally a method for extracting a parametric dependence of the initial aspect ratio of the drive beam from simulations of the SSM process is outlined and used to verify the scaling of the model in [10] with initial aspect ratio.

## SIMULATION SET-UP

Simulations were carried out using the 3D quasi-static particle-in-cell (PIC) code *QuickPIC* [11]. We use a uniform plasma density, $n_p = 7 \times 10^{-14}\,\mathrm{cm}^{-3}$, corresponding to a plasma skin-depth of $c/\omega_p = k_p^{-1} = 200\,\mu\mathrm{m}$, on a grid of $1024 \times 1024 \times 4096$ cells, spanning a volume of $12 \times 12 \times 130\,(c/\omega_p)^3$ in $x$, $y$, and $\xi$ with 4 particles per cell. Here $\xi$ is a co-moving coordinate along the length of the beam, defined in terms of the propagation length, $s$, and longitudinal coordinate in the lab frame, $z$, as $\xi = s - z$. The long proton bunches were initialized with parameters similar to the SPS bunches arriving at AWAKE, as transversely bi-Gaussian beams with root-mean-square radius $\sigma_r = 1.0$, Lorentz factor $\gamma_b = 427$ and equal $x$ and $y$ rms emittances of 3.5 mm mrad, but neglecting the 0.035% momentum spread. The self-modulation seed was achieved by using a sharp longitudinal density step up to the maximum density in the beam profile at $\xi = 0$, placed $2\,\mathrm{c/_p}$ from the front edge of the simulation window. Since we are interested in the general behaviour of elliptical SSM, and since the longitudinal

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simulation window was much shorter than the beam length in AWAKE, a flat-top profile was used in this direction. PIC simulations of long beams, especially in 3D, are known to be highly sensitive to simulation parameters if not properly converged [12]. The simulations here were converged with respect to grid resolution, transverse domain size, particles-per-cell, their anisotropy, and the angular orientation of the beam ellipse in the cartesian grid.

## ASYMMETRY EVOLUTION

Since the wakefield of elliptical beams inherit their asymmetry, the resulting transverse evolution of the beam therefore acquires a strong degree of asymmetry, particularly in the focussed microbunches. As we showed previously in [7], the transverse profile of the beam is no longer well-characterised by only the root-mean-square sizes in transverse directions $x$ and $y$. Instead, here we construct a metric for the magnitude;

$$\bar{V}_\phi[n_b] \equiv \mathrm{E}\left[\left(\frac{V_\phi[n_b](r)}{\mathrm{E}_\phi[n_b^2](r)}\right) n_b(r,\phi)\right] \Big/ \mathrm{E}[n_b] \qquad (1)$$

and complexity;

$$\bar{D}_\phi[n_b] \equiv \mathrm{E}\left[\underset{m>0}{\operatorname{argmax}}\left[n_{b,m}^2(r)\right] n_b(r,\phi)\right] \Big/ \mathrm{E}[n_b], \qquad (2)$$

of the asymmetry using the $m$-th azimuthal Fourier amplitudes, $n_{b,m}(\xi, r)$ of the transverse beam profile, $n_b$ at each longitudinal and radial position ($r$) along the beam. Here, $V_\phi$, E and $\mathrm{E}_\phi$ represent the variance in the azimuthal direction only; the expectation value over the full transverse profile; and that over the azimuthal direction only, respectively.

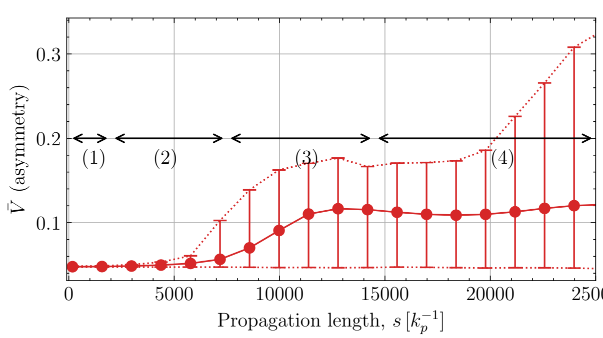


Figure 1: Evolution of the asymmetry metric, $\bar{V}$, with propagation length, $s$ for a beam with initial $h = 0.6$. The central dots, upper ends and lower ends of the vertical bars at each $s$ correspond to the mean, maximum and minimum values in the simulation window, respectively.

Figures 1 and 2 show the evolution of this metric over the course of the self-modulation process of a beam with aspect ratio $h = 0.6$. As found in [7], the asymmetry of the focusing slices of the beam tend to increase while that of defocusing ones decreases, leading to a strong variation of these metrics also along $\xi$. The bars in Figs. 1 and 2

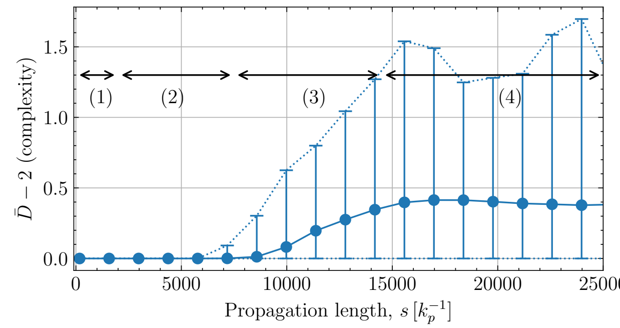


Figure 2: Evolution of the complexity metric, $\bar{D}$, with propagation length, $s$ for a beam with initial $h = 0.6$. The central dots, upper ends and lower ends of the vertical bars at each $s$ correspond to the mean, maximum and minimum values in the simulation window, respectively.

show the maximum and minimum values along $\xi$ within the simulated length of the beam, while the dots mark the mean values. Nonetheless, it is seen that the evolution of both the magnitude and complexity of the asymmetry remains nearly unchanged during the fastest-growth stage (2) of the SSM [1], validating our assumption used for the model in [10] (and the results in the next section) of using only the $m = 2$ azimuthal mode in the transverse profile.

## RELATIVE GROWTH RATE

The beam shape parameter, $\nu$ relative to that for the cylindrical case, $\nu_0$ was found to be

$$\nu/\nu_0 = \left[1 - \left(\frac{1}{2} + \mu(r_0)\right)\left(\frac{1-h^2}{1+h^2}\right)^2\right] \qquad \text{with} \qquad (3)$$

$$\mu = \frac{\hat{r}_0^2 I_1(\hat{r}_0)K_3(\hat{r}_0) - 2\hat{r}_0\left[I_1(\hat{r}_0)K_2(\hat{r}_0) + I_2(\hat{r}_0)K_3(\hat{r}_0)\right]}{8K_2(\hat{r}_0)I_2(\hat{r}_0)}, \qquad (4)$$

where $I_m$ and $K_m$ are the modified Bessel functions of order $m$ of the first and second kind, $\hat{r}_0 = k_p\sqrt{2}\sigma_{r,0}$, and $\mu(r_0) \lesssim 1/2$ for $\hat{r}_0 \lesssim 1$. The radius-dependent term, $\mu(r_0)$ is strongly sensitive to the model used to represent the profile of the beam (flat-top radial function). Therefore, it is expected that a more realistic profile will not present the same scaling with $\sigma_{r,0}$. Hence, to test the variation of $\nu/\nu_0$ with aspect ratio, $h$, we use the more general function

$$\nu/\nu_0 = 1 - \hat{\nu}_2\left(\frac{1-h^2}{1+h^2}\right)^p \qquad (5)$$

parameterised by a coefficient $\hat{\nu}_2$ (expected to be $O(1)$) and a power, $p$ (predicted to be 2). Using the expressions derived for the asymptotic behaviour of the longitudinal electric field, $E_z$, in [13] we derive an expression for the instantaneous value of $\nu$ obtained from the evolution of $E_z$ at a maximum along the beam in a simulation of SSM:

$$\nu(s) = \frac{1}{k_p\xi_{\max}}\left[\frac{3}{2}\frac{n_0}{n_{b0}}\frac{\gamma_b m_b}{m_e}\frac{\left(1 + \frac{s}{E_{z,\max}}\frac{\partial E_{z,\max}}{\partial s}\right)^3}{(k_p s)^2}\right], \qquad (6)$$

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where $\xi_{\max}$ is the $s$-dependent position of the chosen maximum, $E_{z,\max}$.

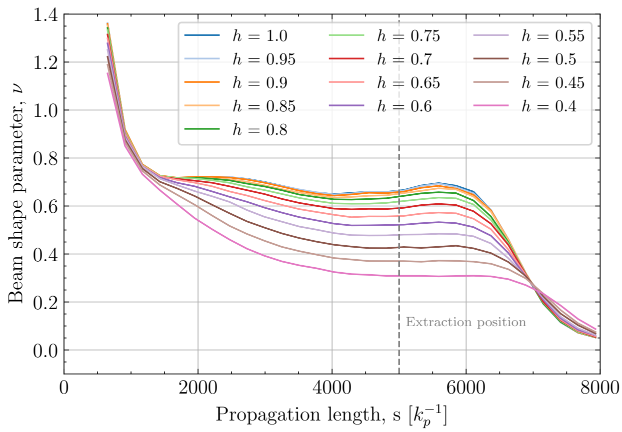


Figure 3: The variation of the extracted beam shape parameter $\nu(s)$ during the growth stage of the SSM of beam as its initial aspect ratio, $h$ is varied while holding charge per unit length and r.m.s radial size ($\sigma_{r,0} = 1.0$) constant. The vertical dotted line shows the position where $\nu$ is extracted from the curves for use in Fig. 4.

Figure 3 shows the extracted instantaneous $\nu$ from the $E_z$ field according to equation (6) for varying intial beam aspect ratio $0.4 < h < 1.0$ and fixed radius $\sigma_{r,0} = 1.0$. Since (6) is asymptotic, it is only valid after $\nu$ reaches a stable value, which is used to determine the extraction position of a nominal value for each curve. To verify the scaling of equation (3), we perform a least-squares fit to the obtained variation of nominal $\nu$ with $h$, as shown in Fig. 4. Comparing the fitted parameters in the top left, is found that at these beam parameters, the model under-estimates the value of $\hat{\nu}_2$ as $\hat{\nu}_2 = 0.8$ compared to $1.05 \pm 0.01$ obtained from the fit.

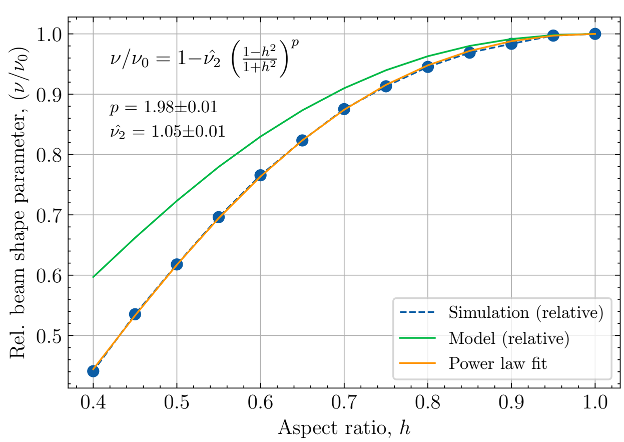


Figure 4: The variation of the extracted relative beam shape parameter $\nu/\nu_0$ during the growth stage of the SSM of beam as its initial aspect ratio, $h$ is varied while holding charge per unit length and r.m.s radial size ($\sigma_{r,0} = 1.0$) constant. The orange curve is a fit performed to a function (5), with best-fit parameters given in the top right.

However, the power-law scaling with $h$ is found to be in good agreement with the model value of $p = 2$ ($1.98 \pm 0.01$ from fit). Further verification with respect to different radii and other beam parameters will be presented in a future work.

## CONCLUSION

A comparison of predictions of the envelope-based theoretical model for seeded self-modulation (SSM) of an elliptical beam that we described in [10] with 3D simulation results was presented. First, a metric was constructed to verify the minimal magnitude and complexity of the azimuthal beam profile used in the model during the fastest growth stage of the seeded-self-modulation against simulation results. Second, a method to extract an instantaneous value for a parameter quantifying the dependence of the SSM on the beam shape from simulations was derived. Finally, values extracted this way were compared to predictions from the model. While the model under-estimates the the overall change to the relative growth rate of the SSM (attributed to use of a simple, non-Gaussian radial profile), the scaling with aspect ratio shows excellent agreement. Future work will examine the robustness of the scaling with varying beam parameters and seeding methods for the SSM.

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