# MONTE CARLO SIMULATION OF TOUSCHEK EFFECTS IN A LINAC BEAM *

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*Abstract*

We present a Monte Carlo method implemented in the code `elegant` [1] for simulating Touschek scattering effects in a linac beam. The local scattering rate and the distribution of scattered electrons can be obtained from the code. In addition, scattered electrons can be tracked to the end of the beamline and the local beam loss rate and beam halo information recorded. This information can be used for beam collimation system design.

## INTRODUCTION

The Touschek effect is a single Coulomb scattering effect between charged particles in a beam. For a relativistic beam, a small change of transverse momentum results in a much larger change of longitudinal momentum, as the momentum change is increased by the Lorentz factor $\gamma$. When the change exceeds the machine's momentum acceptance, the scattered particle is lost.

The Touschek effect is well understood in storage rings and has been largely ignored for linac beams in the past due to the negligible loss rate. However, this is not the case for an intense electron bunch with ultra-low emittance and very short bunch length that passes through a linac with a very high repetition rate, as occurs in proposed Energy Recovery Linacs (ERLs) [2]. This is a concern for a possible ERL upgrade of the APS, since the radiation shielding of the ring is not designed for high continuous loss rates [3].

Previously, a preliminary theoretical analysis [4] was performed using Piwinski's formula [5]. To better determine the scattered electron distribution and determine precisely the electron loss rate and loss position, we have included a Monte Carlo simulation method in `elegant` for studying the Touschek scattering effect.

In this paper, we review the theory of Touschek scattering, then describe the method used for the Monte Carlo simulation. The scattering rates calculated from the Monte Carlo simulation and Piwinski's formula are compared. The strategy used for calculating beam loss rate and location in `elegant` is then explained. Finally, we give an application example to a proposed APS ERL lattice design.

## THEORY DESCRIPTION

In the center-of-mass (CM) system [1], the probability of one of the two encountered electrons being scattered into a solid angle $d\Omega^*$ is given by the differential Møller cross section [6]

$$\frac{d\sigma^*}{d\Omega^*} = \frac{r_e^2}{4\gamma^{*2}}\left[\left(1+\frac{1}{\beta^{*2}}\right)^2 \frac{4-3\sin^2\Theta^*}{\sin^4\Theta^*} + \frac{4}{\sin^2\Theta^*} + 1\right], \tag{1}$$

where $r_e$ is the classical electron radius; $\gamma^*$ and $\beta^*$ are the relative energy and velocity of scattered electrons in the CM system, respectively; $\Theta^*$ is the angle between the momenta before and after scattering; and $d\Omega^* = \sin\Theta^* d\Theta^* d\Psi^*$.

The total scattering rate $R$ is given by the integration over all possible scattering angles and over all electrons in the bunch. In the CM system,

$$R^* = 2\int |v^*|\sigma^*\rho(\vec{x}_1^*)^*\rho(\vec{x}_2^*)^* dV^*, \tag{2}$$

where $v^*$ is the scattered electrons' velocity, $\sigma^*$ is the total Møller cross section, $\vec{x}^* = (x^*, y^*, z^*, p_x^*, p_y^*, p_z^*)$, $\rho(x_i^*)^*$ is the electron phase-space density, and $dV = dx^*dy^*dz^*dp_{x1}^*dp_{y1}^*dp_{z1}^*dp_{x2}^*dp_{y2}^*dp_{z2}^*$. $\sigma^*$ is integrated over the solid angle $d\Omega^*$ with $\Theta^* \in (0, \frac{\pi}{2}]$, $\Psi^* \in [0, 2\pi]$:

$$\sigma^* = \int_0^{2\pi}\int_0^{\pi/2} \frac{d\sigma^*}{d\Omega^*}\sin\Theta^* d\Theta^* d\Psi^*. \tag{3}$$

The reason for $\Theta^* \in (0, \frac{\pi}{2}]$ is that, if one electron is scattered into the region $0 < \Theta^* \le \frac{\pi}{2}$, then the other is scattered into the region $\frac{\pi}{2} \le \Theta^* < \pi$. The factor "2" in Equation (2) includes both regions.

For the problem we are interested in, we assume that $p_x \ll p_z$ and $p_y \ll p_z$, which means that the Lorentz transformation is mainly taking place along the $z$ direction, and $\sigma^*$ is parallel to the $z^*$-axis. Transforming to the laboratory coordinate system gives

$$|v|\sigma = \frac{|v^*|}{\gamma}\frac{\sigma^*}{\gamma} \tag{4}$$

and

$$R = 2\int |v|\sigma\rho(\vec{x_1})\rho(\vec{x_2})dV, \tag{5}$$

with

$$dV = dx_\beta dy_\beta d\Delta z dx'_{\beta 1}dx'_{\beta 2}dy'_{\beta 1}dy'_{\beta 2}d\Delta p_1 d\Delta p_2. \tag{6}$$

Ignoring coupling, for an electron bunch with Gaussian distribution, the electron's density in phase space $(x_\beta, x'_\beta, y_\beta, y'_\beta, \Delta z, \Delta p/p_0)$ is given by

$$\rho = \frac{N\beta_x\beta_y}{8\pi^3\sigma_z\sigma_p\sigma_{x\beta}^2\sigma_{y\beta}^2}\exp\left\{-\frac{\Delta z^2}{2\sigma_z^2} - \frac{1}{2\sigma_p^2}\frac{\Delta p^2}{p_0^2}\right\} \times \exp\left\{-\frac{x_\beta^2+(\alpha_x x_\beta+\beta_x x'_\beta)^2}{2\sigma_{x\beta}^2} - \frac{y_\beta^2+(\alpha_y y_\beta+\beta_y y'_\beta)^2}{2\sigma_{y\beta}^2}\right\}, \tag{7}$$

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[1] For clarity, we use (*) to denote all quantities in the CM system, as opposed to quantities in the laboratory coordinate system.

where $N$ is the total number of electrons in the bunch; $\beta_{x,y}$ and $\alpha_{x,y}$ are the local optical functions; $\sigma_{x\beta,y\beta}$ are the transverse beam sizes for on-momentum electrons; $\sigma_z$ is the bunch length; and $\sigma_p$ is the energy spread. A scattering event happens when the following conditions are satisfied

$$\begin{aligned} \Delta z_1 &= \Delta z_2, \\ x_{\beta 1} + D_x \frac{\Delta p_1}{p_0} &= x_{\beta 2} + D_x \frac{\Delta p_2}{p_0}, \\ y_{\beta 1} + D_y \frac{\Delta p_1}{p_0} &= y_{\beta 2} + D_y \frac{\Delta p_2}{p_0}, \end{aligned} \tag{8}$$

where $D_{x,y}$ are the dispersion functions.

Equation (5) is a general expression for the Coulomb scattering rate. The Touschek scattering rate (beam loss rate) is $R_T = R(|\Delta p/p_0| > \delta_m)$, where $\delta_m = \Delta p_m/p_0$ is the machine's local momentum aperture, i.e., the momentum aperture relevant to particles scattered at a specific location.

## MONTE CARLO SIMULATION

The method used in `elegant` for simulating electron scattering processes is modified from S. Khan's work for BESSY II [7, 8]. For each simulated scattering event, a total of 11 random numbers are generated and used to determine the position of the scattering event $(x, y, z)$; the momenta of electron 1 $(x'_1, y'_1, \Delta p_1)$; the momenta of electron 2 $(x'_2, y'_2, \Delta p_2)$; and the scattering angles $(\Theta^*, \Psi^*)$. Dispersion corrections are included in the process of assignment. The electron's momenta after scattering are calculated and saved. We compute the integral in Equation (5) using Monte Carlo integration with $N$ uniform distributed samples in the n-dimensional volume $V$, e.g.,

$$\int_V f(\vec{x})d\vec{x} \approx \frac{V}{N}\sum_{i=1}^{N} f(\vec{x_i}). \tag{9}$$

The average scattering rate is thus given by

$$R_M(|\delta| > \delta_m) = \frac{V}{N}\sum_{k=1}^{M}\left[\frac{v^*}{\gamma^2}\frac{d\sigma^*}{d\Omega^*}\sin\Theta^*\rho(\vec{x_1})\rho(\vec{x_2})\right]_k, \tag{10}$$

where $N$ is the total number of simulated scattering events; M is the total number of scattered electrons with $|\delta| > \delta_m$ (one scattering event may generate one or two electrons with $|\delta| > \delta_m$, and they are counted individually); and $V$ is the total volume in $(x, y, z, x'_1, y'_1, dp_1, x'_2, y'_2, dp_2, \Theta^*, \Psi^*)$ space from which the events are selected. When $M$ is large enough, the simulated scattering rate converges to the analytical scattering rate, see Figure 1. In `elegant`, we use $5 \cdot 10^6$ as the default value of $M$.

Figure 2 shows the excellent agreement in the local scattering rates calculated from Piwinski's formula [5] and our Monte Carlo simulation. The lattice used here is the turn-around arc (TAA) cell of the APS-ERL lattice [9].

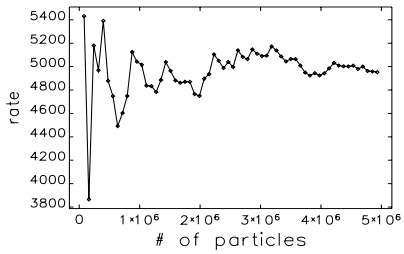



Figure 1: Simulated scattering rate vs. number of simulated scattered electrons.

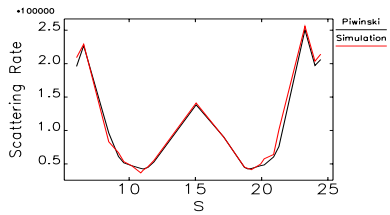



Figure 2: Local Touschek scattering rate: Piwinski formula (black) and Monte Carlo simulation (red).

## BEAM LOSS CALCULATION

To simulate beam loss due to Touschek scattering, we combined Piwinski's formula and the Monte Carlo method in `elegant`. First, the entire beamline is divided into $N_s$ short segments by inserting `TSCATTER` elements as separators. For each segment, we use Piwinski's formula to calculate the total scattering rate (integrated scattering rate $\int R_{Piwinski,i}$) for that segment. The scattered electron distribution at each `TSCATTER` element is generated from the Monte Carlo simulation. Each simulated scattered electron represents many electrons with a local scattering rate $r_i$. Let the total rate each represents for that *segment* be $R_i$, given by

$$R_i = \frac{r_i}{\sum r_i,} \times \int R_{Piwinski}. \tag{11}$$

The beam loss rate and location can then be calculated through tracking those scattered electrons through the beamline and recording all lost electrons, with $R_i$ providing a weight for each simulated electron's contribution to the total loss rate.

As we pointed out in the previous section, to obtain a reliable scattering rate and a smooth scattered electron distribution, $M$ has to be large enough ($\sim 5 \cdot 10^6$). We also require a large number of segments, $N_s$. That means that we need to track $M \times N_s$ electrons, which requires significant CPU time. This motivated a search for an alternative strategy to reduce the simulation burden. We examined the scattering rate that each simulated electron represents and, not surprisingly, found a large variation. Some simulated electrons represent very likely scattering events, while

some represent very low probability events. We sorted all simulated electrons by the associated scattering rate. Figure 3 illustrates the sum of the scattering rate ($\sum r_i$) vs. the number of simulated electrons ($\sum_i$).

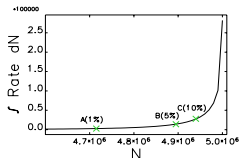



Figure 3: Integrated scattering rate vs. number of simulated electrons. Electrons are sorted with increasing associated scattering rate.

From this plot we can see that about $5\%$ of simulated electrons represent about $99\%$ of the scattering rate, which means that we may track a small portion of simulated electrons and get a fairly good beam loss information. Figure 4 compares the computed loss rate for tracking scattered electrons with $95\%$, $99\%$, and $100\%$ of the total scattering rate, respectively. It's clear that the differences are small and that this strategy can greatly speed up simulation with little sacrifice of accuracy. In `elegant` we use 99% as a default value.

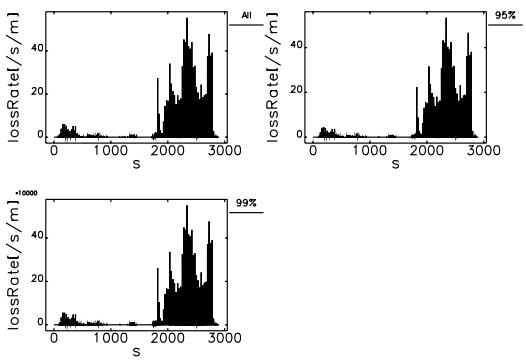
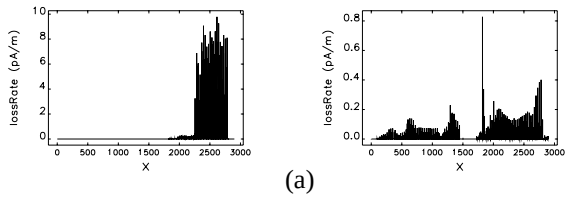



Figure 4: Simulated loss rate vs. position for various values of the scattering rate cut-off.

## APPLICATION TO APS-ERL DESIGN

APS has an eye on a future ERL upgrade, but we are concerned about beam loss issues since the APS tunnel is already built. The allowed beam loss in the APS ring is about 170 pA / 1100 m [10], i.e., 0.15 pA/m. We applied the Monte Carlo simulation to the APS-ERL lattice [9]. We assumed geometric rms emittances of 22 pm, 0.01% rms energy spread, 0.6 mm rms bunch length, 77 pC/bunch, and a 1.3 GHz repetition rate (giving 100 mA average current). Figure 5 shows the beam loss rate from Touschek scattering without sextupole correction in the TAA section. This is well above what is tolerable. A sextupole correction along the TAA section was then added to the lattice by maximizing the minimum value of the local momentum aperture. Figure 5 shows that this significantly reduces the beam loss rate, so that the average loss rate (0.018 pA/m) is well below the desired value.




(a) (b)

Figure 5: Simulated loss rate vs. position for APS-ERL without sextupole correction in TAA (a) and with optimized sextupole correction (b).

## CONCLUSION

A Monte Carlo simulation of Touschek scattering for a single-pass system (linac or transport line) was implemented in `elegant`. We compared the simulated local scattering rate with the rate from Piwinski's formula and found good agreement that confirms our simulation is correct. We also developed a strategy to speed up the simulation by tracking only those simulated scattered particles that represent the bulk of the scattering events. The application to a proposed APS-ERL lattice shows that the Touschek scattering effect is serious in an intense high-brightness electron beam. An optimized sextupole correction must be employed to lower the beam loss to a safe value. This simulation can also provide information for beam collimator design and verify the collimator system's efficacy.

## REFERENCES

[1] M. Borland, "elegant: A Flexible SDDS-Compliant Code for Accelerator Simulation," Advanced Photon Source LS-287, September 2000.

[2] M. Tigner, Nuovo Cimento **37**, 1228 (1965).

[3] M. Borland *et al.*, NIM A 582, 54 (2007).

[4] A. Xiao and M. Borland, "Touschek effect calculation and its application to a transport line," Proc. of PAC2007, 3453 (2007).

[5] A. Piwinsiki, "The Touschek effect in strong focusing storage rings," DESY 98-179, Nov. 1998.

[6] W. T. Ford, A. K. Mann and T. Y. Ling, "Beam Polarization Effects in High Energy Electron-Positron Storage Rings," SLAC-158, 1972.

[7] S. Khan, "Simulation of the Touschek Effect for BESSY II: A Monte Carlo Appproach," Proc. of EPAC1994, 1192 (1994).

[8] S. Khan, private communication.

[9] M. Borland *et al.*, Proc. of AccApp'07, 196 (2007).

[10] "Advanced Photon Source Safety Assessment Document, APS-3.1.2.1.0 Rev. 3, July 2006.