STUDY OF IBS EFFECTS FOR HIGH-BRIGHTNESS LINAC BEAMS *

A. Xiao†, ANL, Argonne, IL 60439, USA

Abstract

Intrabeam scattering (IBS) may become an issue for linac-based fourth-generation light sources such as X-ray free-electron lasers and energy recovery linacs (ERLs), both of which use high-brightness electron beams with extremely small emittance and energy spread. Any degradation of this extremely high beam quality could significantly reduce the machine's performance. We present here a strategy first used in the code `elegant` [1] for simulating IBS effects for high-brightness linac beams. We also present an application to a possible ERL upgrade of the Advanced Photon Source.

INTRODUCTION

Particles in a beam exchange energy between transverse and longitudinal oscillations due to Coulomb scattering. Depending on the scattering angles, the process leads to a diffusion in beam size (intrabeam scattering or IBS) or beam loss (Touschek effect).

The theory of IBS is discussed in several publications [2, 3]. A number of codes (e.g., ZAP [4], MAD-X [5]) have been developed for calculating the beam size growth rates. In the past, particle densities were not very high, so the growth times were much longer than the time spent traversing a typical linac. Thus, codes were designed for the stored beam case only with a constant beam energy. Linac-based fourth-generation light sources, such as X-ray free-electron lasers and Energy Recovery Linacs (ERLs [6]), require a high-brightness electron beam with extremely small emittance and energy spread. Any degradation of the beam quality could significantly reduce the machine's performance. Since the IBS growth time becomes much shorter for a high-brightness beam, IBS effects may become an issue even for a linac beam.

To investigate this issue, we modified the IBS calculation in `elegant` to include vertical dispersion effects and added the ability to handle acceleration. We applied our code to a proposed APS-ERL [7, 8] upgrade lattice also. Our results show that the IBS effects is moderate with the designed beam parameters.

CALCULATION OF IBS GROWTH RATES

A detailed formalism for intrabeam scattering, taking into account the variation of lattice functions with azimuth, has been developed by Bjorken and Mtingwa [3]. The expression of emittance growth rate τ_d in the direction d (x, y, or z) is given by (3.4) in [3] as:

$$\frac{1}{\tau_d} = \frac{\pi^2 c r_0^2 m^3 N ln\Lambda}{\gamma\Gamma} f,$$

$$f = \left\langle \int_0^\infty \frac{\lambda^{1/2} d\lambda}{\sqrt{|A|}} \left\{ TrL^d Tr[A^{-1}] - 3Tr[L^d(A)^{-1}] \right\} \right\rangle, \tag{1}$$

where c is the speed of light, r_0 is the classical particle radius, m is the particle mass, N is the number of particles per bunch (or in the beam for unbunched case), $ln\Lambda$ is a Coulomb logarithm, γ is the Lorentz factor, Γ is the 6-dimensional invariant phase-space volume of the beam,

$$\begin{aligned}
\Gamma_B &= (2\pi)^3(\beta\gamma)^3 m^3 \varepsilon_x \varepsilon_y \sigma_p \sigma_z \qquad (bunched) \\
\Gamma_U &= 4\pi^{5/2}(\beta\gamma)^3 m^3 \varepsilon_x \varepsilon_y \sigma_p (2\pi R) \quad (unbunched),
\end{aligned} \tag{2}$$

and $A = (L + \lambda I)$, with

$$\begin{aligned}
L &= L^x + L^y + L^z, \\
L^x &= \frac{\beta_x}{\varepsilon_x} \begin{pmatrix} 1 & -\gamma\phi_x & 0 \\ -\gamma\phi_x & \gamma^2(\frac{D_x^2}{\beta_x^2} + \phi_x^2) & 0 \\ 0 & 0 & 0 \end{pmatrix}, \\
L^y &= \frac{\beta_y}{\varepsilon_y} \begin{pmatrix} 0 & 0 & 0 \\ 0 & \gamma^2(\frac{D_y^2}{\beta_y^2} + \phi_y^2) & -\gamma\phi_y \\ 0 & -\gamma\phi_y & 1 \end{pmatrix}, \\
L^z &= \frac{\gamma^2}{\sigma_p^2} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix}.
\end{aligned} \tag{3}$$

Here, $\phi_{x,y} = D'_{x,y} + \frac{\alpha_{x,y} D_{x,y}}{\beta_{x,y}}$; $\varepsilon_{x,y}$ and $\sigma_{p,z}$ are beam distribution related quantities; and $\beta_{x,y}, \alpha_{x,y}, D_{x,y}, D'_{x,y}$ are local optical functions.

Equation (3), which includes vertical dispersion effects, is used in `elegant` for calculating the beam size growth rate. We found there are missing terms in MAD-X in the expressions for $a_{x,y}$ and $b_{x,y}$ used in formula (8) in [9], as confirmed by the developer [10]. The following equations show the differences between a_x in `elegant`,

$$\begin{aligned}
a_x &= 2\gamma^2 \left(\frac{H_x}{\varepsilon_x} + \frac{H_y}{\varepsilon_y} + \frac{1}{\sigma_p^2} \right) - \frac{2\beta_x}{\varepsilon_x} - \frac{\beta_y}{\varepsilon_y} \\
&\quad - \frac{\beta_x H_y}{H_z \varepsilon_y} + \frac{\beta_x}{H_x \gamma^2} \left(\frac{2\beta_x}{\varepsilon_x} - \frac{\beta_y}{\varepsilon_y} - \frac{\gamma^2}{\sigma_p^2} + \frac{6\beta_x}{\varepsilon_x}\gamma^2\phi_x^2 \right)
\end{aligned} \tag{4}$$

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† xiaoam@aps.anl.gov

and a_x in MAD-X,

$$\begin{aligned} a_x &= 2\gamma^2 \left(\frac{H_x}{\varepsilon_x} + \frac{H_y}{\varepsilon_y} + \frac{1}{\sigma_p^2} \right) \\ &- \frac{\beta_x H_y}{H_z \varepsilon_y} + \frac{\beta_x}{H_x \gamma^2} \left(\frac{2\beta_x}{\varepsilon_x} - \frac{\beta_y}{\varepsilon_y} - \frac{\gamma^2}{\sigma_p^2} \right) \end{aligned} \tag{5}$$

with

$$H_{x,y} = \frac{D_{x,y}^2 + \beta_{x,y}^2 \phi_{x,y}^2}{\beta_{x,y}}.$$

To further improve accuracy, we also moved the Coulomb logarithm $ln\Lambda$ into the average bracket in Equation (1), since it varies with location also.

To test the code, we simulated the emittance growth rate using the APS1nm [11] lattice and compared our results with ZAP and MAD-X. The beam parameters used in simulation are given in Table 1 (varied parameters are show in a bold face).

Table 1: Parameters used in Code Comparisons

Index	Energy GeV	N_b 10^{10}	coupling	σ_p 10^{-3}	σ_z mm
0	**3.0**	2.0	0.01	1.11	6
2	**5.0**	2.0	0.01	1.11	6
4	**7.0**	2.0	0.01	1.11	6
5	7.0	**2.0**	0.01	1.11	6
7	7.0	**6.0**	0.01	1.11	6
9	7.0	**10.0**	0.01	1.11	6
10	7.0	2.0	0.01	**1.11**	6
12	7.0	2.0	0.01	**3.0**	6
14	7.0	2.0	0.01	**5.0**	6
15	7.0	2.0	0.01	1.11	**2**
17	7.0	2.0	0.01	1.11	**6**
19	7.0	2.0	0.01	1.11	**10**
20	7.0	2.0	**0.01**	1.11	6
22	7.0	2.0	**0.03**	1.11	6
24	7.0	2.0	**0.05**	1.11	6

Figure 1 shows results from `elegant`, ZAP, and MAD-X. It shows that our results agree with the other two codes very well in a large range of parameters. The differences due to the missing terms are minor and negligible in our test run, as are the differences due to variation in $ln\Lambda$.

To verify the calculation with non-zero vertical dispersion, we performed the following test: first we set $\epsilon_x = \epsilon_y = \epsilon_0$ and computed the horizontal growth rate; then we exchanged the horizontal and vertical plane optical functions and computed the vertical growth rate. The resulting vertical growth rate should be the same as the original horizontal growth rate. Figure 2 shows simulation results before and after exchange of the optical functions, demonstrating that our vertical dispersion implementation is correct.

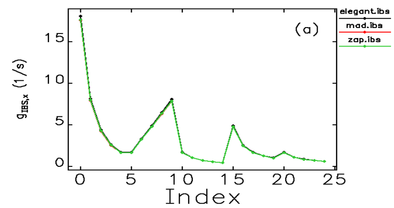


Figure 1: Comparison of calculated IBS growth rate from `elegant`, MAD-X, and ZAP. See Table 1 for the meaning of the Index.

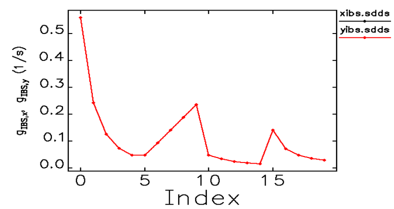


Figure 2: Demonstration that the vertical dispersion calculation is correct (xibs.sdds and yibs.sdds have their horizontal and vertical optical functions exchanged).

IBS GROWTH RATES FOR ACCELERATING BEAM

To calculate the IBS growth rates for a beam with acceleration is straightforward. First, we calculate the IBS growth rates locally with local beam energy and emittance, then integrate over the beam's path length s. Note that, since there are no synchrotron oscillations in a linac, the formula for unbunched beam should be used and results in a factor of 2 increase in the longitudinal growth rate [12]:

$$\frac{1}{\tau_z} [Unbunched] = 2 \frac{1}{\tau_z} [Bunched] \,. \tag{6}$$

The evolution of beam size is simulated through tracking using a specially designed element, `IBSCATTER`. First the entire beamline is divided into several segments by inserting `IBSCATTER` elements between selected existing elements. The normalized beam emittance is assumed constant within each segment, and the integrated IBS growth rates are calculated using the input beam parameters (for the first `IBSCATTER` element) or the parameters at the exit of the previous `IBSCATTER` element. The scattering action takes place at each `IBSCATTER` and the beam parameters are updated. This is done either by inflating the beam phase-space coordinates by factors that provide the equivalent size changes, or by adding appropriately scaled random values to the simulation particle coordinates. This is continued until the end of the beamline. Since the growth rates depend on beam size, the user needs to perform tests to determine how many `IBSCATTER` elements are needed. For example, in a linac the IBS growth rates are much higher in the low-energy region compared to the high-energy region.

Because the IBS growth rates are energy dependent, special caution is needed for calculations for accelerating beam. At the beginning of acceleration, the beam energy varies significantly. Figure 3 illustrates the energy change along the first sector of an APS-ERL design [7, 8]. We don't have an analytical method to calculate the IBS growth rate in this case. The integration is done using the local IBS rate at each element and the distance between elements. In such a case, the user needs to split their accelerating cavity into several pieces, so that γ has no large changes between elements. We split our rf cavities into 100 pieces. It's not necessary to put an `IBSCATTER` element after each element, since the effects of IBS on the beam size will only manifest themselves some distance downstream. The code will take care of beam size change vs. energy in computing the local IBS growth rate.

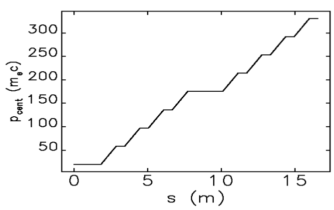


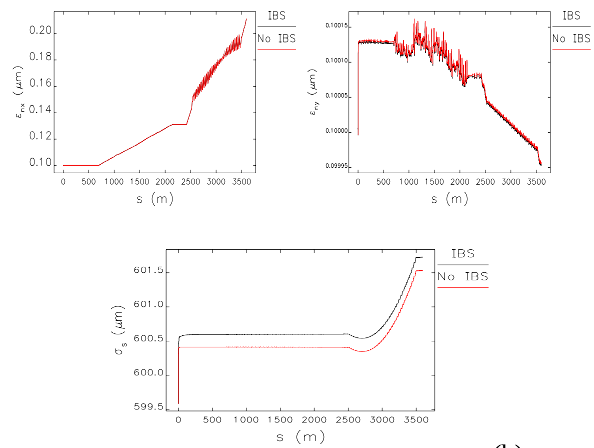
Figure 3: Energy vs. s at the first sector of APS-ERL.

EXAMPLE

In this section we present an example of an IBS calculation for an APS-ERL design. We assumed that the 10 MeV beam from the pre-injector has normalized emittances of 0.1 μm, 0.1% rms energy spread, 0.6 mm rms bunch length, and 19 pC/bunch. Figure 4 shows the beam size evolution as the beam passes through the beamline to the exit of the APS. We see a moderate but noticeable bunch lengthening due to IBS effects. But the normalized emittance dilution is minor compared to the emittance dilution from other effects, such as quantum excitation.

CONCLUSION

A more general IBS growth rate calculation including vertical dispersion effects and beam energy variation was added to `elegant`. The code was benchmarked with ZAP and MAD-X for vertical dispersion equal to zero and against itself by exchanging the horizontal and vertical plane parameters. Insertion of many scattering elements, `IBSCATTER`, allows simulation along a transport line or a linac. An application was made to a possible APS-ERL upgrade. Results show that the IBS effects are moderate with the proposed beam parameters.



(a)

(b)

Figure 4: Evolution of beam dimensions for a proposed APS-ERL lattice, with and without IBS. (a) Effective normalized emittance. (b) Bunch length.

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