# SRF CAVITY IMPERFECTION STUDIES USING ADVANCED SHAPE UNCERTAINTY QUANTIFICATION TOOLS*

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*Abstract*

The deviations of a SRF cavity from the design shape may results in significant impact on cavity performance and wakefields that could lead to unexpected effects in beam dynamics. Yet, most of these deviations are unknown in the final cavity installation because of the complicated process of assembly and tuning. It is desirable to be able to infer for such distortions using measurable RF quantities. With these data, the cavity performance can be analyzed and realistic tolerance criteria may be implemented in the cavity design and manufacture for quality assurance. To perform such analyses, SLAC has developed advanced Shape Determination Tools, under the SciDAC support for high performance computing, that recover the real cavity shape by solving an inverse problem. These tools have been successfully applied to analyze shape deviations of many SRF cavities, and identified the cause of unexpected cavity behavior. The capabilities and applications of these tools are presented.

## INTRODUCTION

The SRF cavities differ from original shape due to manufacture error and the tuning for the accelerating mode to achieve the right frequency and flat field. The deformation of the cavity leads to changes in higher-order mode frequencies, field distribution and wakefield damping, and may result in beam instabilities. It is important to understand the shape deviations of the real cavity from the design shape so that undesirable side effects due to shape deviation can be minimized in the manufacture of new cavities. Although direct measurements of the cavity shape are not feasible after the cavity is installed in the cryomodule and tuned, some of the RF quantities can be measured and can be used to evaluate the cavity shape deviations. SLAC has developed a set of shape uncertainty quantification tools based on SLAC's parallel finite element software that can recover the real cavity shape by solving an inverse problem using measurable RF quantities as input data.

We formulate the shape determination problem as a PDE constraint optimization problem, where the constraint is the Maxwell eigenvalue problem, the objective function is the weighted summation of the least squares difference of the modeled and measured RF quantities, and the inversion variables are the unknown shape deviations.

We present capabilities of the shape uncertainty quantification tools on two real applications: 1) the shape determination of the TDR cavity; and 2) identifying the cause of the BBU in one of the CEBAF 12-GeV upgrade prototype cavities.

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## METHODS

We pose the shape determination problem as a nonlinear least squares optimization problem

$$\begin{aligned} \underset{\mathbf{e}_j, k_j, \mathbf{d}}{\text{minimize}} \quad & \sum_i \alpha \left(f_i - \bar{f}_i\right)^2 + \sum_i \beta \left(Q_i - \bar{Q}_i\right)^2 \\ \text{subject to} \quad & \mathbf{K}\mathbf{e}_j + ik_j\mathbf{W}\mathbf{e}_j - k_j^2\mathbf{M}\mathbf{e}_j = \mathbf{0} \\ & \mathbf{e}_j^T\mathbf{M}\mathbf{e}_j = 1 \end{aligned}$$

where α and β are weighting constants, $\mathbf{e}_j$ eigenvector, and $k_j$ eigenvalue for the *j*th mode, $f_i$ are mode frequencies, and $Q_i$ are external Q value for the *i*th mode, and **d** the represents the unknown shape parameters.

To solve the optimization problem, we follow a gradient based approach. The resulted nonlinear problem is solved using Gauss-Newton method, in which the required eigenvalue and eigenvector sensitivities are computed with a discrete adjoint approach. Inverse shape determination problem is generally ill-posed and rank deficient. To remedy this we use regularization methods such as truncated singular value decomposition (T-SVD), and Tikhonov regularization. The nonlinear optimization algorithm typically converges within a handful of nonlinear iteration. Each nonlinear iteration requires solutions of the forward eigenvalue problem, the adjoint problems, and evaluations of inversion equations. The forward eigenvalue problem is solved using Omega3P, and adjoint problems are solved using direct solvers such as MUMPS.

## SHAPE DETERMINATION OF TDR CAVITY

The shape determination tool is used to infer for the unknown cavity deviations of a TDR cavity (Fig. 1). The following RF quantities are used as input in the objective function: 9 monopole frequency value, 36 dipole frequency values, and normalized field values measured at the center of each cell for the 9 monopole modes. These input data were obtained from the TFF-III measurements [1].

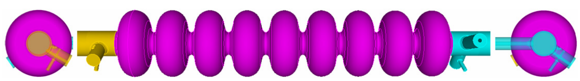

Figure 1: 3D CAD mode of TDR cavity.

The parametrization of each cell is shown in Fig. 2. There are five sections along each cell profile *(dt1, dt2, dt3, dt4, dr)* each of these are unknown shape deviations parameters. In addition, changes in cell length (*dz*) and changes in cell iris *(da)* are also parametrized [2]. In total, inversion variable has 62 unknown parameters.

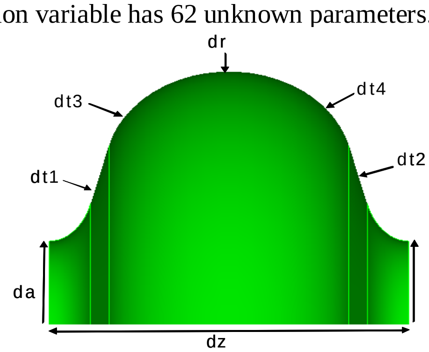


Figure 2: Definitions of shape parameters for a single cell.

The shape determination problem is solved using different regularization methods (Tikhonov regularization, and T-SVD), and different mesh discretization. In all of the inversion cases, the algorithm successfully reduced the error in the frequencies, and field values to the noise level. Figure 2 shows the frequency errors, which is the differences between calculated and measured, for the original design and the recovered cavity using the shape determination tools. The details of the analysis can be found in Ref. [3].

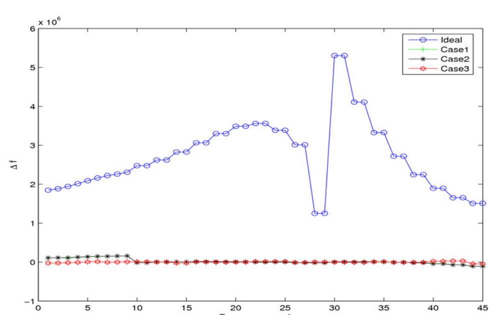


Figure 3: Frequency misfit for optimized and original cavities.

## JEFFERSON LAB HIGH GRADIENT PROTOTPYE CRYOMODULE

Beam based instability studies for JLab high gradient prototype cryomodule indicated that the beam breakup (BBU) threshold was well below the design value [4, 5]. Frequency spectrum peaked by the off-sided beam power shows that the cause is due to abnormal high Q modes in one of the cavities. The cause of this abnormality and future impact on the BBU couldn't be resolved due to limitations of information from the measurements. Omega3P [6] was used to compute $Q_{ext}$ values of these three HOM modes in an idea cavity shape, and they do not agree well with measurements of the real cavity. To understand the cause of this abnormality we used the shape determination tool developed as SLAC to solve for the unknown shape deviations.

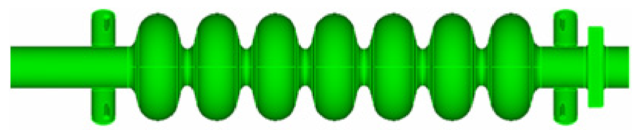

Figure 4: The 7-cell high gradient (HG) cavity model used for Omega3P simulations.

To descritize the unknown shape deviations we have used similar shape parametrization as in the TDR cavity shape determination problem. To recover the unknown shape, we used measured frequencies and $Q_{ext}$ values of the monopole modes and the first two dipole bands, which include the 7 monopole frequencies, 12 dipole frequencies, and 6 $Q_{ext}$ values.

Inversion algorithm successfully reduced the frequency differences between the recovered shape and the measurement of the real cavity to the noise level. Figure 5 shows the frequency error for the ideal and deformed cavities. Figure 6 shows the geometry difference of the deformed shape and ideal cavity. Although the inversion result is not unique due to ill-posedness and rank deficiency of the problem, inversion studies confirmed that the length of the deformed cavity is significantly shorter than the ideal cavity. The difference is found to be 8-mm. This result was confirmed by the cavity QC data.

Figure 7 shows the $Q_{ext}$ results of the deformed cavity. The deformed cavity not only reproduced high $Q_{ext}$ modes, it also reproduced the low $Q_{ext}$s for the modes around 2160 MHz which agree well with the measurements.

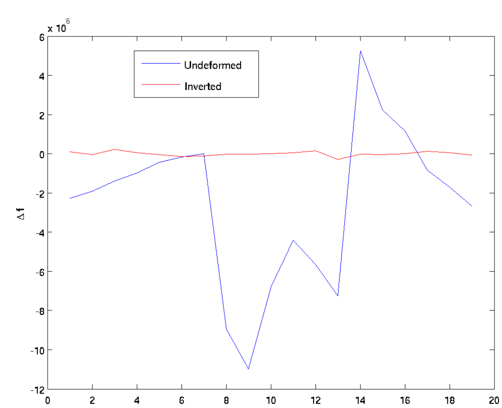


Figure 5: Frequency misfit for deformed and original cavities.

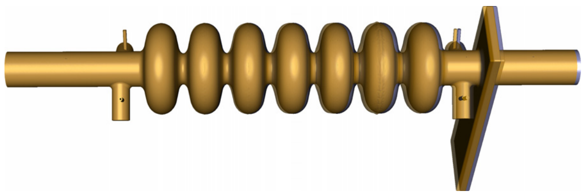

Figure 6: Deformed (goldenrod) and ideal (lightgrey) cavities.

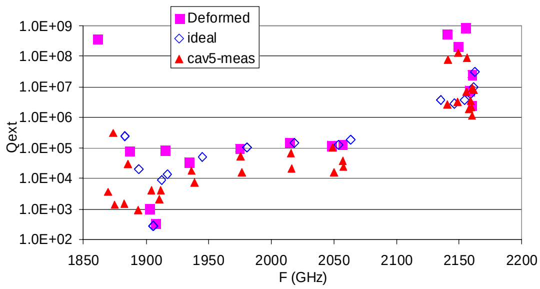


Figure 7: Qext for the ideal and deformed cavities.

The field distributions of the three abnormal modes in the deformed cavity are significantly different from that in the ideal cavity (Fig. 8). In the ideal cavity, fields are symmetrically distributed along the cavity, while in the deformed cavity they are heavily tilted toward the right side. The fields in HOM coupler region (left side) are more than one order of magnitude lower than the original cavity. This has resulted in ineffective damping. Because the fields of the other HOM modes were not altered as significantly by the deformation, no significant changes in the Qext of these modes were observed in the RF measurements.

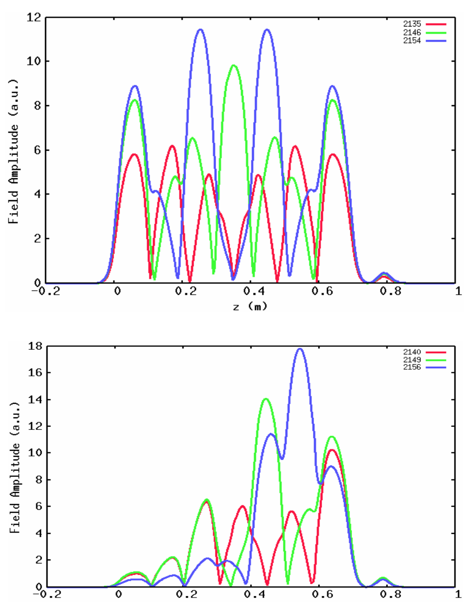


Figure 8: Comparison of electric field of the abnormal modes. Top: ideal design, bottom: deformed cavity with high $Q_{ext}$ modes.

## SUMMARY

Using the state of the art PDE constraint optimization techniques, and high performance computing tools, SLAC has developed shape uncertainty quantification tools to solve for the unknown shape deviations of the accelerator cavities using measurable RF quantities. We have used these tools to recover the unknown shape deviations of a TDR cavity from measured RF quantities. And these tools have been successfully used to determine the cause of the high abnormal $Q_{ext}$ values which caused unexpected BBU in one of the CEBAF 12 GeV upgrade cavities. It was found using the shape determination tool that the leading cause of the high Qext in that cavity was due to large errors in the cell lengths, which resulted in a total cavity length shortage of 8-mm. This result has been confirmed by the cavity QC data.

## REFERENCES

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