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INTRABUNCH MOTION WITH BOTH IMPEDANCE AND BEAM-BEAM USING THE CIRCULANT MATRIX APPROACH

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Abstract

In high-intensity high-brightness circular colliders such as the CERN LHC, coherent beam-beam effects and impedance cannot be treated independently. Coherent beam-beam dipole modes can couple with higher order head-tail modes and lead to the transverse mode coupling instability of colliding beams. This mechanism has been analysed in detail in the past through the eigenvalues, which describe the evolution of the beam oscillation mode-frequency shifts. In this contribution, the transverse mode coupling instability of colliding beams is studied using the eigenvectors, which describe the evolution of the intrabunch motion. As this instability exhibits several mode couplings and mode decouplings, the evolution of the intrabunch motion reveals quite some interesting features (such as a propagation of the traveling-wave not only from the head to the tail but also from the tail to the head and similar intrabunch signals for some mode coupling and mode decoupling), which are compared to past predictions in the presence of impedance only.

INTRODUCTION

A transverse mode coupling instability (TMCI) can be observed in the presence of impedance only [1], impedance and tune spread [2], impedance and beam-beam [3], electron cloud [4], impedance and space charge [5]. These instabilities are usually studied analytically with the linearized Vlasov equation, ending up with an eigenvalue system to solve, and in particular looking at the evolution of the eigenvalues as a function of the bunch intensity. However, in the presence of nonlinearities or when higher-order modes are involved, this becomes quite difficult, if not impossible, and the coupling between the modes cannot be directly measured (or simulated) anymore. Another important (and always accessible) observable is the intrabunch motion, which can be also computed analytically thanks to the eigenvectors. The case of an impedance only was already discussed in some detail in Refs. [6] [7], where it could be seen that the oscillation amplitude of the head of the bunch is significantly higher than the tail just above the TMCI intensity threshold. Increasing the intensity further beyond the threshold shifts the peak oscillation amplitude to the tail of the bunch. The purpose of this paper is twofold: first, study the intrabunch motion for the case of the CERN LHC with its transverse impedance only, using the circulant matrix approach [8] [9] and compare to previous studies [6] [7]; second, extend this type of analysis to the case where one beam-beam head-on interaction is added (see Table 1).

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Table 1: Machine, beam and numerical parameters for the studies with beam-beam (or impedance only)

Energy [TeV]	7
Bunch population [10^{11}]	1.2 (or scanned)
Bunch length [cm]	7.6
Rel. momentum spread [10^{-4}]	1.1
Trans. tune	0.31
Long. tune [10^{-3}]	2.1
Number of interaction points	1 (or 0)
Beam-beam model	Round, Gaussian and linearised
Number of slices	80
Number of rings	40
Slicing	Equidistant
Impedance / wake model	LHC 2022 flat top [11]

INTRABUNCH MOTION WITH IMPEDANCE ONLY

In the case of the transverse impedance only, the beam oscillation mode-frequency shifts as a function of the bunch intensity for the CERN LHC are depicted in Fig. 1, where it can be deduced that the TMCI intensity threshold appears at about 4.45×10^{11} protons. The evolution of the transverse intrabunch motion as a function of the bunch intensity is revealed in Fig. 2, where a similar behaviour as in previous studies [6] [7] can be observed. As predicted, we see first the signal mainly in the head with a signal in the tail growing with bunch intensity. We see, also as predicted, a fixed point (in the tail) at the start of the TMCI, which then disappears for higher bunch intensities. It is replaced by a traveling-wave with an amplitude growing from the head to the tail: it is worth stressing that when we mention "traveling-wave" here, we refer to the propagation of the maximum of the signal amplitude. After these careful checks, it can be concluded that the circulant matrix approach leads to the same results obtained in the past with the PyHEADTAIL macroparticle tracking code [10] [6] and the Vlasov formalism [7]. The combined effect of the impedance and beam-beam is discussed in the next section.

INTRABUNCH MOTION WITH BOTH IMPEDANCE AND BEAM-BEAM

In the case of both the transverse impedance and one beam-beam head-on interaction, the usual TMCI plots for the CERN LHC are depicted in Fig. 3, as a function of the beam-beam parameter. Considering round beams without crossing angle and neglecting the hourglass effect, the beam-

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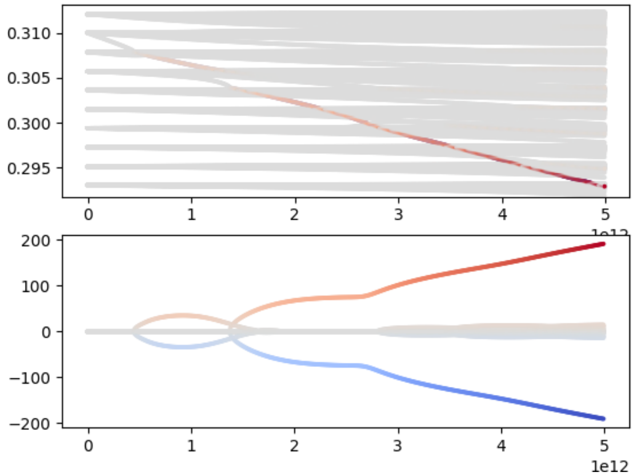

Figure 1: Real part (top) and imaginary part (bottom) of the beam oscillation mode-frequencies as a function of the bunch intensity, for the case of the CERN LHC transverse impedance only.

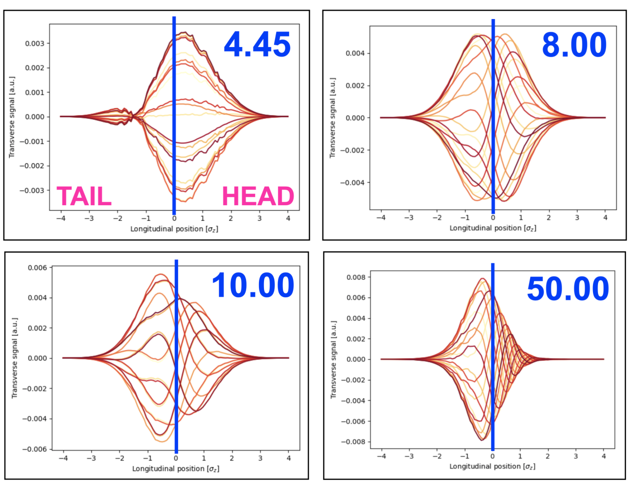

Figure 2: Signals of transverse intrabunch motion with impedance only for 4 different bunch intensities: (top left) TMCI intensity threshold of 4.45×10^{11} protons; (top right) 8.00×10^{11} protons; (bottom left) 10.00×10^{11} protons; (bottom right) 50.00×10^{11} protons.

beam parameter is given by

$$\xi = \frac{N r_0}{4\pi \epsilon_n}, \quad (1)$$

with N the bunch intensity (number of particles), r_0 the classical particle radius and ϵ_n the normalised transverse emittance. The impact of the impedance is kept constant by considering identical bunch intensities for all computations and the transverse emittance is varied to obtain the corresponding beam-beam parameter. It is worth stressing that the case $\xi = 0$ in Fig. 3 does not mean zero intensity, it means infinite emittance (since we keep the intensity constant) and this explains why the beam-beam σ and π modes for $\xi = 0$ appear slightly below the tune of 0.31 as they are slightly shifted by the impedance.

This configuration exhibits two mode couplings and decouplings. The first mode coupling and decoupling come from the interaction between the beam-beam π-mode and the head-tail mode $m = -1$, while the second mode coupling and decoupling come from the interaction between the beam-beam σ-mode and the head-tail mode $m = +1$. This behaviour correspond to the behaviour already obtained in [3] for a configuration without synchro-betatron coupling due to the beam-beam interaction.

The evolution of the transverse intrabunch motion as a function of the beam-beam parameter is revealed in Fig. 4 for beam 1. Between the first mode coupling and mode decoupling, i.e. between the beam-beam parameters 3.16×10^{-3} and 3.87×10^{-3}, the evolution of the intrabunch motion is very similar to the one predicted from impedance only (see previous section and Ref. [7] where the case of mode decoupling was also discussed). Indeed, we can observe first the head-dominated signal, with the node in the tail at the mode coupling threshold and then a traveling-wave from head to tail, ending up at the mode decoupling threshold with a signal very close to the symmetric one from the mode coupling (as predicted in Ref. [7]). What is new here is that when the second mode coupling starts, the intrabunch motion is tail-dominated (with a signal very close to the one at the threshold of the first mode decoupling) and then there is a traveling-wave with an amplitude growing from the tail to the head, to end up at the threshold of the second mode decoupling with a signal very close to the initial one at the threshold of the first mode coupling. Therefore, in the presence of several mode couplings and decouplings, the maximum amplitude of the traveling-wave does not necessarily increase from the head to the tail but may also increase from the tail to the head. In addition similar intrabunch signals can be observed for some mode coupling and mode decoupling. Therefore, without the knowledge from the picture of the mode couplings from the eigenvalues, it would be impossible to say if the pictures from Fig. 4 correspond to mode coupling or mode decoupling.

Looking now at the beam 2, the corresponding signals of transverse intrabunch motion are depicted in Fig. 5, where it can be seen that the first two signals (linked to the mode coupling and decoupling between the beam-beam π-mode and the head-tail mode $m = -1$) are 180-degree out-of-phase with the corresponding beam 1 signals (compare the traces with the same colour), while the following two signals (linked to the mode coupling and decoupling between the beam-beam σ-mode and the head-tail mode $m = +1$) are in-phase with the corresponding beam 1 signals (see Fig. 4). The phase relation between the beams is therefore preserved for the σ and π modes when coupled with higher order head-tail modes.

CONCLUSION

A transverse mode coupling instability can be analysed either through the eigenvalues or the eigenvectors. In some cases, these two approaches lead to the same conclusions and

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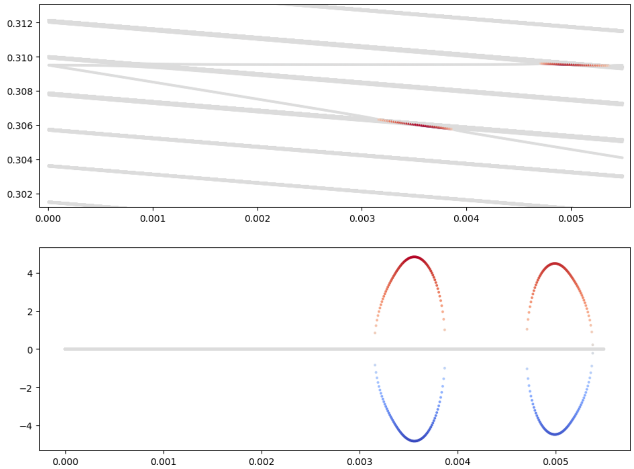

Figure 3: Usual TMCI plots with the real part (top) and imaginary part (bottom) of the eigenvalues as a funtion of the beam-beam parameter, for the case of both impedance and beam-beam.

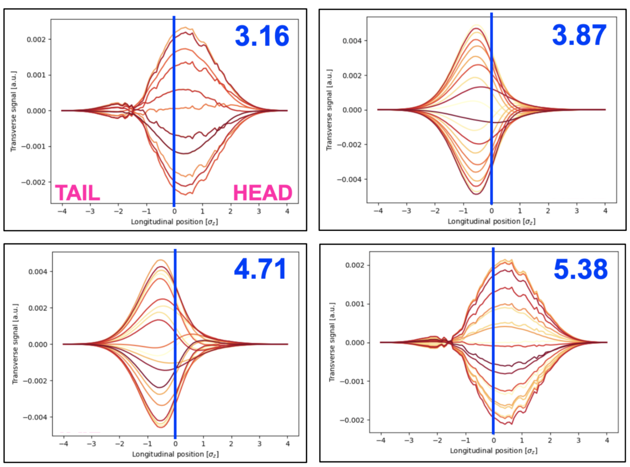

Figure 4: Signals of transverse intrabunch motion, for beam 1, with both impedance and beam-beam for the 4 beam-beam parameters corresponding to the TMCI intensity thresholds of Fig. 3: (top left) 3.16×10^{-3}; (top right) 3.87×10^{-3}; (bottom left) 4.71×10^{-3}; (bottom right) 5.38×10^{-3}.

one or the other approach can be used in the presence of only one mode coupling and decoupling (if any). Indeed, for the impedance-induced TMCI studied in the past, the intrabunch motion was observed to be head-dominated "just above" the TMCI threshold. Therefore, if a head-dominated signal is observed in simulations or measurements, one could then deduce that the beam is "just above" the TMCI threshold, while if the signal is in the tail, one could deduce that the beam is "well above" the TMCI threshold. However, in the presence of several mode couplings and decouplings (as e.g. discussed in this paper), the situation is more involved and it is not possible anymore to tell, from a single picture of intrabunch motion only, if it corresponds to mode coupling or mode decoupling. To be able to reconstruct what really

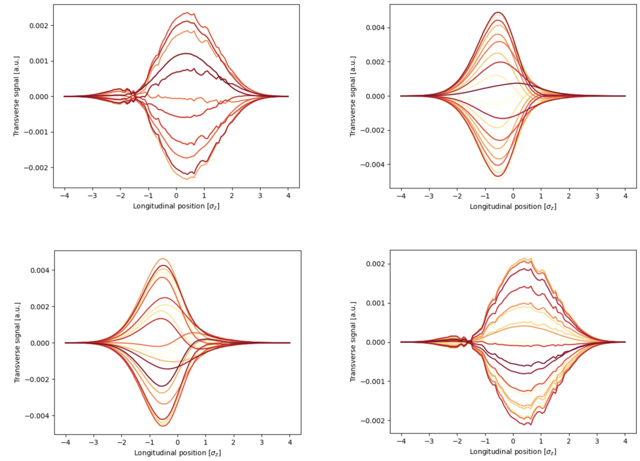

Figure 5: Same as Fig. 4 but for beam 2.

happens, the intrabunch motion needs be carefully studied as a function of the bunch intensity [7].

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[10] PyHEADTAIL, https://github.com/PyCOMPLETE/PyHEADTAIL

[11] https://impedance.web.cern.ch/