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RADIATION OF A PARTICLE MOVING ALONG A HELICAL TRAJECTORY IN A RESISTIVE-WALL CYLINDRICAL WAVEGUIDE*

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Abstract

The radiation field of a particle moving on a helical trajectory in a cylindrical waveguide with resistive walls is calculated. The deformation of the energy spectrum of radiation as a result of the finite conductivity of the walls is investigated.

INTRODUCTION

The helical motion of a charged particle considered in this work simulates the operation of a helical undulator, which is widely used as a source of circularly polarized synchrotron radiation and FEL designs [1, 2]. Insertion of the helically moving particle in a cylindrical waveguide converts the radiation energy spectrum from continuous to discrete [3]. With an appropriate selection of parameters, it becomes possible to concentrate most of the radiation power at one frequency and, thereby, create a source of monochromatic radiation. In work [3], however, an ideal waveguide was used as a model. For a more accurate determination of the structure characteristics, it is necessary to consider the finite conductivity of the waveguide walls.

Usually the problem has been solved numerically, using simulation codes [1, 2, 4], or asymptotically [5-8]. An attempt of an analytical solution was made in [9]. Here the explicit expressions for the radiation fields are presented.

STATEMENT OF THE PROBLEM

Consider a relativistic point charge q with longitudinal velocity V and revolution frequency ω_0, moving along the helical trajectory in the resistive-wall cylindrical waveguide with inner radius b. The charge density ρ and current $\vec{j}$ are given in the forms:

$$\rho(r,\varphi,z,t) = q\,\frac{\delta(r-a)}{\sqrt{ra}}\,\delta(\varphi-\omega_0 t)\delta(z-Vt)$$

$$\vec{j}(r,\varphi,z,t) = (\omega_0 a\vec{e}_\varphi + V\vec{e}_z)\rho(r,\varphi,z,t) \qquad (1)$$

where $\vec{e}_\varphi$, $\vec{e}_z$ are unit vectors in the cylindrical coordinates r,φ,z and a orbit radius. The electromagnetic properties of a metal wall are determined by the dielectric $\varepsilon_1 = \varepsilon_0 + j\,\sigma/\omega$ and magnetic $\mu_1 = \mu_0$ (ε_0 and μ_0 are dielectric and magnetic permeability of vacuum) permeability of the wall material.

The search of a solution is performed in the form of a superposition of a particular solution $\vec{E}_n^0, \vec{H}_n^0$ of the inhomogeneous Maxwell equations and the general solution $\vec{E}_n^i, \vec{H}_n^i$ of the homogeneous Maxwell equations in the form of multipole expansions of TM and TE components of a point charged particle radiation fields:

$$\vec{E} = \sum_{n=1}^{\infty}\{\vec{E}_n^0 + \vec{E}_n^i\},\ \vec{H} = \sum_{n=1}^{\infty}\{\vec{H}_n^0 + \vec{H}_n^i\}, \qquad (2)$$

$$\vec{E}_n^0 = \vec{E}_n^{0,TM} + \vec{E}_n^{0,TE},\ \vec{H}_n^0 = \vec{H}_n^{0,TM} + \vec{H}_n^{0,TE}$$
$$\vec{E}_n^i = \vec{E}_n^{i,TM} + \vec{E}_n^{i,TE},\ \vec{H}_n^i = \vec{H}_n^{i,TM} + \vec{H}_n^{i,TE} \qquad (3)$$

The solutions are based on the vector functions containing the Bessel J_n and Hankel $H_n^{(1)}$ functions of the first kind:

$$\vec{e}_J = \{(\alpha r)^{-1} n J_n(\alpha r),\ j J_n'(\alpha r),\ 0\}\,exp(j\psi_n),$$
$$\vec{e}_H = \left\{(\alpha r)^{-1} n H_n^{(1)}(\alpha r),\ j H_n^{(1)'}(\alpha r),\ 0\right\} exp(j\psi_n),$$
$$\alpha = \begin{cases} \alpha_1 = \sqrt{\omega^2\varepsilon_1\mu_1 - k^2}, & in\ metal \\ \alpha_0 = \sqrt{\omega^2/c^2 - k^2}, & in\ vacuum \end{cases} \qquad (4)$$

In Eq. (4): α is the transverse wavenumber and $\psi_n = k(z - vt) + n(\varphi - \omega_0 t)$ is the phase factor with $k = (\omega - n\omega_0)/V$ being the longitudinal wavenumber.

PARTICULAR SOLUTION

As a particular solution of the inhomogeneous Maxwell equations, one takes the solution for the radiation of a particle moving along a helical trajectory in free space:

$$\vec{E}_n^0 = \vec{E}_n^{0,TM} + \vec{E}_n^{0,TE},\ \vec{H}_n^0 = \vec{H}_n^{0,TM} + \vec{H}_n^{0,TE} \qquad (5)$$

with

$$\vec{E}_n^{0,TM} = \begin{Bmatrix} \vec{E}_{H,n}^{0,TM} \\ \vec{E}_{J,n}^{0,TM} \end{Bmatrix} = \begin{cases} A_{H,n}^{0,TM} rot\ \vec{e}_H, & r > a \\ A_{J,n}^{0,TM} rot\ \vec{e}_J, & r < a \end{cases},$$

$$\vec{H}_n^{0,TM} = \begin{Bmatrix} \vec{H}_{H,n}^{0,TM} \\ \vec{H}_{J,n}^{0,TM} \end{Bmatrix} = \begin{cases} B_{H,n}^{0,TM} \vec{e}_H, & r > a \\ B_{J,n}^{0,TM} \vec{e}_I, & r < a \end{cases}$$

$$\vec{E}_n^{0,TE} = \begin{Bmatrix} \vec{E}_{H,n}^{0,TE} \\ \vec{E}_{J,n}^{0,TE} \end{Bmatrix} = \begin{cases} A_{H,n}^{0,TE} \vec{e}_H, & r > a \\ A_{J,n}^{0,TE} \vec{e}_J, & r < a \end{cases},$$

$$\vec{H}_n^{0,TE} = \begin{Bmatrix} \vec{H}_{H,n}^{0,TE} \\ \vec{H}_{J,n}^{0,TE} \end{Bmatrix} = \begin{cases} B_{H,n}^{0,TE} rot\ \vec{e}_H, & r > a \\ B_{J,n}^{0,TE} rot\ \vec{e}_J, & r < a \end{cases} \qquad (6)$$

Amplitudes $A_{H(J),n}^{0,TM(TE)}$, $B_{H(J),n}^{0,TM(TE)}$ remain undefined. To determine them, one should use the boundary conditions that establish a connection between the fields on both sides of the surface $r = a$ containing charges $\rho = q$ and currents

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$\vec{j} = q\{0,\ \omega_b a,\ v\}$ [10, 11]. The conditions for the discontinuity of fields on the surface $r = a$ are reduced to two systems of equations (for TM (7) and TE (8) modes):

$$\begin{aligned} &1)\quad E_{H,z}^{0,TM} - E_{J,z}^{0,TM} = 0 \\ &2)\quad -E_{H,\varphi}^{0,TM} + E_{J,\varphi}^{0,TM} = 0 \\ &3)\quad -H_{H,\varphi}^{0,TM} + H_{J,\varphi}^{0,TM} = q\chi_n j_{n,z}^{TM} \\ &4)\quad \varepsilon_0\left(E_{H,r}^{0,TM} - E_{J,r}^{0,TM}\right) = q\chi_n \rho_n^{TM} \\ &5)\quad H_{H,r}^{0,TM} - H_{J,r}^{0,TM} = 0 \end{aligned} \tag{7}$$

$$\begin{aligned} &1)\quad -E_{H,\varphi}^{0,TE} + E_{J,\varphi}^{0,TE} = 0 \\ &2)\quad H_{H,z}^{0,TE} - H_{J,z}^{0,TE} = q\chi_n j_{n,\varphi}^{TE} \\ &3)\quad -H_{H,\varphi}^{0,TE} + H_{J,\varphi}^{0,TE} = q\chi_n j_{n,z}^{TE} \\ &4)\quad \varepsilon_0\left(E_{H,r}^{0,TE} - E_{J,r}^{0,TE}\right) = q\chi_n \rho_n^{TE} \\ &5)\quad H_{H,r}^{0,TE} - H_{J,r}^{0,TE} = 0 \end{aligned} \tag{8}$$

In Eqs. (7) and (8), the normalized components of currents $j_{n,z}^{TM}$, $j_{n,z}^{TE}$, $j_{n,\varphi}^{TE}$ ($j_z^{TM} + j_z^{TE} = V$) and charges ρ_n^{TM}, ρ_n^{TE} ($\rho_n^{TM} + \rho_n^{TE} = 1$), responsible for the formation of TM and TE modes n[th] harmonic, and weight factor χ_n [11] are introduced. Each of systems Eqs. (7) and (8) contains five equations and four amplitudes to be determined. The components of the currents and charges are determined from the compatibility conditions for all equations included in systems Eqs. (7) and (8): $j_z^{TE} = kn\omega_0/\alpha_0^2$, $j_\varphi^{TE} = \omega_0 a$, $\rho_n^{TE} = n\omega_0\omega/c^2\alpha_0^2$. Below are the final expressions for the amplitudes:

$$\begin{aligned} A_{n,J}^{0,TM} &= -jq\frac{\pi}{2}\frac{a\chi_n}{\alpha_0 c^2\varepsilon_0}(V\omega - c^2 k)H_n^{(1)}(\alpha_0 a), \\ A_{n,H}^{0,TM} &= -jq\frac{\pi}{2}\frac{a\chi_n}{\alpha_0 c^2\varepsilon_0}(V\omega - c^2 k)J_n(\alpha_0 a) \\ B_{n,J}^{0,TM} &= -j\varepsilon_0\omega A_{n,J}^{0,TM},\ B_{n,H}^{0,TM} = -j\varepsilon_0\omega A_{n,H}^{0,TM} \\ A_{n,J}^{0,TE} &= jq\frac{\pi}{2}\frac{a^2\chi_n\omega\omega_0}{c^2\varepsilon_0}H_n^{(1)}(\alpha_0 a), \\ A_{n,H}^{0,TE} &= jq\frac{\pi}{2}\frac{a^2\chi_n\omega\omega_0}{c^2\varepsilon_0}J_n'(\alpha_0 a) \\ B_{n,J}^{0,TE} &= -j\,A_{n,J}^{0,TE}/\omega\mu_0,\ B_{n,H}^{0,TE} = -j\,A_{n,H}^{0,TE}/\omega\mu_0 \end{aligned} \tag{9}$$

COMPLETE SOLUTION

The total radiation field of a particle in the inner region of waveguide ($0 \le r \le b$) is presented in the form of the sum of the general solution of Maxwell's equations with indefinite weight amplitudes and the above determined particular solution (radiation field of a particle in free space (6, 9)) of the same equations:

$$\begin{aligned} \vec{E}^{in} &= \vec{E}^{1,TM} + \vec{E}^{1,TE} + \vec{E}^{0,TM} + \vec{E}^{0,TE} \\ \vec{H}^{in} &= \vec{H}^{1,TM} + \vec{H}^{1,TE} + \vec{H}^{0,TM} + \vec{H}^{0,TE} \end{aligned} \tag{10}$$

Fields $\vec{E}^{0,TM}$, $\vec{E}^{0,TE}$ are determined by Eq. (6), (9), and fields $\vec{E}^{1,TM}$, $\vec{E}^{1,TE}$ are sought in the form:

$$\begin{aligned} &\vec{E}^{1,TM} = A_n^{1,TM} rot\ \vec{e}_J,\ \vec{H}^{1,TM} = B_n^{1,TM}\vec{e}_J, \\ &B_n^{1,TM} = -jA_n^{1,TM}\varepsilon_0\omega, \\ &\vec{E}^{1,TE} = A_n^{1,TE}\,\vec{e}_J,\ \vec{H}^{1,TE} = B_n^{1,TE} rot\ \vec{e}_J, \\ &B_n^{1,TE} = -j\,A_n^{1,TE}/\mu_0\omega. \end{aligned} \tag{11}$$

The field in the wall of the waveguide is represented as

$$\vec{E}^{out} = \vec{E}^{2,TM} + \vec{E}^{2,TE},\ \vec{H}^{out} = \vec{H}^{2,TM} + \vec{H}^{2,TE} \tag{12}$$

$$\begin{aligned} &\vec{E}^{2,TM} = A_n^{2,TM} rot\ \vec{e}_H,\ \vec{H}^{2,TM} = B_n^{2,TM}\vec{e}_H, \\ &B_n^{2,TM} = -jA_n^{2,TM}\varepsilon_1\omega, \\ &\vec{E}^{2,TE} = A_n^{2,TE}\,\vec{e}_H,\ \vec{H}^{2,TE} = B_n^{2,TE} rot\ \vec{e}_H, \\ &B_n^{2,TE} = -j\,A_n^{2,TE}/\mu_1\omega. \end{aligned} \tag{13}$$

The further is matching the tangential components of the electric and magnetic fields Eqs. (10) and (12) on the waveguide wall. This leads to a system of four linear equations for the amplitudes $A_n^{1,TM}$, $A_n^{1,TE}$, $A_n^{2,TM}$ and $A_n^{2,TE}$ with solution:

$$\begin{aligned} \hat{A} &= \hat{A}_1 + \hat{A}_2 \\ \hat{A}_{1,2} &= \{A_{n_{1,2}}^{1,TM}, A_{n_{1,2}}^{1,TE}, A_{n_{1,2}}^{2,TM}, A_{n_{1,2}}^{2,TE}\} \\ A_{n_1}^{1,TM} &= jq\frac{\pi}{2}a\chi_n C_u\tilde{J}_n\frac{W_{\varepsilon\mu}}{\alpha_0 c^2\varepsilon_0 D}, \\ A_{n_1}^{1,TE} &= qa\chi_n kn\mu_0\frac{C_u\alpha_1^2\alpha_{01}H_n^2\tilde{J}_n\omega^2}{\alpha_0 c^2 D}, \\ A_{n_1}^{2,TM} &= -qa\chi_n b\frac{C_u\alpha_0^2\alpha_1^2\tilde{J}_n I_\mu\omega^2}{\alpha_0 c^2 D}, \\ A_{n_1}^{2,TE} &= qa\chi_n kn\mu_1\frac{C_u\alpha_1\alpha_{01}^2 H_n J_n\tilde{J}_n\omega^2}{c^2 D}, \\ A_{n_2}^{1,TM} &= -qa^2\alpha_1^2\alpha_{01}\chi_n\frac{\tilde{J}_n' H_n^2 kn\omega\omega_0}{c^2\varepsilon_0 D}, \\ A_{n_2}^{1,TE} &= -jq\frac{\pi}{2}\chi_n ka^2\omega\omega_0\tilde{J}_n'\frac{W_{\mu\varepsilon}}{c^2\varepsilon_0 D} \\ A_{n_2}^{2,TM} &= -qa^2\chi_n kn\omega\omega_0\alpha_0\alpha_1\alpha_{01}\frac{\tilde{J}_n' H_n J_n}{c^2\varepsilon_0 D}, \\ A_{n_2}^{2,TE} &= q\mu_1\chi_n ba^2\omega^3\omega_0\alpha_0^2\alpha_1^2\frac{\tilde{J}_n' I_\varepsilon}{c^2\varepsilon_0 D} \end{aligned} \tag{14}$$

In Eq. (14) the following notations are introduced:

$$\begin{aligned} &J_n = J_n(\alpha_0 b), H_n = H_n^{(1)}(\alpha_1 b),\ C_u = V\omega - c^2 k \\ &\tilde{J}_n = J_n(\alpha_0 a), \widetilde{H}_n = H_n^{(1)}(\alpha_0 b),\ \alpha_{01} = \alpha_0^2 - \alpha_1^2 \\ &\left.\begin{matrix} I_\varepsilon \\ I_\mu \end{matrix}\right\} = -\alpha_1 J_n' H_n\begin{Bmatrix}\varepsilon_0 \\ \mu_0\end{Bmatrix} + \alpha_0 J_n H_n'\begin{Bmatrix}\varepsilon_1 \\ \mu_1\end{Bmatrix} \\ &\left.\begin{matrix} Y_\varepsilon \\ Y_\mu \end{matrix}\right\} = -\alpha_1\widetilde{H}_n' H_n\begin{Bmatrix}\varepsilon_0 \\ \mu_0\end{Bmatrix} + \alpha_0\widetilde{H}_n H_n'\begin{Bmatrix}\varepsilon_1 \\ \mu_1\end{Bmatrix} \\ &\left.\begin{matrix} W_{\varepsilon\mu} \\ W_{\mu\varepsilon} \end{matrix}\right\} = k^2 n^2\alpha_{01}^2 H_n^2 J_n\widetilde{H}_n - b^2\alpha_0^2\alpha_1^2\begin{Bmatrix}Y_\varepsilon I_\mu \\ Y_\mu I_\varepsilon\end{Bmatrix}\omega^2 \\ &D = k^2 n^2\alpha_{01}^2 H_n^2 J_n^2 - b^2\alpha_0^2\alpha_1^2 I_\varepsilon I_\mu\omega^2 \end{aligned} \tag{15}$$

The roots of dispersion equation $D = 0$ Eq. (15) determines the discrete eigenfrequencies where the real components are the resonant frequencies, and imaginary ones are the modes attenuation coefficients. In the limit of an ideal waveguide ($\varepsilon_1 \to \infty$) the nonzero in Eq. (14) remain the amplitudes $A_{n_1}^{1,TM}$ and $A_{n_2}^{1,TE}$, correspond to the fields propagating inside the waveguide:

$$\begin{aligned} A_{n_1}^{1,TM} &= jq\frac{\pi}{2}\frac{a\chi_n C_u H_n^{(1)}(\alpha_0 b)J_n(\alpha_0 a)}{\alpha_0 c^2\varepsilon_0 J_n(\alpha_0 b)}, \\ A_{n_2}^{1,TE} &= -jq\frac{\pi}{2}\omega\omega_0\frac{a^2\chi_n H_n^{(1)'}(\alpha_0 b)J_n'(\alpha_0 a)}{c^2\varepsilon_0 J_n'(\alpha_0 b)}. \end{aligned} \tag{16}$$

The complete solution for an ideal waveguide is the sum of the general solution Eq. (11) and (16) and the particular solution Eqs. (6) and (9). For comparison with existing solutions for an ideal waveguide [3, 12, 13], the obtained limiting solution was expanded in terms of the eigenfunctions of an ideal waveguide. The presence of an additional pole

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at $\alpha_0 = 0$ in the obtained solution is detected. To eliminate its contribution, the function χ_n is given the form

$$\chi_n = a^{-1}\left\{1 - 4^n\Gamma^2(n)n\frac{J_n(\alpha_0 b)J_n'(\alpha_0 b)}{(\alpha_0 b)^{2n-1}}\right\},$$
$$\Gamma(n) = (n-1)!, \qquad (17)$$

which does not distort the contributions of the poles at $\alpha_0^2 = j_{nm}^2$, $J_n(j_{nm}b) = 0$ (TM modes) and $\alpha_0^2 = \nu_{nm}^2$, $J_n'(\nu_{nm}b) = 0$ (TE modes). Passing to the space-time representation (integration over frequency in the sense of the principal value) in both cases gives the same result.

The radiation energy of the multipole, accumulated in the waveguide is determined by the formula (cross terms are neglected due their small contribution):

$$\mathcal{E}_n = 2\pi\int_{-\infty}^{\infty} W_n(\omega)d\omega, \qquad (18)$$
$$W_n(\omega) = \frac{2\pi}{V}\int_0^b\left\{\varepsilon_0\left|\vec{E}^{1,TM} + \vec{E}^{1,TE}\right|^2 + \mu_0\left|\vec{H}^{1,TM} + \vec{H}^{1,TE}\right|^2\right\}rdr, \qquad (19)$$

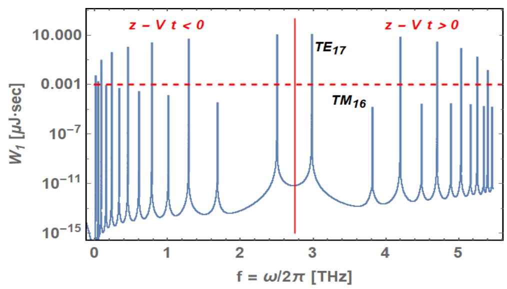

Figure 1: Spectral distribution of dipole mode ($n = 1$) radiation in a copper waveguide; forward (right) and back (left) radiation.

where $W_n(\omega)$ is a spectral energy density. The relationships between the undulator parameter K, the undulator period l_u, and the Lorentz factor γ of a particle with an orbit radius $a = l_u K/2\pi\gamma$, longitudinal velocity $V = c\{1 - (1 + K^2)/2\pi\gamma^2\}$, and rotation frequency $\omega_0 = 2\pi V/l_u$ are used. In the calculations (Fig. 1-3), the following parameter values were used: $b = 1\ cm, K = 0.42,\ l_u = 8\ cm, \gamma = 29.35, q = 10\ pC$ and $\sigma = 58 \cdot 10^6 \Omega^{-1}m^{-1}$ (copper). Figure 1 shows the spectral distribution of the dipole mode ($n = 1$), calculated by these parameters, providing terahertz radiation both for ahead of the particle ($z - Vt > 0$) and in the opposite direction ($z - Vt < 0$). The peaks above the level $1\ nJ \cdot sec$ constitutes the TE modes. Another row with peaks less than $1\ nJ \cdot sec$ corresponds to TM modes. As in an ideal waveguide, the TE modes prevail. Figure 2 compares the spectra of resonant frequencies for an ideal and copper waveguide, presented in the space-time domain. The fields in this representation, as in [3], are obtained by integrating the spectral distribution of the field components Eqs. (11) and (14) over frequency using the contributions from residues (at $D = 0$) on the complex plane. The frequency overlapping and profile repetition at a somewhat low level (due to finite conductivity) takes place. In both cases, the $TE_{1,7}$ mode accumulates most of the energy.

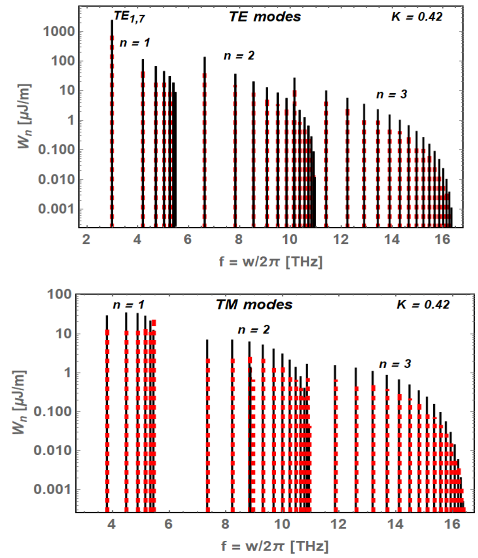

Figure 2: Spectral distribution (space-time domain, forward direction, $z - Vt \to 0$) of first three multipoles ($n = 1,2,3$) stored radiation energy for TE (top) and TM (bottom) modes in a copper (red, dashed) and ideal (black, solid) waveguides.

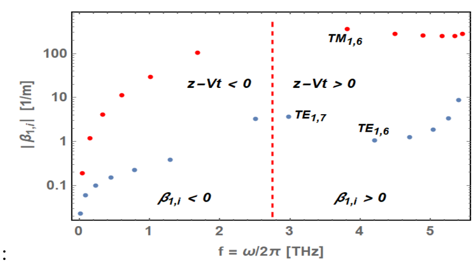

Figure 3: Damping factors of TE (blue) and TM (red) modes at resonant frequencies for $n = 1$.

For a resistive waveguide, an important parameter is the mode damping factor $\beta_{n,i} = Im\{\omega_{n,i}\}/V$, where $\omega_{n,i}$ are the roots of the equation $D = 0$. As can be seen from Fig. 3, TM modes have significantly higher attenuation than TE modes. The backward radiation is attenuated more weakly. Mode $TE_{1,6}$ has the minimum attenuation in the forward direction.

CONCLUSION

The obtained exact solution, in contrast to the case of an ideal waveguide, has no singularities at the critical (TE modes, space-time domain) and resonant (TE and TM modes, frequency domain) frequencies. Along with the greater prevalence of TE modes (due to higher attenuation of TM modes) than in an ideal waveguide, the maximum value of the amplitude of the dominant TE mode is limited due to the finite conductivity of the walls.

The solution presents a realistic picture of radiation and creates wide opportunities for research on optimizing the parameters of the structure, depending on its purpose.

The solution can be extended to two-layer (by the method described in [9]) and multilayer (by analogy with [14]) waveguides

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